Trajectory Tracking Control for Differential-Drive Mobile Robot by a Variable Parameter PID Controller

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***Abstract*—Differential-drive mobile robots are most commonly used in industrial applications among wheeled mobile robots. Therefore, this paper presents a method to design a variable parameter PID controller for a differential-drive mobile robot following NURBS trajectory with a desired time-varying velocity. First, the robot's nonlinear kinematic error model is established, from which linearized around the desired angular velocity to obtain a linear error equation. Then, the variable parameter PID controller is designed to control the robot to follow the NURBS trajectory with minor error under the condition of time-varying velocity. The controller coefficients are selected through simulation and experiment to achieve the minor kinematic error. A platform robot is designed and built to demonstrate the proposed controller. Simulation and experimental results are presented to illustrate the effectiveness of the proposed controller. Therefore, it is possible to apply this result to control mobile robots in industrial applications.**

***Index Terms*—Differential drive mobile robot, PID controller, kinematic error, NURBS curve, trajectory tracking**

I. Introduction

In recent years, Wheeled Mobile Robots (WMR) have played an increasingly crucial role in modern industries because of their efficiency, flexibility, and highly mobilisable [1]-[5]. Meanwhile, the Differential-Drive Mobile Robot (DDMR) is a popular choice due to its simplicity of construction and ease of control [6], [7]. The problem of control of DDMR has attracted a lot of research interest in robotics [8]-[14]. The main problem of DDMR is to control its quick movement precisely following the desired trajectory [15]. Although advanced controllers have shown great potential, PID is still the favourite in industrial robot control applications, especially in the case of trajectory tracking control problems for the robot [9], [10], [15]-[18]. Some researchers developed the PID controller for DDMR, where the fuzzy PID controller has been proposed in [15] to control the robot when the system response has the error and the error rate. To track a moving target, a nonlinear PID controller is designed in [19] to control a DDMR where the controller parameters are tuned by a trial and error technique and the obtained controller whose K_P, K_I, and K_D parameters are constant. However, when the trajectory is complex and the desired velocity varies, the time-invariant parameter PID controller does not efficiently control the system because the accuracy is not high [5], [9]. A DDMR cannot be asymptotically stabilized to an arbitrary point by a smooth, time-invariant controller [20], [21]. To overcome the above problems, in this study, we design a PID controller whose parameters vary as a function of the tracking error which primarily ensures that the robot adheres to its predefined desired trajectory is complex with the velocity is continuously changing. Also, one problem regarding the robot's trajectory is that it is necessary to smooth the bends to avoid sudden changes in the robot's speed. Research in this area currently has the following methods: Dubins [20] using arcs to fillet the folded segments; using B-spline to path planning [22]; Path smoothing using Clothoids [23] or use Bézier curve continuous [24] etc. That is why in our work, the desired motion trajectory of the robot is designed by the NURBS curve [25], [26] to avoid sudden changes in the robot's speed, as shown presented above.

This paper is organized as follows. First, the robot's nonlinear kinematic error model was established, from which linearized it around the desired velocity to obtain a linear error equation. Then, a parameter-varying PID controller is designed to control the DDMR to track the NURBS trajectory with minor errors, while the desired velocity is time-varying. Finally, the designed controller was installed on a practical mobile platform for general NURBS trajectory tracking with minimal error, and the results were also verified by experimental.

II. Kinematic Error Model of Differential Drive Mobile Robot

A. Kinematic Model

Consider a DDMR moving along the trajectory ξ with pure rolling and without lateral slip are derived in the global coordinate system $\vartheta_f\{O_f x_f y_f z_f\}$ as shown in Fig. 1. The relationship linear and angular velocities of

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the robot to the angular velocities of the wheels are determined by [25, 27, 28]:

$$\begin{cases} V_G = \dfrac{v_2 + v_1}{2} = r\dfrac{(\omega_2 + \omega_1)}{2} \\ \dot{\theta} = \dfrac{v_2 - v_1}{2\ell} = r\dfrac{(\omega_2 - \omega_1)}{2\ell} \end{cases} \tag{1}$$

here r, 2ℓ is the radius of the wheel and the distance between the two wheels, while V_G and $\dot{\theta}$ are the linear and angular velocities of the DDMR with $v_1 = r\omega_1$ and $v_2 = r\omega_2$ are the velocities of the left and right wheel, respectively.

Thus, the kinematic model of the DDMR in $\vartheta_f\{O_f x_f y_f z_f\}$ is given by:

$$\dot{\mathbf{q}}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t)) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_G(t) \\ \dot{\theta}(t) \end{bmatrix} \tag{2}$$

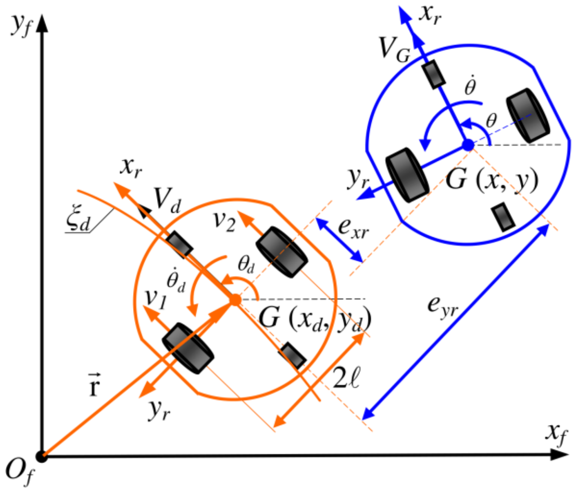

Figure 1. Illustration of the kinematic error relationship of the DDMR in the global coordinate system

B. Kinematic Error Model

The error model describes the variation in position and orientation of the DDMR when moving along the desired trajectory, defined by the error vector **e**:

$$\mathbf{e}(t) = \mathbf{q}_d(t) - \mathbf{q}(t) = \begin{bmatrix} e_x & e_y & e_\theta \end{bmatrix}^T \tag{3}$$

where $\mathrm{q}_d(t) = \begin{bmatrix} x_d(t) & y_d(t) & \theta_d(t) \end{bmatrix}^T$ is the desired trajectory of the DDMR in the global coordinate system $\vartheta_f\{O_f x_f y_f z_f\}$.

With $\vartheta_r\{Gx_r y_r\}$ is the coordinate system associated with the DDMR, transform $\mathbf{e}(t)$ from ϑ_f to ϑ_R of the DDMR:

$$\mathbf{e}_r = \begin{bmatrix} e_{xr} & e_{yr} & e_{\varphi r} \end{bmatrix}^T = \mathbf{R}^T\mathbf{e} \tag{4}$$

where $\mathbf{R}(\theta(t)) = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Derivative of equation (4) we have:

$$\dot{\mathbf{e}}_r = \begin{bmatrix} \dot{e}_x\cos\theta(t) + \dot{e}_y\sin\theta(t) + \dot{\theta}(t)e_{yr} \\ -\dot{e}_x\sin\theta(t) + \dot{e}_y\cos\theta(t) - \dot{\theta}(t)e_{xr} \\ \dot{\theta}_d(t) - \dot{\theta}(t) \end{bmatrix} \tag{5}$$

Combining equation (5) and equation (2, 3) we have:

$$\dot{\mathbf{e}}_r = \begin{bmatrix} \dot{e}_{xr} \\ \dot{e}_{yr} \\ \dot{e}_{\theta r} \end{bmatrix} = \begin{bmatrix} V_d(t)\cos e_{\varphi r} - V_G(t) + \dot{\theta}(t)e_{yr} \\ V_d(t)\sin e_{\varphi r} - \dot{\theta}(t)e_{xr} \\ \dot{\theta}_d(t) - \dot{\theta}(t) \end{bmatrix} \tag{6}$$

Rewrite equation (6) as a matrix form:

$$\begin{aligned} \dot{\mathbf{e}}_r &= \begin{bmatrix} 0 & \dot{\theta}(t) & 0 \\ -\dot{\theta}(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{xr} \\ e_{yr} \\ e_{\theta r} \end{bmatrix} + \\ &+ \begin{bmatrix} \cos e_{\theta r} & 0 \\ \sin e_{\theta r} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_d(t) \\ \dot{\theta}_d(t) \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V_G(t) \\ \dot{\theta}(t) \end{bmatrix} \end{aligned} \tag{7}$$

Equation (7) is a nonlinear error model of the DDMR. To apply the PID control law to the DDMR, we linearize the equation (7) along the reference trajectory $\dot{\theta}(t) \approx \dot{\theta}_d(t)$, one finally obtains:

$$\begin{cases} \cos e_{\theta r} \approx 1 \\ \sin e_{\theta r} \approx e_{\theta r} \end{cases} \tag{8}$$

Substituting equation (8) into equation (7) we have a linear error model of the DDMR:

$$\dot{\mathbf{e}}_r = \mathbf{A}\mathbf{e}_r + \mathbf{B}\mathbf{u} \tag{9}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & \dot{\theta}_d(t) & 0 \\ -\dot{\theta}_d(t) & 0 & \dot{\theta}_d(t) \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \mathbf{B} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{vmatrix} \quad ;$$

$\mathbf{u} = \begin{bmatrix} e_V & e_{\dot{\theta}} \end{bmatrix}^T = \begin{bmatrix} V_d(t) - V_G(t) & \dot{\theta}_d(t) - \dot{\theta}(t) \end{bmatrix}^T$ is the input vector.

According to [18, 27], the PID control law is selected:

$$\begin{cases} e_V = K_{Px}e_{xr} + K_{Ix}\displaystyle\int e_{xr}dt + K_{Dx}\dfrac{de_{xr}}{dt} \\ e_{\dot{\theta}} = K_{Py}e_{yr} + K_{Iy}\displaystyle\int e_{yr}dt + K_{Dy}\dfrac{de_{yr}}{dt} + B \end{cases} \tag{10}$$

with $B = K_{P\theta}e_{\theta r} + K_{I\theta}\displaystyle\int e_{\theta r}dt + K_{D\theta}\dfrac{de_{\theta r}}{dt}$

The above-built kinematic controller mathematical model Fig. 2 is the PID kinematics controller NURBS curve trajectory tracking of the DDMR. In which the transform block is defined by equation (4), PID controller block is the block of control laws equation (10), the kinematic block is the block of kinematic equation (2). Assume that the wheels and road surface are free of hysteresis; the robot has no longitudinal and transverse slip.

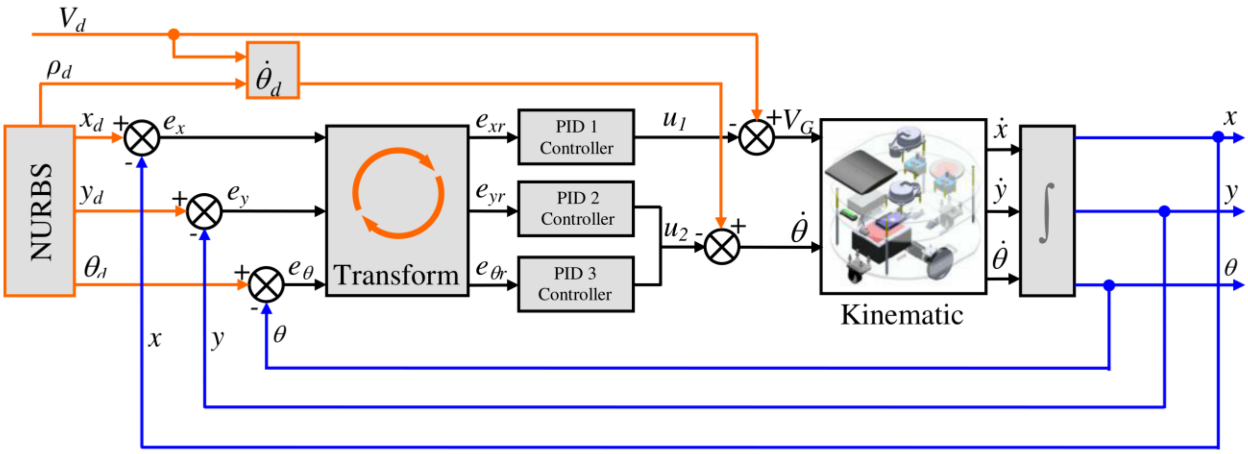

Figure 2. The control configuration that we consider use a variable parametric PID controller for the DDMR tracks the NURBS trajectory with minor error.

III. Setup System Parameters and Determine Controller Coefficients

A. Hardware Architecture

Fig. 3 is the DDMR platform designed and manufactured by this research conducted at the Department of Design of Machines and Robots, Hanoi University of Science and Technology. Shell material: is 5mm thick mica to ensure lightness and rigidity; Drive: Two differential drive wheels connected to integrated DC servo motor with integrated planetary gear reducer with gear ratio 1:14 and encoder 500CPR (2 channels A-B, 500 pulses), IMU (SEN 0386).

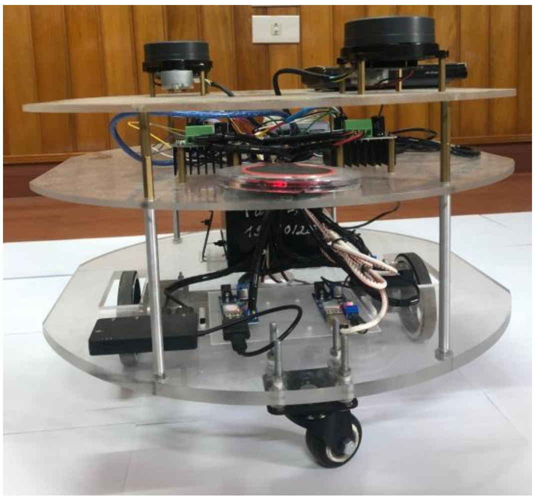

Figure 3. Differential-drive mobile robot prototype.

Hardware: The central processor is an embedded computer Raspberry Pi 4 model b, control module Arduino ATmega 2560 R3; Communication: connect between host and robot by module WIFI WHR-G301N; Robot control software is developed on ROS operating system.

Dimensional of the robot: Distance between the two wheels of the DDMR, $2\ell = 0.3$ (m) and the radius of the two wheels r = 0.0475 (m).

B. Design of Moving Trajectory of Robots by the NURBS Curve

In this study, we use the NURBS curve to design the robot's motion trajectory [4], [11], [25], [26] with the data of the interpolated points synthetic in Table I and the desired motion trajectory of the DDMR is described in Fig. 4 a.

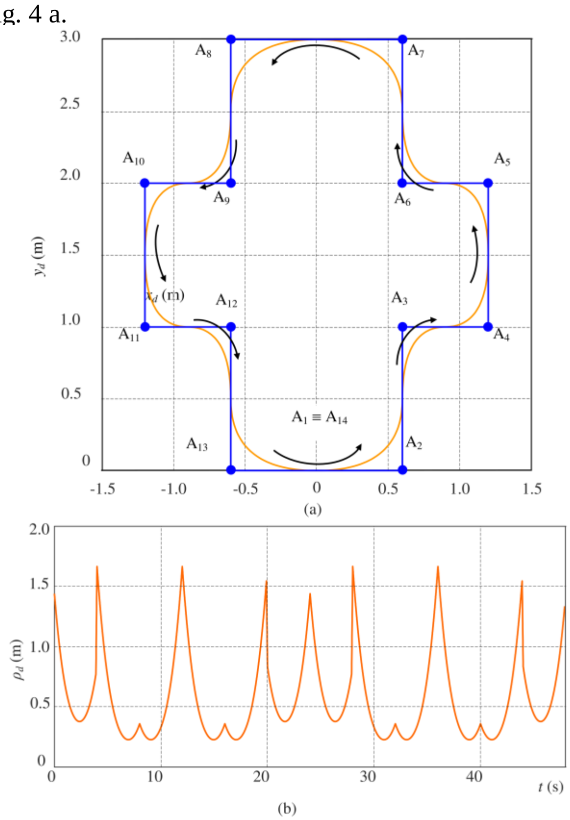

Figure 4. The robot's NURBS motion trajectories with (a) Desired move trajectory and (b) Radius of curvature of the motion trajectory.

TABLE I. INTERPOLATION POINT DATA OF THE ROBOT NURBS MOTION TRAJECTORY

A$_1$	***A***$_2$	***A***$_3$	***A***$_4$	***A***$_5$
$\begin{bmatrix}0\\0\end{bmatrix}$	$\begin{bmatrix}0.6\\0.0\end{bmatrix}$	$\begin{bmatrix}0.6\\1.0\end{bmatrix}$	$\begin{bmatrix}1.2\\1.0\end{bmatrix}$	$\begin{bmatrix}1.2\\2.0\end{bmatrix}$
A$_6$	***A***$_7$	***A***$_8$	***A***$_9$	***A***$_{10}$
$\begin{bmatrix}0.6\\2.0\end{bmatrix}$	$\begin{bmatrix}1.2\\3.0\end{bmatrix}$	$\begin{bmatrix}-0.6\\3.0\end{bmatrix}$	$\begin{bmatrix}-0.6\\2.0\end{bmatrix}$	$\begin{bmatrix}-1.2\\2.0\end{bmatrix}$
A$_{11}$	***A***$_{12}$	***A***$_{13}$	***A***$_{14}$	
$\begin{bmatrix}-1.2\\1.0\end{bmatrix}$	$\begin{bmatrix}-0.6\\1.0\end{bmatrix}$	$\begin{bmatrix}-0.6\\0.0\end{bmatrix}$	$\begin{bmatrix}-0.6\\0.0\end{bmatrix}$	

C. Determine Linear and Angular Velocities of the Robot

The desired velocity of the DDMR $V_d(t)$ is determined from the NURBS motion trajectory ξ in Fig. 4a through equation (11) bellows:

$$V_d(t) = \frac{\Delta S}{\Delta t} = \frac{\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}}{t_i - t_{i-1}} \tag{11}$$

and the desired angular velocity θ_d around the G point on the DDMR is determined by:

$$\theta_d(t) = \frac{V_d(t)}{\rho(t)} \tag{12}$$

Here $\rho_i \in [\rho_{\min}, \rho_{\max}]$ is the radii of ξ with $i = 1,2 \,..n$ and given by:

$$\rho_i = \left| \frac{\left(\dot{x}_i^2 + \dot{y}_i^2\right)^{3/2}}{\dot{x}_i \ddot{y}_i - \dot{y}_i \ddot{x}_i} \right| \tag{13}$$

Here $\begin{cases} \dot{x}_i = \dfrac{\Delta x}{\Delta t} = \dfrac{x_i - x_{i-1}}{t_i - t_{i-1}} \\ \dot{y}_i = \dfrac{\Delta y}{\Delta t} = \dfrac{y_i - y_{i-1}}{t_i - t_{i-1}} \end{cases}$ $\begin{cases} \ddot{x}_i = \dfrac{\Delta \dot{x}}{\Delta t} = \dfrac{\dot{x}_i - \dot{x}_{i-1}}{t_i - t_{i-1}} \\ \ddot{y}_i = \dfrac{\Delta \dot{y}}{\Delta t} = \dfrac{\dot{y}_i - \dot{y}_{i-1}}{t_i - t_{i-1}} \end{cases}$

From formula (13) and desired trajectory interpolation data, we have Fig. 4b is the radii of the desired NURBS trajectory tracking of the DDMR.

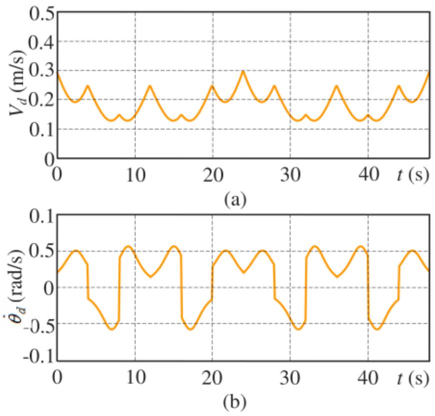

Figure 5. The desired linear and angular velocities as the DDMR moves on ξ.

From formulas (11, 12) and Fig. 4b, we have the desired linear velocity V_d and desired angular velocity θ_d of the DDMR when the robot moves to the NURBS trajectory tracking ξ with Δt = 1 (s) and the maximum allowable velocity $V_{d\max}$ = 0.3 (m/s). Fig. 5 shows the desired velocities for the DDMR.

D. Determine the Coefficients of the Controller

According to [16], [17], the PID parameters (K_P, K_I, K_D) depend on the tracking error e and can be modelled by this formula:

$$\begin{cases} K_P = a_1 + a_2 e \\ K_I = b_1 - b_2 e \\ K_D = c_1 + c_2 e \end{cases} \tag{14}$$

where $e = \sqrt{e_x^2 + e_y^2}$ is the tracking error of the DDMR when moving along the desired NURBS trajectory ξ. The parameters a_i, bi, ci (i = $1,2$) are real positive constant dependent on an error function.

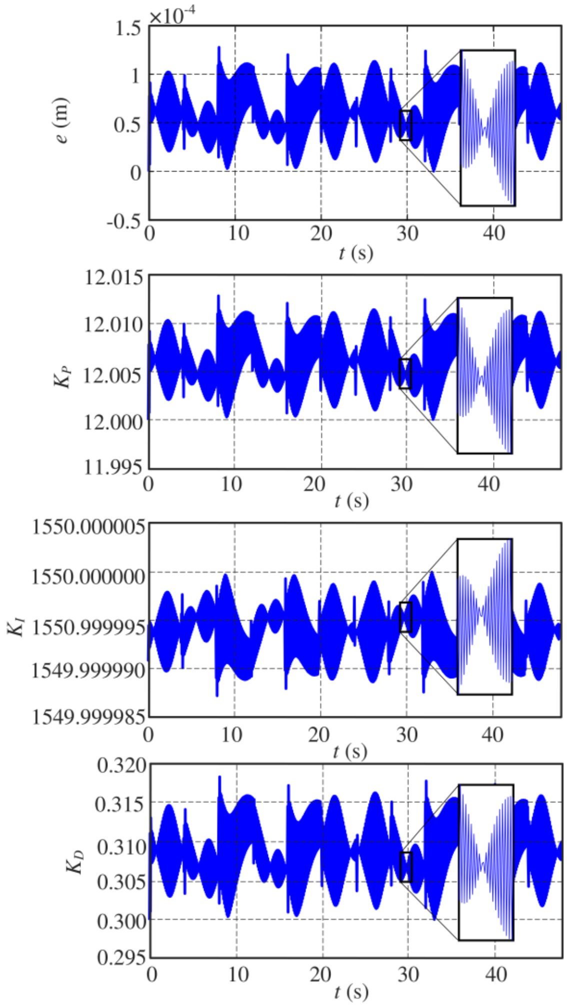

Figure 6. The change of parameters K_P, K_I, K_D according to the error $\boldsymbol{e}$.

The coefficients a_i, b_i, c_i (i = $1,2$) of the PID controller are determined by trial and error technique to minimize error when the controlled robot moves to follow the desired NURBS trajectory ξ. Table II below is the data of the selected controller coefficients after investigation. Fig. 6 depicts the change over time of the control parameters K_P, K_I, K_D with the coefficients a_i, b_i, c_i in Table II so that the NURBS trajectory tracking error is minimal.

TABLE II. THE COEFFICIENTS OF THE PID PARAMETERS

a_1	a_2	b_1	b_2	c_1	c_2
12	100	1550	0.1	0.3	144

IV. RESULTS AND DISCUSSION

With the setting parameters including: (1) The desired NURBS trajectory ξ, (2) The constants a_i, b_i and c_i of the parameters K_P, K_I, K_D of the PID controller, (3) Dimensions of the DDMR, (4) Hardware structure and the designed controller in Fig. 2. After executing the simulation in Simulink/Matlab, the actual motion trajectory of the robot by a PID controller with variable parameters is shown in Fig. 7 and Table III is the data error at the numbered positions on Fig. 7 (positions that change direction and change velocity as the robot follows the desired NURBS trajectory). In Table III, we can see that the maximal position error (e_{max}) of the G point on the robot error is less than 0.1114 (mm), while the maximum position error ($e_{\theta max}$) of the robot is less than 0.1163°. Fig. 8 depicts the position and orientation error of the robot during the robot tracks the NURBS trajectory. The points, numbered 0 through 11 in Fig 7, correspond to points 0 through 11 in Table III and Fig. 8.

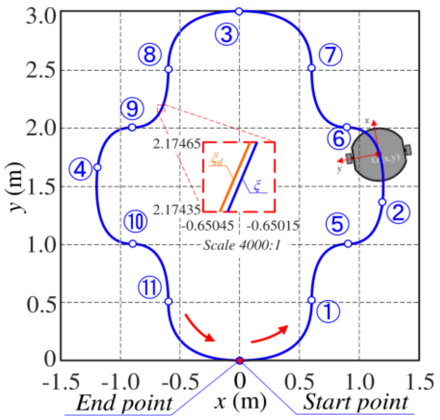

Figure 7. The movement trajectory of the DDMR.

TABLE III. POSITION AND POSTURE ERRORS AT THE NUMBERED POINTS IN FIGURE 6

Point	**0**	**1**	**2**	**3**	**4**	**5**
e_x (mm) e_y (mm)	-0.0493 -0.0203	0.0855 - 0.0007	- 0.1017 0.0115	0.0403 -0.0590	0.0993 -0.0112	-0.0025 0.1163
e_θ (°)	0.003	0.0380	- 0.006	-0.002	-0.0060	-0.1025
e (mm)	0.0533	0.0855	0.1023	0.0714	0.0999	0.1163
Point	**6**	**7**	**8**	**9**	**10**	**11**
e_x (mm) e_y (mm)	0.0024 0.1103	0.0150 0.0002	-0.0932 0.0007	0.0024 -0.1122	-0.0024 -0.1114	-0.0156 -0.0002
e_θ (°)	0.1023	- 0.0711	0.0381	-0.1023	0.1023	-0.0711
e (mm)	0.1103	0.0150	0.0932	0.1122	0.1114	0.0156

From Table III and Fig. 8, it can be found that: (i) The robot is controlled around the desired points from the inside to the outside of the desired trajectory; (ii) At the points marked in Fig. 7 are the inflexion points of the moving trajectory. Therefore, the DDMR will change direction to follow the desired trajectory, and these are the points with the maximal tracking errors. In the whole process of the tracking, the robot's direction continuously changes within the angular error of approximately 0.01°.

Fig. 9 compares the actual versus the desired values of linear and angular velocities, where the solid line (blue colour) provides the actual robot velocities. In contrast, the orange line shows the desired values of linear and angular velocities to robot follow the desired trajectory. It can see that the actual velocities of the robot changes in the step close to the desired point form to the robot can track the desired trajectory with a minor error. The angular velocity ω_1, ω_2 of the two wheels driven to produce the actual V_G and $\dot{\theta}$ of the robot are shown in Fig. 10.

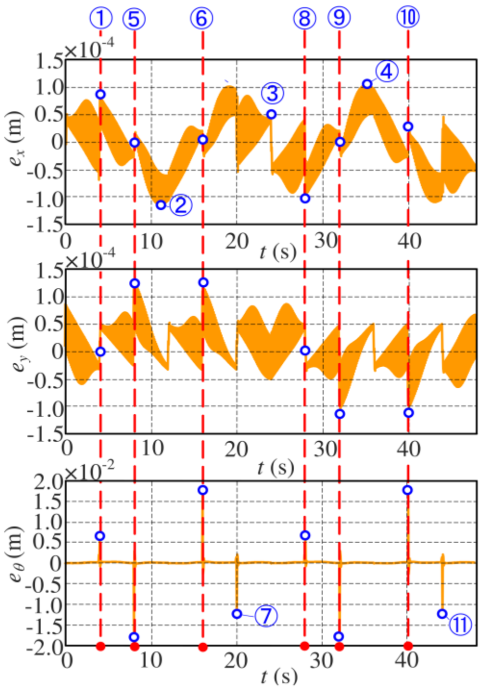

Figure 8. The position error of the G-point and the posture error of the DDMR.

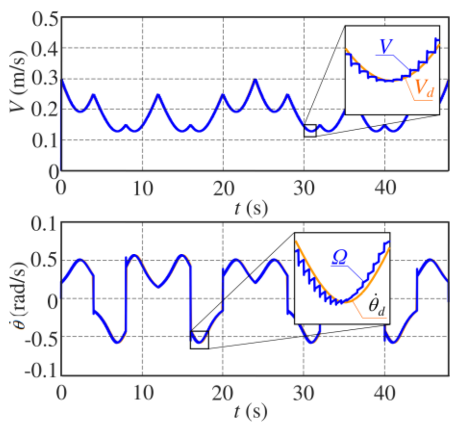

Figure 9. Linear and angular velocities of the DDMR.

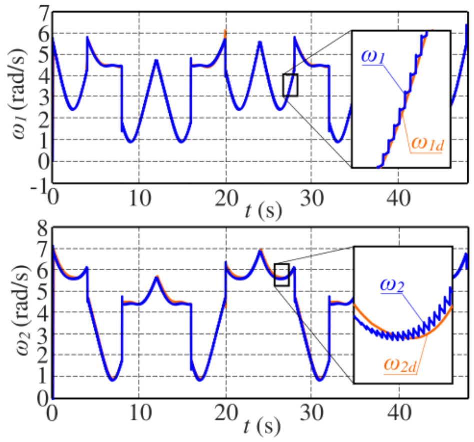

Figure 10. The actual angular velocity of the two wheels.

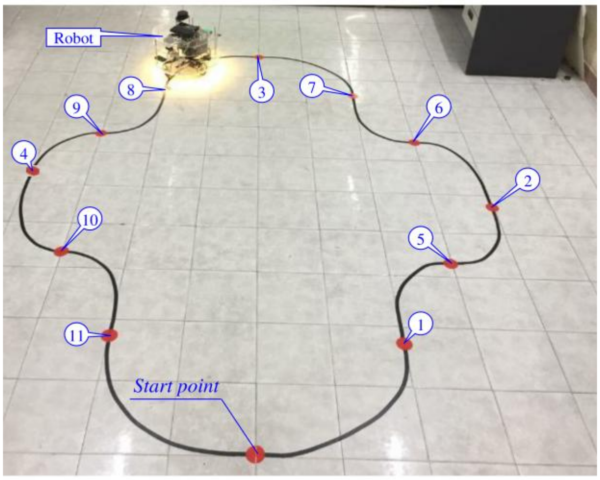

Figure 11. Experimental setup of the DDMR.

The tracking error between the simulation in Matlab versus the practical given in Table IV with the experimental setup of the DDMR depicted as shown in Fig. 11. The experimental design in Fig. 11 is made based on the calculated data according to Fig. 4. Practical measurement data position (x_G, y_G) and robot posture angle (θ) are determined through IMU and encoder.

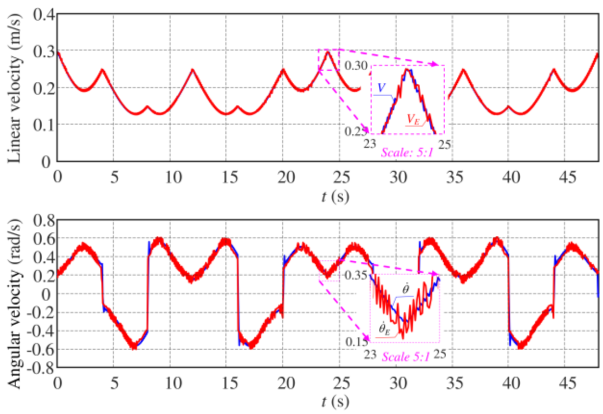

Figure 12. Linear and angular velocities of the DDMR between simulation and experiment.

Fig. 11 and Table IV show that the position error of the DDMR when moving in practical versus the simulation calculation ranges from 0 cm to approximately 1.2 cm, while the maximal posture error is about 5°. The cause of errors between experimental and simulation is the variation of linear and angular velocities of the DDMR at the inflexion points, as shown in Fig. 12. In Fig. 12, the blue line is the simulation result, while the red line is the measured value. The simulation and experiment errors of linear and angular robot velocities of are described in Fig. 13. In addition, the phenomenon of wheel slip versus the road surface hasn't been considered in this study also the causes of the position and posture error of DDMR in practice.

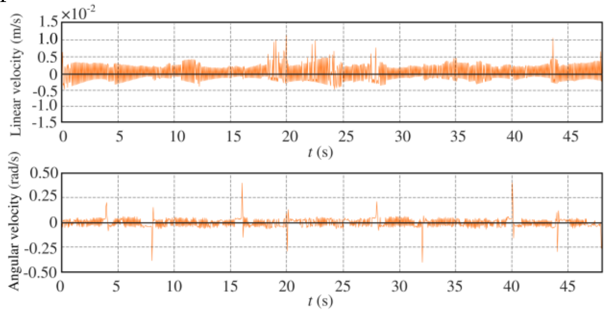

Figure 13. The errors of the linear and angular velocities of the DDMR between simulation and experiment.

The result shows that the controller PID with the time-varying parameter developed by this research successfully tracks the general NURBS trajectory with minor errors.

TABLE IV. POSITION AND POSTURE ERRORS AT THE NUMBERED POINTS IN FIGURE 8 AND 9

Point	**0**	**1**	**2**	**3**	**4**	**5**
e_x (mm) e_y (mm)	0.0000 0.0000	2.372 - 4.774	-5.923 -6.117	3.302 - 6.700	11.454 0.876	-2.558 4.621
e_θ (°)	0.0000	0.066	-0.035	-2.811	-1.218	-1.130
e (mm)	0.0000	5.178	8.515	7.469	11.487	10.616
Point	**6**	**7**	**8**	**9**	**10**	**11**
e_x (mm) e_y (mm)	9.578 - 4.714	6.643 - 9.286	3.761 5.514	5.077 2.982	5.988 9.022	-4.684 1.134
e_θ (°)	1.966	- 4.488	1.248	-0.984	4.809	- 4.252
e (mm)	10.675	11.417	6.675	10.318	10.8284	4.8194

In Table IV: $e_x = e_{Sx} - e_{Ex}$, $e_y = e_{Sy} - e_{Ey}$ and $e_\theta = e_{S\theta} - e_{E\theta}$ with subscripts S, E are simulated and experimental values, respectively.

V. CONCLUSION

From the above results of calculation, simulation, experiment and discussion, this study achieves the following points:

a) Time-varying PID controller based on a function of tracking error has been developed for DDMR tracks the NURBS trajectory which effectively improves performance with a minor error this is different from previous studies.

b) An investigation method has been proposed to determine the time-varying coefficients of the PID controller with a variable set value. The proposed controller design method can be done similarly for any trajectory with minimum control action and perfect orientation.
c) A series of comparative experiments are conducted to demonstrate the remarkable performance of the proposed method in terms of higher accuracy and efficiency.

Therefore, can apply the research results of this paper to control service robots or industrial autonomous robots such as AGVs.

In addition, the problems of dynamics, interaction between the robot and the environment will be considered as part of our future research goals.

CONFLICT OF INTEREST

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

AUTHOR CONTRIBUTIONS

Nguyen Hong Thai and Trinh Thi Khanh Ly made the initiative idea, theoretical modelling, implementation plan, and manuscript writing. Author Hoang Thien conducts a simulation, while author Le Quoc Dzung performs an experiment and consults with authors Nguyen Hong Thai on significant issues. All authors discussed the results, reviewed and approved the final version of the manuscript. Completed the manuscript was thanks to the contributions of all authors.

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