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Research Paper

TOLERANCES ALLOCATION OF RGGR SLIDER CRANK MECHANISM USING GILBERT-MOORE ENCODING METHOD

S R Madan[1]*, Sushant Mahendru[2] and Anjali Mahendru[3]

Corresponding Author: **S R Madan, ✉ sunilraj.madan@rediffmail.com*

The output of a mechanism is always different from desired one due to the existence of different types of errors. These errors can be categorized as mechanical and structural errors. For an accurate synthesis, total error consisting of structural and mechanical errors, should be considered simultaneously. The tolerances in link lengths as well as clearances in joints introduce appreciable mechanical error. It is desired that total error (sum of structural and mechanical error) should be small as far as possible in desired range of path/function generation with desired probabilities. In this proposed work, an attempt has been made to allocate the tolerance in link lengths of a five bar RGGR spatial slider crank mechanism using weighted least square approach (Gilbert-Moore encoding procedure for distribution of probability) in such a way that total error may be controlled in a prescribed range.

Keywords: Mechanical error, Structural error, Total error Mechanism, Gilbert-moore probability distribution, Actual link length, Nominal link length

INTRODUCTION

The synthesis of mechanism is usually carried out either by precision point approach or by using optimization techniques. It is always required that that at precision points, the output of mechanisms should be as far as possible equal to or very close to desired output. However, there is always a deviation between actual and desired output. This deviation is known as error. The error is either structural error or mechanical error. Sutherland and Roth (1974) used an improved least square method for designing function generating mechanisms. Chen and Chen (1974) used Marquaredt's compromise for synthesizing function generating mechanism. Bhakthvachalm and

[1] Mech. Engg. Depdtt., M.I.T.M College, UJJAIN, Madhya Pradesh, India.
[2] Kuwait National Petroleum Company, MAB, Kuwait.
[3] Alghanim International, Kuwait.

Kimbrell (1974) have suggested the method of synthesizing the mechanisms for path generation by minimization of error function, with equality and inequality constraints. Giascient *et al.* (1979) used penalty function approach for synthesizing the mechanism for path generation and function generation. Baumgrtion and Fixmer (1979) used the probability theory consideration of effect of manufacturing tolerances on four bar path generation mechanism. It is often required to control the error (structural /total error) in a certain region of path/function generation with desired probability scheme. If accuracy greater than the accuracy required in a specified region of the operating range of the mechanism with the given probability scheme is required, then the weighted least square method seems to be tailor-made for the problems is possible. However, no rational procedure is available in the literature which enables the selection of most proper "weight" connected with desired probability. In this paper concept of Gilbert-Moore distribution of probability (Richard, 1998) is used to develop a weighted least square method for total error synthesis of a mechanism in which the error is required to be specifically controlled in the prescribed region of the range of path generation . The Gilbert-Moore encoding is a simple and is used widely in communication engineering for encoding the signals. The occurrence of event E(k), i.e., probability of occurrence of event E(k) can be defined in terms of some function. This function will give the measure of uncertainty.

The Gilbert-Moore encoding procedure can be conveniently applied to the synthesis of mechanisms with some modification. Assuming each error equation as an element, the probability of its contribution in the total error is determined. Example problem is also presented to elaborate the theory.

FORMULATION

According to Chen and Chen (1974), the displacement equation of mechanism can be written as:

$$b_2^2 + b_4^2 - b_3^2 + 2 \cdot b_2(b_1 \cdot \text{Sin}\Theta_i - b_5 \cdot \text{Cos}\Theta_i) - 2 \cdot b_1 \cdot b_4 \cdot \text{Sin}\varnothing_i - 2 \cdot b_2 \cdot b_4 \cdot \text{Sin}\Theta_i \cdot \text{Sin}\varnothing_i + b_1^2 - b_5^2 = 0 \quad ...(1)$$

The actual output $\varnothing_i$ of the linkages corresponding to input Θ_i at the i^{th} precision point may not correspond to the desired value $\varnothing_{di}$, but may differ slightly from it. Then the structural error ε_{si} at the i^{th} precision point can be written as:

$$\varepsilon_{si} = \varnothing_i - \varnothing_{di} \quad ...(2)$$

Considering small deviations (tolerances) Δb_1, Δb_2, Δb_3, Δb_4, Δb_1 and Δb_5 in nominal link dimension b_1, b_2, b_3, b_4 and b_5 the actual link lengths may be written as:

$$\overline{b_1} = b_1 + \Delta b_1$$

$$\overline{b_2} = b_2 + \Delta b_2$$

$$\overline{b_3} = b_3 + \Delta b_3$$

$$\overline{b_4} = b_4 + \Delta b_4 \text{ and}$$

$$\overline{b_5} = b_5 + \Delta b_5 \quad ...(3)$$

Owing to these link length deviations the output angle at the i^{th} point will deviate from $\varnothing_i$ by an amount $\Delta\varnothing_i$, which is termed as mechanical error. Therefore if b_1, b_2, b_3, b_4 and b_5 are replaced by $\overline{b_1}$, $\overline{b_2}$, $\overline{b_3}$, $\overline{b_4}$ & $\overline{b_5}$ respectively in Equation (1), then $\varnothing_i$ has to be replaced by $(\varnothing_i + \Delta\varnothing_i)$. Substituting expression for link lengths from Equation (3) in Equation (1) and simplifying by neglecting higher order terms we gets:

Mechanical error åmi at the ith precision point

$\Delta\varnothing_i = A_1(i)\,\Delta b_1 + A_2(i)\,\Delta b_2 + A_3(i)\,\Delta b_3 + A_4(i)\,\Delta b_4 + A_5(i)\,\Delta b_5$, where

$$A_1(i) = \frac{b_1 + b_2 \cdot \sin\varnothing_i - b_4 \cdot \varnothing_i}{A_6(i)}$$

$$A_2(i) = \frac{b_2 + b_1 \cdot \sin\varnothing_i - b_5 \cdot \cos\varnothing_i - b_4 \cdot \sin\Theta_i \cdot \sin\varnothing_i}{A_6(i)}$$

$$A_3(i) = -\frac{b_3}{A_6(i)}$$

$$A_4(i) = \frac{b_4 - b_1 \cdot \sin\varnothing_i - b_2 \cdot \sin\Theta_i \cdot \sin\varnothing_i}{A_6(i)}$$

$$A_5(i) = \frac{-b_5 - b_2 \cdot \cos\Theta_i}{A_6(i)}$$

and

$$A_6(i) = b_1 \bullet b_4 \bullet \cos\varnothing_i + b_2 \bullet b_4 \bullet \cos\varnothing_i \bullet \sin\Theta_i \quad ...(4)$$

Then the total error ε_i in the output at the i^{th} point is given by

$$\varepsilon_i = \varepsilon_{ti} = \varepsilon_{si} + \varepsilon_{mi} \quad ...(5)$$

APPLICATION TO MECHANISM SYNTHESIS

A path generating mechanism is required to generate the given displacement as closely as possible in the specified range. However, quite often but not always the closeness of the generated displacement with desired one is required in few specified regions of operating range, e.g., in timing devices, measuring instruments, Geneva mechanism, etc., because the output in their regions may only be of particular importance and more frequently 'measured'. A mechanism which generates the given path closely in specified region or otherwise may have a little larger overall error, may, therefore, be preferred. A large but finite number of precision points can be selected and desired output at these points. In order to apply Gilbert-Moore theory of distribution of probability scheme (P) may be associated with the errors at these precision points. Since the output is more frequently measured at these precision points lying in critical regions, the probability of transmission of error associated with these points should be low. So it is possible to associate a probability scheme to the generated equation given as:

$$w_1 = 1/2 \bullet x \bullet P(a_1)$$

$$w_2 = x \bullet P(a_1) + 1/2 \bullet x \bullet P(a_2) w_2$$

$$w_3 = x \bullet P(a_1) + x \bullet P(a_2) + 1/2 \bullet x \bullet P(a_3)$$

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$$w_{(n-1)} = x \bullet P(a_1) + x \bullet P(a_2) + x \bullet P(a_3) + ... + 1/2 \bullet x \bullet P(a_{n-1})$$

$$w(n) = x \bullet P(a_1) + x \bullet P(a_2) + x \bullet P(a_3) + ... + x \bullet P(a_{n-1}) + 1/2 \bullet x \bullet P(a_n) \quad ...(6)$$

where $P(a_1) = P(a_2) = P(a_3) = ...\, P(a_{n-1}) = P(a_n)$

= 1/number of selected design or precision points

= $1/n$ Also

$$w_1 + w_2 + w_3 + ... + w_{(n-1)} + w(n) = 1 \quad ...(7)$$

These equations provide the value of x hence weight at design/precision points.

The error ε_{ci} or ε_i with probability point can be obtained at i^{th} precision as

$$\varepsilon_{ci} = w_i \bullet \varepsilon_{ti} \qquad ...(8)$$

Where w_i is the weight must be related to probability P_i are associated with error ε_{ti}. The determination of tolerances Δb_1, Δb_2, Δb_3, Δb_4 and Δb_5 are done by minimizing the function of errors The function selected for minimizing is:

$$F = \sum_{i=1}^{n} \left(w_i \cdot \varepsilon_{ti} \right)^2 \qquad ...(9)$$

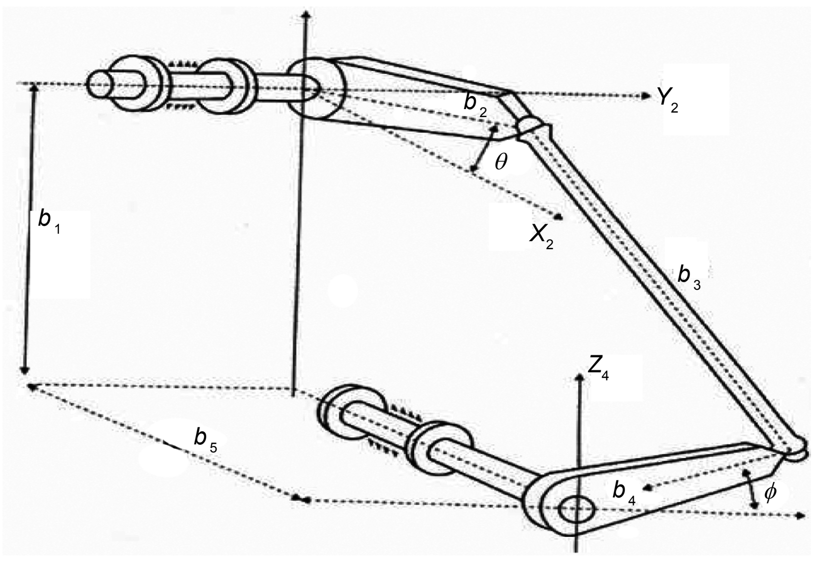
SYNTHESIS PROCEDURE

Structural error synthesis of the linkages is carried out separately using any of the available standard technique. This gives nominal link lengths of linkages. A finite number of precision points are selected and structural error ε_{si} on these points are calculated by using Equations (1) and (2). Coefficients A_1, A_2, A_3, A_4 and A_5 determined by Equation(4) and determined total error åti or åi on selected design (precision) points in terms of unknown link tolerances Δb_1, Δb_2, Δb_3, Δb_4 and Δb_5. The probability scheme (P) associated with transmission of error at various points may be calculated using Equations (6) and (7), which can be treated as weight on these precision points for formulation of the function F, and function F may be minimized by partially differentiating it with respect to different link deviation and equating each derivative to zero. These results in five linear simultaneous equations which may be solved by Gauss elimination method to determine unknown link length deviations (tolerances). Substitute the value of link length deviations in total error equation to get total error at each selected precision point.

Example

Design and allocate the tolerances to a spatial five bar RGGR mechanism whose revolute axes Z_1 and Z_4 are orthogonal but non intersecting to each other. As shown in Figure 1 in the mechanism is choosen to be unit length.

Figure 1: RGGR Slider Crank Mechanism



Solution

Assuming all initial link dimensions of unit to start with iteration converges to the solution.

$b_1 = 1.0000$, $b_2 = 1.1283$, $b_3 = 0.9515$, $b_4 = 0.8716$ and $b_5 = 0.9971$

(From F Y Chen and Venlin Chen, JET – 2.74)

Given or select six input angles in degree on six precision points are (Figure 2).

$\Theta_1 = 0.0°$, $\Theta_2 = 10.0°$, $\Theta_3 = 20.0°$, $\Theta_4 = 30.0°$, $\Theta_5 = 40.0°$, $\Theta_6 = 45.0°$

The desired out put angle related to input angles in degree are

$\varnothing_{d1} = 30.0°$, $\varnothing_{d2} = 38.50°$, $\varnothing_{d3} = 47.42°$, $\varnothing_{d4} = 57.57°$, $\varnothing_{d5} = 71.30°$, $\varnothing_{d6} = 85.95°$

The actual output in angles are

$\varnothing_1 = 30.001°$, $\varnothing_2 = 38.495°$, $\varnothing_3 = 47.427°$, $\varnothing_4 = 57.568°$, $\varnothing_5 = 71.289°$ and $\varnothing_6 = 85.9830$

Thus the corresponding structural error may be determined as:

$\varepsilon_{s1} = -0.001°$, $\varepsilon_{s2} = 0.005°$, $\varepsilon_{s3} = -0.007°$, $\varepsilon_{s4} = 0.002°$, $\varepsilon_{s5} = 0.011°$, $\varepsilon_{s5} = -0.033°$

Using Equations (3), (4) and (5) the total error comes out as follows,

$\varepsilon_{t1} = 42.799\Delta b_1 + 9.9694\Delta b_2 - 72.192\Delta b_3 + 28.189\Delta b_4 - 161.574\Delta_5 - 0.001$

$\varepsilon_{t2} = 45.893\Delta b_1 + 15.870\Delta b_2 - 66.806\Delta b_3 + 8.938\Delta b_4 - 147.82\Delta b_5 + 0.005$

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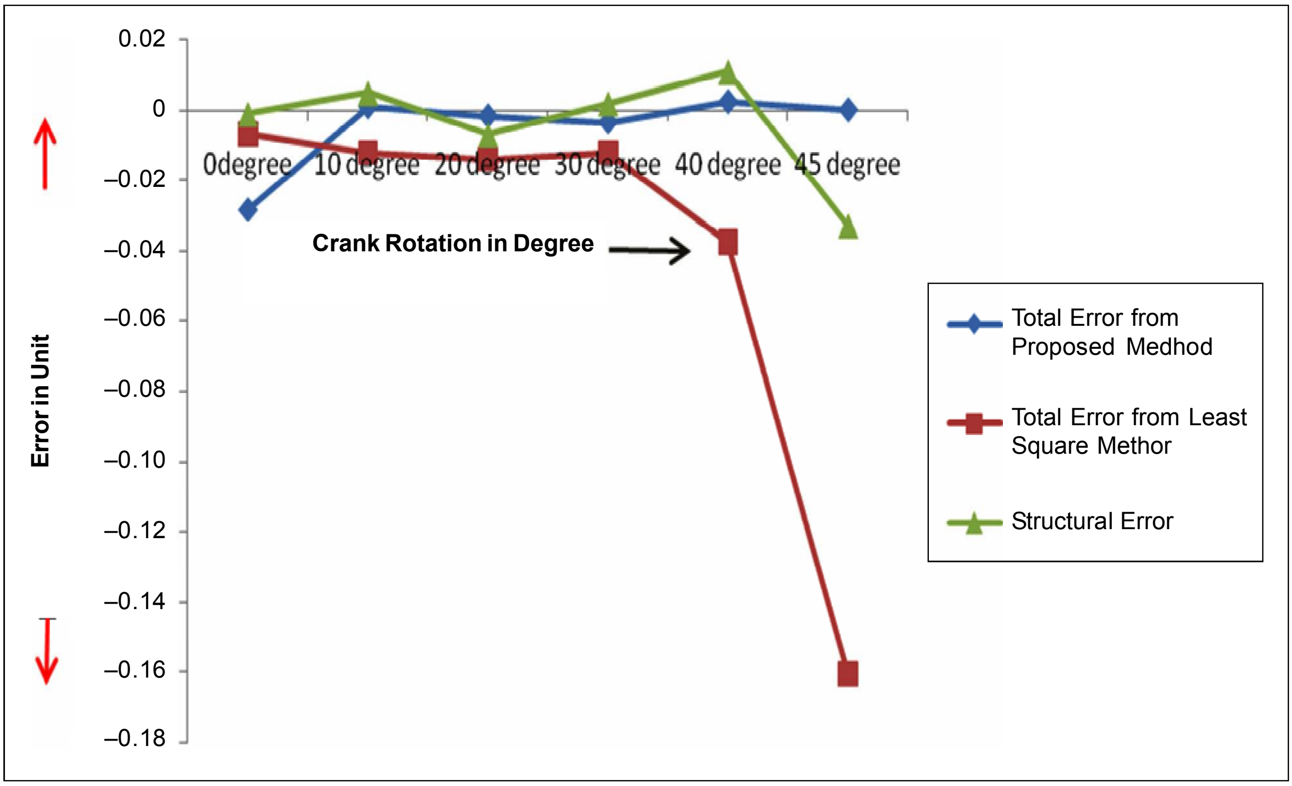
$\varepsilon_{t6} = 484.72\Delta b_1 + 269.29\Delta b_2 - 496.75\Delta b_3 - 481.28\Delta b_4 - 936.843\Delta_5 - 0.033$ ...(10)

The probability of occurrence of error at each point is same and is equal to 1/6.



By Gillbert Moore encoding procedure the weights at each point can be determined as:

$$w_1 = \frac{P_1 \cdot x}{2} = \frac{1 \cdot x}{6 \cdot 2} = \frac{x}{12}$$

$$w_2 = P_1 \cdot x + \frac{P_2 \cdot x}{2} = \frac{1 \cdot x}{6} + \frac{1 \cdot x}{6 \cdot 2} = \frac{3x}{12}$$

$$w_3 = P_1 \cdot x + P_2 \cdot x + \frac{P_3 \cdot x}{2} = \frac{1 \cdot x}{6} + \frac{1 \cdot x}{6} + \frac{1 \cdot x}{6 \cdot 2} = \frac{5x}{12}$$

--- = ----- ----- ----- ----- ----- ----- ---

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$$w_6 = P_1 \cdot x + P_2 \cdot x + P_3 \cdot x + P_4 \cdot x + P_5 \cdot x + \frac{P_6 \cdot x}{2} = \frac{11x}{12} \quad ...(11)$$

The sum of all weight (in respect of probability) should be equal to 1.

Hence $x/12 + 3x/12 + 5x/12 + 7x/12 + 9x/12 + 11x/12 = 1$

We get $x = 0.3333$ ...(12)

From Equations (9), (10), (11) and (12) we get function F and by partially differentiating this function F with respect to Δb_1, Δb_2, Δb_3, Δb_4 and Δb_5

We get

$\delta F/\delta\Delta b_1 = -45603.141\Delta b_1 + 32107.326\Delta b_2 - 46889.66\Delta b_3 - 44916.8277\Delta b_4 - 88785.191\Delta b_5 - 0.0495 = 0$

$\delta F/\delta\Delta b_2 = -32107.32\Delta b_1 + 14014.982\Delta b_2 - 25991.28\Delta b_3 - 24889.76\Delta b_4 - 49338.61\Delta b_5 - 0.27386 = 0$

$\delta F/\delta\Delta b_3 = -46889.66\Delta b_1 - 25991.28\Delta b_2 - 2849.088\Delta b_3 + 46026.072\Delta b_4 + 91302.51\Delta b5 - 0.0506 = 0$

$\delta F/\delta\Delta b_4 = -44916.827\Delta b_1 - 24899.76\Delta b_2 + 46026.07\Delta b_3 + 44339.412\Delta b_4 + 87108.66\Delta b_5 + 5.09874 = 0$

$\delta F/\delta\Delta b_5 = -88785.191\Delta b_1 - 49338.618\Delta b_2 + 91302.519\Delta b_3 + 9192.148\Delta b_4 + 17282.44\Delta b_5 + 0.09565 = 0$ ...(13)

Equation (13) is solved by Gauss elimination method, we get

$\Delta b_1 = -0.0275$ unit, $\Delta b_2 = 0.0392$ unit, $\Delta b_3 = -0.0062$ unit, $\Delta b_4 = 0.0032$ unit and $\Delta b_5 = -0.00134$ unit

By substituting the values of Δb_1, Δb_2, Δb_3, Δb_4 and Δb_5 in Equation (10) we get,

$\varepsilon_{t1} = -0.0283$ unit, $\varepsilon_{t2} = 0.0106$ unit, $\varepsilon_{t3} = -0.0015$ unit, $\varepsilon_{t4} = -0.0033$ unit, $\varepsilon_{t5} = 0.0025$ unit, $\varepsilon_{t6} = 0.000219$ unit

CONCLUSION

- It is observed from the result that weighted least square approach performs excellent for the spatial or non planer mechanisms.
- Consideration of mechanical error along with the structural error, it is observed that the maximum total error is than the maximum structural error.
- The total error at the last points is almost zero, which is advantage of weighted least square approach.
- If the number of points under consideration is increased then the mechanism is expected to perform better.

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