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Research Paper

POSITION ANALYSIS OF SPATIAL 3-RPS PARALLEL MANIPULATOR

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Parallel manipulators increase quality because of their large payload carrying capacity, high stiffness, high accuracy and good precision in positioning. Several parallel manipulators with 6-degree of freedom were developed, but 6-dof manipulators have complicated analysis and moreover distinct applications require less than 6-dof such as surgical manipulators, 3-dof sensors and flight simulations, etc. Therefore manipulators with less than 6-dof have been developed. They are called low-dof parallel manipulators like 3-dof, 4-dof or may be 5-dof. A intend position analysis of spatial 3-RPS parallel manipulator consists of three identical limbs, Base platform, mobile platform, revolute joints, prismatic joints and spherical joints. In the present task, it is intend to carryout, the position analysis of a spatial 3-RPS parallel manipulator. The position analysis of forward kinematic equations and inverse kinematic equations of 3-RPS spatial parallel manipulator have to be analyzed and simplified by using Sylvester dialytic elimination technique and vector analysis. All the possible solutions of this initial analysis are calculated by changing the limb lengths through linear actuators.

Keywords: 3-RPS spatial parallel manipulator, Position analysis, Identical limbs, 3-dof, Sylvester dialytic elimination technique, Vector analysis

INTRODUCTION

Spatial multi-degree of freedom mechanisms are known as parallel mechanisms. The motion characteristics of parallel mechanisms may be planar, Spherical [or] spatial type parallel manipulators. The kinematic structure of parallel manipulator is made up of a closedloop mechanism. In planar parallel mechanisms, type of each joint must be revolute [R] or prismatic[p] and mobility of each joint must be one. In planar parallel mechanisms, revolute joint[R] axes are perpendicular to the plane of motion and prismatic joint[P]axes lie on the plane of motion. In spherical parallel

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mechanisms, only revolute joints[R] are to be permissible and all the revolute joint axes must be inter-sect at a common point. In spatial parallel mechanisms, the motion of the parallel manipulator cannot be characterized as planar motion or spherical motion, means the position and orientation of mobile platform freely in 3-dimensional space. The parallel mechanisms are sometimes called platform manipulators because the external load can be shared by linear actuators or proportional actuators. The properties of parallel mechanisms are high stiffness, high accuracy, high speed , low inertia, large payload capacity and good precision in position. The applications of parallel mechanisms are such as airplane simulators, mining machines, walking machines adjustable articulated trusses, pointing devices, robot manipulator, micro manipulator, Tunnel borer, barbette of warship, satellite surveillance platform, parallel machine tools surgical manipulators, three degree of freedom sensors, etc. For general purpose Robot the parallel manipulators with 6-dof were developed for different applications. But 6-dof parallel manipulators such as Stewart-Gough platform type and Stewart universal type test machines (Gough and Whitehall, 1962; and Stewart, 1965) are suffer the difficult in kinematic analysis and complex in mechanical design. Moreover, several applications require less than 6-dof.Therefore, the parallel mechanisms with less than 6-dof (Lee and Shah, 1987; Han and Lung-Wen, 2003; Sameer and Lung-Wen, 2003; Liu and Shengchia, 2004; Yuefa and Lung-Wen, 2004; Yangmini and Qingsong, 2005; Mohan Rao and Mallikarjuna Rao, 2006; Yangmin and Qingsong, 2006; Yi *et al.*, 2008; and Yi *et al.*, 2009) have been developed. They are called low-dof parallel mechanisms. The low degree of freedom parallel mechanisms may be 5-dof, 4-dof [or]3-Dof.

There are 3-DoF spatial manipulators, called rotational parallel manipulators that allow the moving platform to rotate about a fixed point. Some manipulators, called translational parallel manipulators provide the moving platform with pure translational motion and may be more useful in the fields of automated assembly and machine tools as alternatives to serial manipulator systems. Some other manipulators allow the platform to rotate and translate. Lee and Shah (1988) developed a low-cost driving simulator using the 3-RPS manipulator. Kinematic and dynamic characteristics of the 3-RPS manipulator have been studied by Lee and Shah (1988), Song and Zhang (1995), Yang *et al.* (1996), Han and Lung-Wen (2003) and Yuefa and Lung-Wen (2004). Song and Zhang (1995) and Joshi and Tsai (2002) determined two singular positions of the 3-RPS manipulator from the Jacobian, determined by making use of the theory of reciprocal screws, while the other direct singular positions of the manipulator were determined by Joshi and Tsai (2002) and Liu and Cheng (2004). Kinematics of a high precision three degree of freedom manipulator were determined by Mohan Rao and Mallikarjuna Rao (2009).

The design of spatial parallel mechanisms have to be developed for different applications that require both rotational and translational motions of the mobile platform. The symmetrical 3-dof spatial parallel manipulator with 3-identical limbs are to be selected, and each limb connects fixed base to moving platform by revolute [R] joint, prismatic [P] joint

and spherical [S] joint in sequence. The moving platform has three degree of freedom that is two degrees of orientation freedom and one degree of translation freedom.

In the present work, the position equations of direct and inverse kinematic analysis of 3-PS spatial parallel manipulator have to be analyzed and simplified by using Sylvester dialytic elimination technique. The motion of the mobile platform is to be constrained by actuating the 3-actuators separately using electric drive technology. The motion of the 3-actuators must be controlled by micro processors and micro controllers.

GEOMETRY DESCRIPTION OF A SPATIAL 3-RPS PARALLEL MANIPULATOR

The Spatial 3-RPS parallel manipulator consists of a mobile platform [S], i.e., Figure 1 connected to a fixed base [B], i.e., Figure 2 by three identical supporting limbs with symmetrical kinematic structure. That means in the selected closed loop structure the number of limbs, number of joints, type of joints, number of actuated joints all are identical and equal to the mobility of the mobile platform. Each limb connects by means of a fixed base[B] to the mobile platform [S] by a revolute joint [R], a prismatic joint [P], a spherical joint [S] in sequence as in Figure 3, where the prismatic joint is activated by linear actuator. Thus the mobile platform connects to the base platform by means of three identical revolute, prismatic and spherical linkages (Figure 4).

In the fixed base platform of a selected spatial 3-RPS parallel manipulator the Cartesian co-ordinate reference frame $O\{x, y, z\}$ is considered at the centroid of the fixed triangular base platform $\Delta B_1, B_2, B_3$. Similarly the mobile co-ordinate reference frame $P\{u, v, w\}$ is considered at the centroid of the triangular moving platform $\Delta S_1, S_2, S_3$. For simplicity of geometry, let us consider X-axis

Figure 1: 3-DoF Spatial Parallel Manipulator

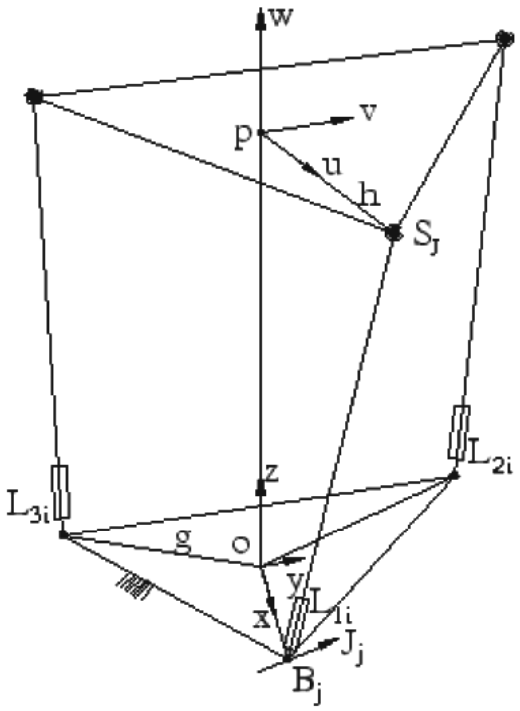


Figure 2: Geometry of Fixed Base Platform

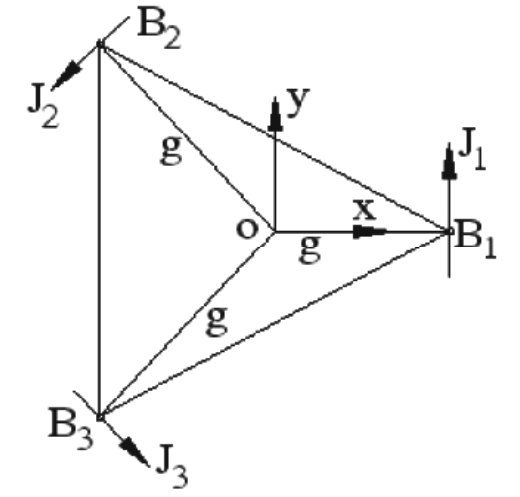


Figure 3: Position Geometry of Spatial Parallel Manipulator

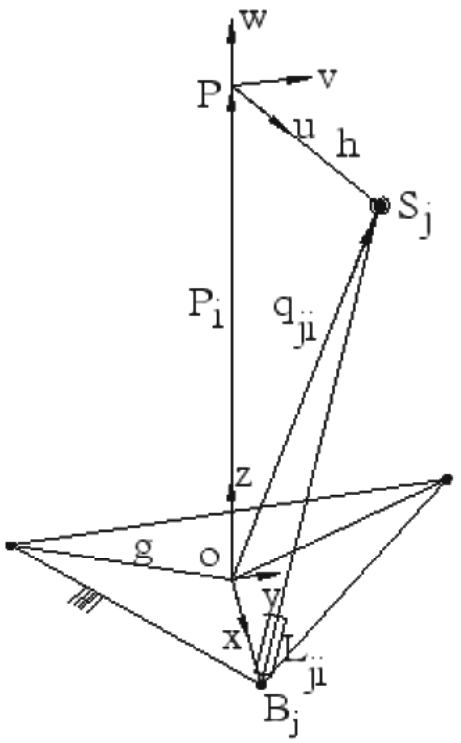
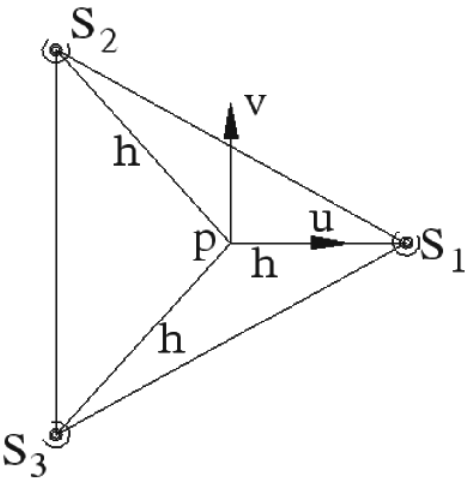


Figure 4: Geometry of Moving Platform



as in the direction of $\overrightarrow{OB_1}$ and the *u*-axis as in the direction of $\overrightarrow{PS_1}$. The distance between corresponding vertices of the fixed platform and mobile platform is denoted by $\overrightarrow{L_jL_i}$, that means after activation of each prismatic joints the distance between corresponding vertices of fixed and moving platforms are $\overrightarrow{L_jL_i}\ |_{j=1,2,3}$ and $i = 1, 2, 3, 4, \text{etc.}$, as in Fin(2).

MOBILITY OF A 3-RPS SPATIAL PARALLEL MANIPULATOR

The mobility of a spatial 3-RPS parallel manipulator are the number of independent translations and rotations of the mobile platform or the number of input parameters required to specify the configuration of a spatial parallel manipulator completely.

The mobility equation depends on number of links, number of joints and type of joints incorporated in the selected spatial parallel manipulator. The spatial parallel manipulator consists of 8-links, 3-revolute joints,3-prismatic joints and 3-spherical joints. The mobility of spatial parallel manipulator can be calculated by using Kutz-bach Criterion.

$$F = \lambda(n - j - 1) + \sum_i f_i$$

F: mobility of a spatial parallel manipulator

λ: Degrees of freedom of space in which a mechanism is intended to function.

λ = 6 for spatial mechanisms [degree of freedom of space with in which the manipulatoroperates]

n: Number of links in a mechanism, including fixed link

j: Number of joints in a mechanism

f_i: Degrees of relative motion permitted by joint *i*.

Therefore $F = 6\,(8 - 9 - 1) + (3 + 3 + 3 \times 3)$ $= -12 + 15 = 3$

POSITION ANALYSIS OF SPATIAL 3-RPS PARALLEL MANIPULATOR

The position vector of vertices $B_j|_{j=1,2,3}$ and $S_j|_{j=1,2,3}$ with respect to fixed base frame centroid $O\{x, y, z\}$ and mobile frame centroid $P\{u, v, w\}$ can be expressed as follows:

$$OB_1 = g_1 = [g \quad 0 \quad 0]^T$$

$$OB_2 = g_2 = \left[-\frac{1}{2}g \quad \frac{\sqrt{3}}{2}g \quad 0\right]^T \qquad ...(1)$$

$$OS_1 = h_1 = [h \quad 0 \quad 0]^T$$

$$OS_2 = h_2 = \left[-\frac{1}{2}h \quad \frac{\sqrt{3}}{2}h \quad 0\right]^T$$

$$OS_3 = h_3 = \left[-\frac{1}{2}h \quad -\frac{\sqrt{3}}{2}h \quad 0\right]^T \qquad ...(2)$$

Let $\vec{u}, \vec{v}, \vec{w}$ be represents three unit vectors along u, v and w-axis of the mobile platform reference coordinate frame $P\{u, v, w\}$. Then the rotation matrix can be expressed in terms of direction cosines of $\vec{u}, \vec{v}, \vec{w}$ are

$${}^oR_p = \begin{bmatrix} x.u & x.v & x.w \\ y.u & y.v & y.w \\ z.u & z.v & z.w \end{bmatrix} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix}$$

The Position vector of mobile frame vertices $S_j|_{j=1,2,3}$ with respect to fixed base frame centroid $O\{x, y, z\}$ can be expressed as:

$$\overrightarrow{OS_j} = \overrightarrow{OP} + \overrightarrow{PS_j}$$

That means $\overrightarrow{OP_i}|_{i=1,2,3,4,\text{etc.}}$, and $\overrightarrow{P_iS_j}|_{j=1,2,3}$ and $i = 1, 2, 3, 4$, etc.

Represent position vector and rotation matrix of a reference point $P_i|_{i=1,2,3,4,\text{etc.}}$, on a mobile platform respectively at the ith specified position and orientation with respect to the fixed reference frame $O\{x, y, z\}$.

i.e., $\overrightarrow{OS_{ji}} = \overrightarrow{OP_i} + \overrightarrow{P_iS_j}$

$$\overrightarrow{q_{ji}} = [P_{xi} \quad P_{yi} \quad P_{zi}]^T + {}^o R_{pi} \cdot h_j^{uvw} \qquad ...(3)$$

where $\overrightarrow{OP_i}$ is the transformation from the moving frame to the fixed frame can be described by position vector $\overrightarrow{OP_i} = [P_{xi} \quad P_{yi} \quad P_{zi}]^T$ and rotation matrix

$${}^oR_{pi} = \begin{bmatrix} u_{xi} & v_{xi} & w_{xi} \\ u_{yi} & v_{yi} & w_{yi} \\ u_{zi} & v_{zi} & w_{zi} \end{bmatrix}^T \qquad ...(4)$$

substituting Equation (2) and Equation (4) into Equation (3) then we can obtain position kinematic equations of mobile frame vertices with respect to fixed base frame centroid $O\{x, y, z\}$, i.e.,

$$\overrightarrow{q_{1i}} = \begin{bmatrix} P_{xi} + hu_{xi} \\ P_{yi} + hu_{yi} \\ P_{zi} + hu_{zi} \end{bmatrix}$$

Similarly

$$\overrightarrow{q_{2i}} = \begin{bmatrix} P_{xi} - \frac{1}{2}hu_{xi} + \frac{\sqrt{3}}{2}hv_{xi} \\ P_{yi} - \frac{1}{2}hu_{yi} + \frac{\sqrt{3}}{2}hv_{yi} \\ P_{zi} - \frac{1}{2}hu_{zi} + \frac{\sqrt{3}}{2}hv_{zi} \end{bmatrix}$$

$$\overrightarrow{q_{3i}} = \begin{bmatrix} P_{xi} - \frac{1}{2}hu_{xi} - \frac{\sqrt{3}}{2}hv_{xi} \\ P_{yi} - \frac{1}{2}hu_{yi} - \frac{\sqrt{3}}{2}hv_{yi} \\ P_{zi} - \frac{1}{2}hu_{zi} - \frac{\sqrt{3}}{2}hv_{zi} \end{bmatrix} \qquad ...(5)$$

REVOLUTE JOINT CONSTRAINTS

Due to revolute joints at $B_j \mid_{j = 1, 2, 3}$ the mechanical constraints are imposed by the revolute joints, so the spherical joints at $S_j \mid_{j=1, 2, 3}$ can only move in the planes defined by the i^{th} linear actuations along the plane of vertices B_jS_j, i.e., OB_jS_j. Therefore the constraints equations are:

$$[q_{1i}]_y = 0$$

$$[q_{2i}]_y = -\sqrt{3}[q_{2i}]x$$

Similarly

$$[q_{3i}]_y = \sqrt{3}[q_{3i}]x \qquad ...(6)$$

That means

$$P_{yi} + hu_{yi} = 0 \qquad ...(7.1)$$

$$P_{yi} - \frac{1}{2}hu_{yi} + \frac{\sqrt{3}}{2}hv_{yi} = -\sqrt{3}\left[P_{xi} - \frac{1}{2}hu_{xi} + \frac{\sqrt{3}}{2}hv_{xi}\right] \qquad ...(7.2)$$

$$P_{yi} - \frac{1}{2}hu_{yi} - \frac{\sqrt{3}}{2}hv_{yi} = \sqrt{3}\left[P_{xi} - \frac{1}{2}hu_{xi} - \frac{\sqrt{3}}{2}hv_{xi}\right] \qquad ...(7.3)$$

adding Equation (7.2) and Equation (7.3) then we have

$$3P_{yi} = -3hv_{xi} \qquad ...(7.4)$$

adding Equation (7.1) and Equation (7.4) then we have

$$\Rightarrow P_{yi} = -hv_{xi} \qquad ...(7.5)$$

Substituting Equation (7.5) into Equation (7.1) then we have

$$-hv_{xi} + hu_{yi} = 0$$

$$\Rightarrow u_{yi} = v_{xi} \ [\because h \neq 0] \qquad ...(7.6)$$

Subtracting Equation (7.3) from Equation (7.2) then we have

$$\sqrt{3}\,hv_{yi} = -\sqrt{3}\,[2P_{xi} - hu_{xi}]$$

$$\Rightarrow h(v_{yi} - u_{xi}) = -2P_{xi}$$

$$\Rightarrow P_{xi} = \frac{1}{2}h(u_{xi} - v_{yi}) \qquad ...(7.7)$$

REVOLUTE JOINT CONSTRAINTS FOLLOWED BY EULER ANGLE REPRESENTATION

The position analysis of a mobile platform with respect to fixed base platform centroid $O\{x, y, z\}$ can be described by direction cosine representation followed Euler angle representation.

Let us consider 3-Euler angles that are

$\emptyset_i$ = Rotating about Z-axis of fixed reference frame

θ_i = Rotating about Y-axis of fixed reference frame

Ψ_i = Rotating about X-axis of fixed reference frame

The 3-Euler angles rotating about Z-X-Y axis of the fixed reference frame in sequence then

^oR_p = Euler ($\emptyset_i$, Ψ_i, θ_i)

$^oR_p = Rot(y, \theta_i).\ Rot(x, \Psi_i).\ Rot(z, \emptyset_i)$

$$^oR_p = \begin{bmatrix} c\theta_i & 0 & s\theta_i \\ 0 & 1 & 0 \\ -s\theta_i & 0 & c\theta_i \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi_i & -s\psi_i \\ 0 & s\psi_i & c\psi_i \end{bmatrix}$$

$$\begin{bmatrix} c\phi_i & -s\phi_i & 0 \\ s\phi_i & c\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{o}R_p = \begin{bmatrix} c\theta_i\, c\phi_i + s\psi_i\, s\theta_i\, s\phi_i \\ c\psi_i\, s\phi_i \\ -s\theta_i\, c\phi_i + s\psi_i\, c\theta_i\, s\phi_i \end{bmatrix}$$

$${}^{o}R_p = \begin{bmatrix} c\theta_i\, s\phi_i + s\psi_i\, s\theta_i\, c\phi_i & s\theta_i\, c\psi_i \\ c\psi_i\, c\phi_i & -s\theta_i \\ s\theta_i\, s\phi_i + s\psi_i\, c\theta_i\, c\phi_i & c\theta_i\, c\psi_i \end{bmatrix} \quad ...(7.8)$$

where *s* and *c* stands for sine and cosine respectively. Then revolute joint constraints in terms of Euler angle representation from Equation (7.1) are:

$$P_{yi} - hc\psi_i\, s\phi_i \quad ...(7.9)$$

From Equation (7.6) are:

$$Tan\phi_i = \frac{s\psi_i\, s\theta_i}{c\psi_i + c\theta_i} \quad ...(7.10)$$

From Equation (7.7) and Equation (7.9) are

$$P_{xi} = \frac{1}{2}h\left(c\theta_i - c\psi_i\, c2\phi_i\right) x \frac{1}{c\phi_i} \quad ...(7.11)$$

Hence 3-constraints on the motion of the moving platform are:

$$\phi_i = Tan^{-1}\left(\frac{s\psi_i\, s\theta_i}{c\psi_i + c\theta_i}\right)$$

$$P_{xi} = \frac{1}{2}h\left(c\theta_i - c\psi_i\, c2\phi_i\right) x \frac{1}{c\phi_i}$$

$$P_{yi} = -hc\psi_i\, s\phi_i \quad ...(8)$$

FORWARD KINEMATIC ANALYSIS

Forward kinematic analysis are obtained by a set of actuated inputs given, then the position and orientation of the mobile platform are resolved by Forward kinematic analysis.

Let $L_{ji} \mid_{j = 1, 2, 3}$ are the vectors of the three actuated input joint variables and

$$\begin{bmatrix} P_{xi} & P_{yi} & P_{zi} & \psi_i & \theta_i & \phi_i \end{bmatrix}^T$$

are the vectors of constrained and unconstrained Cartesian variables which describe the position and orientation of mobileplatform.

Let $\emptyset_{ji} \mid_{j = 1, 2, 3 \text{ and } i = 1, 2, 3, 4, \text{etc.}}$, be the angle made by the i^{th} limb $B_jS_{ji} \mid_{j = 1, 2, 3 \text{ and } i = 1, 2, 3, 4, \text{etc.}}$ with respect to $OB_j \mid_{j = 1, 2, 3}$ then the position vector of spherical joint S_j with respect to fixed coordinate system can be expressed as:

$$\overrightarrow{q_{1i}} = \begin{bmatrix} g - L_{1i}c\phi_{1i} \\ 0 \\ L_{1i}s\phi_{1i} \end{bmatrix}$$

$$\overrightarrow{q_{2i}} = \begin{bmatrix} -\frac{1}{2}(g - L_{2i}c\phi_{2i}) \\ \frac{\sqrt{3}}{2}(g - L_{2i}c\phi_{2i}) \\ L_{2i}s\phi_{2i} \end{bmatrix}$$

$$\overrightarrow{q_{3i}} = \begin{bmatrix} -\frac{1}{2}(g - L_{3i}c\phi_{3i}) \\ -\frac{\sqrt{3}}{2}(g - L_{3i}c\phi_{3i}) \\ L_{3i}s\phi_{3i} \end{bmatrix} \quad ...(9)$$

The position and orientation of the limbs can be determined by the geometric distance between two spherical joints $S_j \mid_{j = 1, 2, 3}$ is constant, i.e., $\left\|\overrightarrow{S_1S_2}\right\|$ or $\left\|\overrightarrow{S_2S_3}\right\|$ or $\left\|\overrightarrow{S_3S_1}\right\| = \sqrt{3}\,h$

$$\Rightarrow \begin{matrix} \left\|\overrightarrow{q_{1i}} - \overrightarrow{q_{2i}}\right\| \\ \left\|\overrightarrow{q_{2i}} - \overrightarrow{q_{3i}}\right\| = \sqrt{3}\,h \\ \left\|\overrightarrow{q_{3i}} - \overrightarrow{q_{1i}}\right\| \end{matrix}$$

$$\Rightarrow L_{1i} L_{2i} c\emptyset_{1i} c\emptyset_{2i} - 2L_{1i} L_{2i} s\emptyset_{1i} s\emptyset_{2i} - 3g L_{1i} c\emptyset_{1i} - 3g L_{2i} c\emptyset_{2i} + (L_{1i}^2 + L_{2i}^2 + 3g^2 - 3h^2) = 0 \quad ...(10)$$

Similarly

$$\Rightarrow L_{2i} L_{3i} c\emptyset_{2i} c\emptyset_{3i} - 2L_{2i} L_{3i} s\emptyset_{2i} s\emptyset_{3i} - 3g L_{2i} c\emptyset_{2i} - 3g L_{3i} c\emptyset_{3i} + (L_{2i}^2 + L_{3i}^2 + 3g^2 - 3h^2) = 0 \quad ...(11)$$

$$\Rightarrow L_{1i} L_{3i} c\emptyset_{1i} c\emptyset_{3i} - 2L_{1i} L_{3i} s\emptyset_{1i} s\emptyset_{3i} - 3g L_{1i} c\emptyset_{1i} - 3g L_{3i} c\emptyset_{3i} + (L_{1i}^2 + L_{3i}^2 + 3g^2 - 3h^2) = 0 \quad ...(12)$$

The above equations are non linear equations and contain 3 unknowns $\{\emptyset_{1i}, \emptyset_{2i}, \emptyset_{3i}\}$

The equations can be solved by Sylvester dialytic elimination technique.

The Syivester dialytic elimination technique is applied to reduce the system of non-linear Equation (10) to polynomial in one variable that is

$$A_1 C\emptyset_{1i} C\emptyset_{2i} + B_1 S\emptyset_{1i} S\emptyset_{2i} + C_1 C\emptyset_{1i} + D_1 C\emptyset_{2i} + E_1 = 0 \quad ...(13)$$

Let $A_1 = L_{1i} L_{2i}$

$B_1 = -2L_{1i} L_{2i}$

$C_1 = -3g L_{1i}$

$D_1 = -3g L_{2i}$

$E_1 = L^2_{1i} + L^2_{2i} + 3g^2 - 3h^2$

Consider $t_{1i} = Tan\left(\frac{\phi_{1i}}{2}\right)$

Then $C\phi_{1i} = \frac{1-t_{1i}^2}{1+t_{2i}^2}$ and $S\phi_{1i} = \frac{2t_{1i}}{1+t_{1i}^2}$ Then

$$(F_1 t^2_{1i} + H_1) t^2_{2i} + (I_1 t_{1i}) t_{2i} + (G_1 t^2_{1i} + J_1) = 0 \quad ...(14)$$

where $F_1 = A_1 - C_1 - D_1 + E_1$

$G_1 = -A_1 - C_1 + D_1 + E_1$

$H_1 = -A_1 + C_1 - D_1 + E_1$

$I_1 = 4B_1$

$J_1 = A_1 + C_1 + D_1 + G_1$

Similarly non-linear Equations (11) and (12) can be written as in polynomial variables are

$$(F_2 t^2_{3i} + H_2) t^2_{2i} + (I_2 t_{3i}) t_{2i} + (G_2 t^2_{3i} + J_2) = 0 \quad ...(15)$$

and

$$(F_3 t^2_{1i} + H_3) t^2_{3i} + (I_3 t_{1i}) t_{3i} + (G_3 t^2_{1i} + J_3) = 0 \quad ...(16)$$

The above three Equations (14), (15) and (16) represent in three unknowns t_{1i}, t_{2i} and t_{3i}.

Elimination of t_{2i} then consider

$$K_1 t^2_{2i} + L_1 t_{2i} + M_1 = 0 \quad ...(17)$$

$$K_2 t^2_{2i} + L_2 t_{2i} + M_2 = 0 \quad ...(18)$$

where,

$K_1 = F_1 t^2_{1i} + H_1$

$L_1 = I_1 t_{1i}$

$M_1 = G_1 t^2_{1i} + J_1$

$K_2 = F_2 t^2_{3i} + H_2$

$L_2 = I_2 t_{3i}$

$M_2 = G_2 t^2_{3i} + J_2$

(17) x K_2 – (18) x K_1 then we have equation

$$(L_1 K_2 - L_2 K_1) t_{2i} + (M_1 K_2 - M_2 K_1) = 0 \quad ...(19)$$

Similarly (17) x M_2 – (18) x M_1 then we have equation

$$(K_1 M_2 - K_2 M_1) t_{2i} + (L_1 M_2 - L_2 M_1) = 0 \quad ...(20)$$

Equations (19) and (20) are two linear equations and one unknown vanishing their elimination then

$$(L_1 K_2 - L_2 K_1)(L_1 M_2 - L_2 M_1) + (M_1 K_2 - M_2 K_1)^2 = 0$$

Once t_{1i} is found, a unique value of t_{3i} can be calculated and then unique value of t_{2i} can be computed. That means the unknowns $\{\emptyset_{1i}, \emptyset_{2i}, \emptyset_{3i}\}$ can be calculated.

By substituting $\emptyset_{1i}$, $\emptyset_{2i}$, $\emptyset_{3i}$ values into $\overrightarrow{q_{1i}}, \overrightarrow{q_{2i}}, \overrightarrow{q_{3i}}$ can be calculated that means $\begin{bmatrix} P_{xi} & P_{yi} & P_{zi} & \psi_i & \theta_i & \phi_i \end{bmatrix}^T$ can be determined.

INVERSE KINEMATIC ANALYSIS

Inverse kinematic analysis are obtained by a location of mobile platform given, Then the limb lengths $L_{ji}\,|_{\,j=1,2,3 \text{ and } i=1,2,3,4, \text{etc.}}$, are resolved by inverse kinematic analysis. Let us consider the unconstrained variables θ_i, ψ_i, P_{zi} then the constrained variables P_{xi}, P_{yi}, ϕ_i can be determined from equations.

$$\phi_i = Tan^{-1}\left(\frac{s\psi_i\, s\theta_i}{c\psi_i + c\theta_i}\right)$$

$$P_{xi} = \frac{1}{2}h\left(c\theta_i - c\psi_i c2\phi_i\right)x\frac{1}{c\phi_i}$$

$$P_{yi} = -hc\psi_i\, s\phi_i$$

Then the limb lengths can be determined by geometry

$$\left\| q_{ji} - g \right\| = L_{ji}\,|_{j=1,2,3 \text{ and } i=1,2,3,4, \text{etc.}}$$

$$\begin{aligned}\Rightarrow d_{1i}^2 &= P_{xi}^2 + P_{yi}^2 + P_{zi}^2 + 2hP_{xi}\left(c\theta_i\, c\phi_i + s\psi_i\, s\theta_i\, s\phi_i\right)\\ &+ 2hP_{yi}c\psi_i\, s\phi_i + 2hP_{zi}\left(s\psi_i\, c\theta_i\, s\phi_i + s\theta_i\, c\phi_i\right)\\ &- 2gP_{xi} - 2gh\left(c\theta_i\, c\phi_i + s\psi_i\, s\theta_i\, s\phi_i\right) + h^2 + g^2\end{aligned} \quad ...(21)$$

Similarly

$$\begin{aligned}d_{2i}^2 &= P_{xi}^2 + P_{yi}^2 + P_{zi}^2 + hP_{xi}\left(c\theta_i\, c\phi_i + s\psi_i\, s\theta_i\, s\phi_i\right)\\ &- hP_{yi}c\psi_i\, s\phi_i - hP_{zi}\left(s\psi_i\, c\theta_i\, s\phi_i - s\theta_i\, c\phi_i\right)\\ &+ \sqrt{3}hP_{xi}\left(s\psi_i\, s\theta_i\, c\phi_i - c\theta_i\, s\phi_i\right) + \sqrt{3}hP_{yi}c\psi_i\, c\phi_i\\ &+ \sqrt{3}hP_{zi}\left(s\theta_i\, s\phi_i + s\psi_i\, c\theta_i\, c\phi_i\right) + gP_{xi}\\ &- \sqrt{3}gP_{yi} - \frac{1}{2}gh\left(c\theta_i\, c\phi_i + s\psi_i\, s\theta_i\, s\phi_i\right)\\ &+ \frac{\sqrt{3}}{2}ghc\psi_i\, s\phi_i + \frac{\sqrt{3}}{2}gh\left(s\psi_i\, s\theta_i\, c\phi_i - c\theta_i\, s\phi_i\right)\\ &- \frac{\sqrt{3}}{2}ghc\psi_i\, c\phi_i + h^2 + g^2\end{aligned} \quad ...(22)$$

$$\begin{aligned}d_{3i}^2 &= P_{xi}^2 + P_{yi}^2 + P_{zi}^2 - hP_{xi}\left(c\theta_i\, c\phi_i + s\psi_i\, s\theta_i\, s\phi_i\right)\\ &- hP_{yi}c\psi_i\, s\phi_i - hP_{zi}\left(s\psi_i\, c\theta_i\, s\phi_i - s\theta_i\, c\phi_i\right)\\ &+ \sqrt{3}hP_{xi}\left(c\theta_i\, s\phi_i - s\psi_i\, s\theta_i\, c\phi_i\right) - \sqrt{3}hP_{yi}c\psi_i\, c\phi_i\\ &- \sqrt{3}hP_{zi}\left(s\theta_i\, s\phi_i + s\psi_i\, c\theta_i\, c\phi_i\right) + gP_{xi}\\ &+ \sqrt{3}gP_{yi} - \frac{1}{2}gh\left(c\theta_i\, c\phi_i + s\psi_i\, s\theta_i\, s\phi_i\right)\\ &- \frac{\sqrt{3}}{2}ghc\psi_i\, s\phi_i + \frac{\sqrt{3}}{2}gh\left(c\theta_i\, s\phi_i - s\psi_i\, s\theta_i\, c\phi_i\right)\\ &- \frac{\sqrt{3}}{2}ghc\psi_i\, c\phi_i + h^2 + g^2\end{aligned} \quad ...(23)$$

By using above equations the relative limb lengths of the manipulator can be determined.

CONCLUSION

In this work the position analysis of spatial 3-RPS parallel manipulator are investigated in this paper. Firstly the mobility of manipulator is analyzed by Kutz-bach criterion. The physical constraints are introduced along the plane of spherical joints and actuated link lengths are considered. The forward kinematics problem is solved through Sylvester dialytic elimination technique. The inverse kinematics solutions are derived in closed form. All the possible solutions of this initial analysis are calculated by changing the limb lengths through linear actuators, and also the position and

orientation equations of the mobile platform with respect to fixed base platform are analyzed in terms of three limb lengths of spatial 3-RPS parallel manipulator.

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