Analysis and Optimisation Strategy on Influence of Knot Vector to NURBS Curve

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***Abstract*—This study investigates the variation in the NURBS shape caused by knot vectors produced by various algorithms to minimise the influence of knot vectors on the NURBS interpolation. The differences in shape variations of NURBS curves computed by knot vectors generated by the Hartley–Judd, Riesenfeld and Rogers algorithms are compared and analysed; an error function is defined to analyse the mathematical features of the differences caused by the three calculations and a weighted average calculation method beyond the classical algorithm is proposed. Additionally, the study introduces a combinatorial algorithm for optimising the computing process and increasing computing efficiency. The experimental results show that the new method produces a more accurate NURBS curve than the classical method does, making it more suitable for CNC interpolation.**

***Index Terms*—CNC interpolation, NURBS, Knot vector, Error analysis**

I. Introduction

NURBS curve is a standard model for designing and calculating curves and surfaces in the technical fields of CAD, CAM, CNC and 3D printing [1]. The knot vector is an important parameter that affects the NURBS shape [2]. However, it can be seen from the literature that there is less research on the knot vector than on other factors affecting the shape, such as weight factor. Three knot calculation methods proposed by Hartley Judd [3], Riesenfeld [4] and Rogers [5] are examples of early research on knot vectors. Since then, almost all NURBS calculations have choose one of these three bases. Recently, there have been reports in the literature on knot vector research. For example, Fusheng Liang [6] investigated a B-spline fitting method for determining the compression error knot vector, Yuhua Zhang [7] investigated a coordinate descent algorithm for knot optimisation, Yohann Audoux and Marco Montemurro [8]-[10] structure optimization and modeling of NURBS curves and surfaces are studied in the literature, and good results are obtained. Others, such as references [11]-[12], investigate the B-spline or NURBS curves insertion knot algorithm. It is clear that these publications are all the result of research that was performed for a special demand, and the methods used are not suitable for CNC interpolation calculation.

CNC interpolation is the core calculation that determines the speed and accuracy of CNC processing. In essence, it is an ‘objective’ calculation, which means that it cannot design a curve such as CAD or plan a machining path such as CAM, but should accurately calculate tool position data planned by cam [13]. As a result, the difficulty of selecting a knot vector for interpolation calculation arises. According to the provisions of the step standard, a cam can output tool position data according to G06.2 as well as control points, weight factors and knot vector values for CNC NURBS interpolation calculation [14], and the type value points for interpolation calculation can also be specified by G01/G02/G03. In either way, NURBS interpolation requires the selection and calculation of the appropriate knot vector. This is especially true when the interpolation cam provides G01/G02/G03 command data. it is necessary to select and calculate the appropriate knot vector.

As mentioned earlier, the methods described in references [3]-[5] are now the most widely used methods for selecting knot vectors. However, there is a dearth of literature comparing the three methods. This study examines it in view of this. Comparing the curves calculated by the three knot vectors under the same control point shows that they are obviously different. This cannot be ignored when calculating CNC interpolation. As a result of the foregoing analysis, during CNC interpolation, the difference in knot vectors results in an interpolation difference, leading to the same interpolation original data being calculated into different target data, resulting in a compatibility problem between CNC systems. As a result, to reduce the possible error, this study uses the weighted average method to generate a new knot vector from the calculation results of three knot vectors.

The first section of this paper is the introduction; The second section introduces the definition of NURBS curve; The third section introduces the calculation method of traditional knot vector; The fourth section analyzes the error of traditional methods and proves that different node vectors will lead to different NURBS curves; In the fifth section, a compromise weighting algorithm is proposed, and the experiments of error analysis and curve

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fitting are carried out and the experiments show that the new method produces NURBS with good approximation characteristics.

II. NURBS Curve Definition

In this section, the fundamentals of the NURBS curves theory are briefly recalled. According to the notation introduced in [4], the parametric implicit form of a NURBS curve is:

$$C(u)=\frac{\sum_{i=0}^{n}\omega_i d_i N_{i,p}(u)}{\sum_{i=0}^{n}\omega_i N_{i,p}(u)},u\in[0,1] \tag{1}$$

which:

$$\begin{cases} N_{i,0}=\begin{cases}1,u_i\le u\le u_{i+1}\\ 0,otherwise\end{cases} \\ N_{i,p}(u)=\dfrac{u-u_i}{u_{i+p}-u_i}N_{i,p-1}(u)+\dfrac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}}N_{i+1,p-1}(u) \end{cases}$$

specify 0/0 = 0.

where p is the degree of the curve, a p-degree NURBS curve is expressed as order p + 1 and the spatial point $d_i(i=0,1,\cdots,n)$ is listed as the control point of the curve, which is the ith weight factor. In this study, the weight factor $\omega_i(i=0,1,\cdots,n)$ is selected by the method of literature[15]. The sub NURBS curve controlled by N + 1 control points requires a knot vector u composed of N + P + 2 knots, which is expressed as follows:

$$U=[u_0,u_1,\cdots,u_p,u_{p+1},\cdots,u_n,u_{n+1},\cdots,u_{n+p+1}],u_i\le u_{i+1}.$$

III. The Mainstream Algorithm of KNOT Vector Selection

In this section, three mainstream and widely used node vector selection algorithms are introduced: Hartley Judd, Riesenfeld and Rogers. As mentioned earlier, there are three widely used algorithms for selecting knot vectors: Hartley Judd, Riesenfeld and Rogers, which are introduced in this section.

A. Riesenfeld Method

According to Riesenfeld's method, the control polygon is approximately regarded as the circumscribed polygon of the spline curve, and the piecewise connection points of the curve correspond to the vertices or edges of the control polygon, which then straightens and normalises them to obtain the parameter sequence of the knot vector [16].

Make the length of each side of the control polygon as follows:

$$l_i=|d_i-d_{i-1}|,i=(1,2,\cdots,n)$$

The total side length obtained by adding the length of each side is:

$$L=\sum_{i=1}^{n}l_i$$

The specific knot vector is determined as follows:

(1) Even knot vector

Riesenfeld assumes that all piecewise connection points of even degree NURBS curves correspond to the midpoint of other edges on the control polygon except for P/2 edges at both ends. According to this method, the knot vectors of common quadratic and quartic NURBS curves are derived as follows:

$$U=[\underbrace{0,0,\cdots,0}_{p+1},\frac{(\sum_{j=1}^{p/2}l_j)+\frac{l_{p/2+1}}{2}}{L},\frac{(\sum_{j=1}^{p/2}l_j)+\frac{l_{p/2+2}}{2}}{L},\cdots,\frac{(\sum_{j=1}^{n-p/2-1}l_j)+\frac{l_{n-p/2}}{2}}{L},\underbrace{1,1,\cdots,1}_{p+1}] \tag{2}$$

It can be seen that each time the number of times the curve increases by two, the two connection points at the beginning and end of the curve move an edge inward.

(2) Odd knot vector

If the knots of a small segment of a NURBS curve are the same as the values of the remaining control points of the control polygon without the start and end P/2 control points, the knot vector of the commonly used cubic NURBS curve obtained by this method is:

$$U=[\underbrace{0,0,\cdots,0}_{p+1},\frac{\sum_{j=1}^{(p+1)/2}l_j}{L},\frac{\sum_{j=1}^{(p+1)/2+1}l_j}{L},\cdots,\frac{\sum_{j=1}^{n-(p+1)/2}l_j}{L},\underbrace{1,1,\cdots,1}_{p+1}]. \tag{3}$$

B. Hartley Judd Method

Hartley and Judd consider that if a NURBS curve of degree p wants to interpolate a vertex, it must be p-heavy, whether it adopts multiple knots or multiple vertices. This shows that the difference between the parameter values of adjacent segment connection points is directly proportional to the distance between adjacent vertices, which is quite different from reality. An obvious alternative is to normalise the sum of the corresponding P edges of the control polygon[16]. That is, the interval length of the domain knot is calculated as follows:

$$u_i-u_{i-1}=\frac{\sum_{j=i-p}^{i-1}l_j}{\sum_{s=p+1}^{n+1}\sum_{j=s-p}^{s-1}l_j},i=p+1,p+2,\cdots,n+1.$$

Thus, the knot value can be obtained:

$$\left.\begin{array}{l} u_i = 0, 0 \le i \le p \\ u_i = \sum_{s=p+1}^{i} (u_s - u_{s-1}) = \dfrac{\sum_{s=p+1}^{i} \sum_{j=s-p}^{s-1} l_j}{\sum_{s=p+1}^{n+1} \sum_{j=s-p}^{s-1} l_j}, \\ \qquad i = p+1, p+2, \cdots, n \\ u_i = 1, n+1 \le i \le n+p+1 \end{array}\right\} \quad (4)$$

C. Rogers Method

According to the method introduced by Rogers[5], the knot vector is related to the side length of the control polygon. For a k-order NURBS curve, the knot vector is selected as:

$$U = [u_1, \cdots, u_k, u_{k+1}, \cdots, u_{n+1}, u_{n+2}, \cdots, u_{n+k+1}], u_i \le u_{i+1}$$

The knot vector is calculated as follows:

$$\left.\begin{array}{l} u_i = 0, 0 \le i \le k \\ u_i = \dfrac{\left(\dfrac{i}{n-k+2}\right) l_{i+1} + \sum_{j=1}^{j} l_j}{\sum_{j=1}^{n} l_j}, 1 \le i \le n-k+1 \\ u_i = n-k+2, n+2 \le i \le n+k+1 \end{array}\right\}$$

In order to keep the Rogers algorithm consistent with the previous two parameters, the order k of the curve is replaced by the degree p, and the number of knot vectors starts from 0. Then there are:

$$\left.\begin{array}{l} u_i = 0, 0 \le i \le p \\ u_i = \dfrac{\left(\dfrac{i-p}{n-p+1}\right) l_{i-p+1} + \sum_{j=1}^{j-p} l_j}{\sum_{j=1}^{n} l_j}, \\ p+1 \le i \le n \\ u_i = 1, n+1 \le i \le n+p+1 \end{array}\right\} \quad (5)$$

IV. Difference Analysis

This section takes the commonly used cubic NURBS curve as an example. Based on the same control polygon, NURBS curves are calculated respectively according to the above three knot vector algorithms for analysis and comparison.

First, determine the control polygon and control vertex of the NURBS curve, and draw the polygon as shown in Fig. 1:

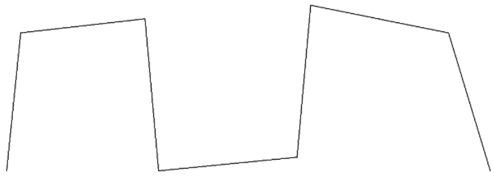


Figure 1. Control polygon

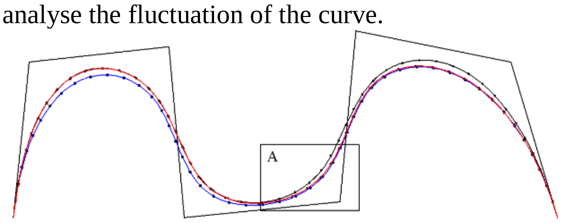
Here, the control points selected by the polygon are: $d_0 = [0,0]$, $d_1 = [1,10]$, $d_2 = [10,11]$, $d_3 = [11,0]$, $d_4 = [21,1]$, $d_5 = [22,12]$, $d_6 = [32,10]$, $d_7 = [35,0]$ respectively. The knot vector of the cubic NURBS curve calculated from eight control points is shown in the following table:

TABLE I. Knot Vector Calculation Results of Different Algorithms

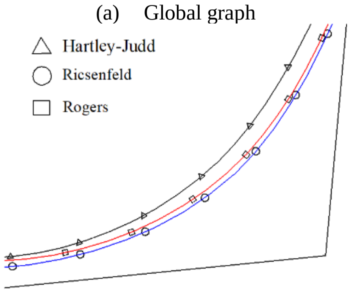
Knot serial number	Hartley Judd	Riesenfeld	Rogers
u0–u3	0.000000	0.000000	0.000000
u4	0.193996	0.265778	0.165001
u5	0.387992	0.419433	0.327240
u6	0.594792	0.559240	0.503317
u7	0.796140	0.712895	0.682164
u8–u11	1.000000	1.000000	1.000000

From the knot vectors shown in Table I, the results of the three methods are different. Calculate the corresponding NURBS curve with the calculated knot vector, as shown in Fig. 2. The curves marked with triangles, rectangles and small circles in the figure are the curves obtained by the Hartley Judd, Rogers and Riesenfeld methods, respectively, where (a) is the global graph of the curve and (b) is the result of enlarging the identification area of a by four times.

According to curve 2 (a), Hartley Judd and Rogers are comparable in the first half, while the Riesenfeld method is significantly different from the first two; Rogers and Riesenfeld are comparable in the second half, while the Hartley Judd method is significantly different from the two. Next, three error curves are drawn by subtracting two by two, and the corresponding variance is calculated to analyse the fluctuation of the curve.



(a) Global graph



(b) Zoom in 4×

Figure 2. NURBS curve

The two curves of Hartley Judd and Rogers are subtracted to obtain the deviation curve, as shown in Fig. 3:

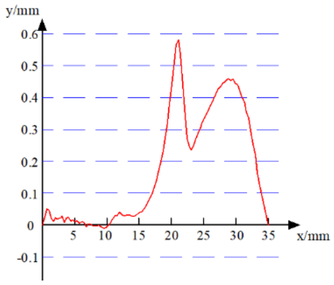


Figure 3. Hartley judd and rogers deviation

The front deviations of the two curves are small, and their proximity in the graph is also high, but there are differences; The deviation in the latter half is large, which also proves that the deviation between curves is real, and the variance of this curve is calculated as follows:

$$s_1^{\ 2} = 0.033183$$

Similarly, subtract Hartley Judd and Riesenfeld curves from Rogers and Riesenfeld curves to obtain two deviation curves, as shown in Fig. 4 and Fig. 5:

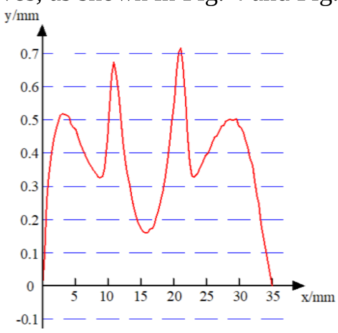


Figure 4. Hartley Judd and Riesenfeld deviation

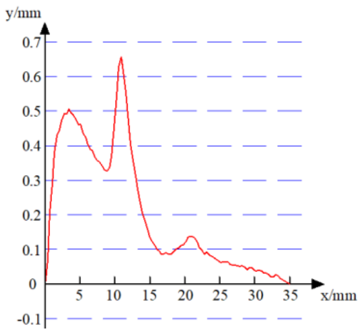


Figure 5. Deviation between Riesenfeld and Rogers

The obtained variances are as follows:

$$s_2^{\ 2} = 0.019508\,,$$

$$s_3^{\ 2} = 0.059002\,.$$

In terms of variance, the variance of Riesenfeld and Rogers is large, which also shows that the deviation between the two curves greatly fluctuates, whereas the variance of Hartley Judd and Riesenfeld is small, which shows that the deviation fluctuation of the two-day curves is small.

Obviously, the deviation between each cubic curve is visible to the eye. As a result, different knot vectors will lead to different NURBS curves. In the interpolation algorithm, how to select the right knot vector has become an important part of drawing NURBS curves.

V. A Weighted Average Method is Used to Calculate the KNOT Vector

A. *Weighted Average Algorithm*

In this section, the weighted average algorithm is proposed to calculate the node vector, and the error analysis and curve fitting experiments are carried out.

The weighted average method assigns different weights to each parameter according to their relative relevance. Set $f_1, f_2, \cdots, f_k$ as the source data to be averaged and the corresponding weight $x_1, x_2, \cdots, x_k$, the calculation formula of the weighted average x is as follows:

$$x = \frac{x_1 f_1 + x_2 f_2 + \cdots + x_k f_k}{\sum_{i=1}^{k} x_i}$$

Let the knot vectors obtained by Hartley Judd, Rogers and Riesenfeld algorithms be U_1, U_2, U_3, respectively, and bring the knot vectors into this formula:

$$U = \frac{x_1 U_1 + x_2 U_2 + x_3 U_3}{x_1 + x_2 + x_3} \tag{6}$$

Take the weights as 1/3 to calculate U. The general calculation flow of the above calculation is shown in Fig. 6:

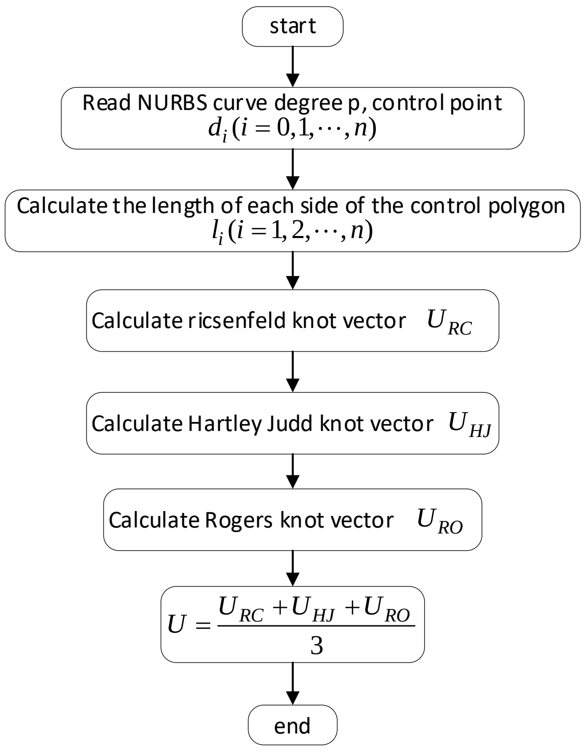


Figure 6. Algorithm flow chart

B. *Speed Increase*

In steps 3–5 of the method shown in Fig. 6, the knot vectors of three traditional methods need to be calculated first, and then the weighted calculation is implemented in step 6, which takes more time than the traditional single calculation of only one knot vector. As a result, two optimisation approaches are given as follows.

(1) Parallel computing

Multi-core is the conventional configuration of computers, and the implementation of parallel computing is also conventional computing. The amount of calculation can be greatly reduced using three processors to calculate the three knot vectors (Riesenfeld, Hartley Judd and Rogers) at the same time and then obtain the new knot vector in a weighted manner.

(2) Three combinatorial optimisation calculation in one

Integrating Riesenfeld, Hartley Judd and Rogers into a new algorithm can avoid repeated calculations and reduce calculation time.

Make the length of each side of the control polygon as follows:

$$l_i = |d_i - d_{i-1}|, i = (1, 2, \cdots, n)$$

The total side length obtained by adding the length of each side is:

$$L = \sum_{i=1}^{n} l_i$$

If the number of times P is 3, the following algorithm can be obtained:

$$\left.\begin{array}{l} u_i = 0, 0 \le i \le p \\ u_i = \begin{bmatrix} a_1 & \cdots & a_{i-3} & a_{i-2} & 0 \\ a_1 & \cdots & a_{i-3} & \dfrac{(i-3)}{n-2} a_{i-2} & 0 \\ b_1 & \cdots & b_{i-3}k & b_{i-2}k & b_{i-1}k \end{bmatrix} \begin{bmatrix} l_1 \\ \vdots \\ l_{i-2} \\ l_{i-1} \end{bmatrix}, \\ p+1 \le i \le n \\ u_i = 1, n+1 \le i \le n+p+1 \end{array}\right\} \quad (7)$$

which:

$$a_j = \frac{1}{L}(j = 0, 1, \cdots, i-2),$$

$$b_j = \begin{cases} 1, (j = 1, i-1) \\ 2, (j = 2, i-2) \\ 3, (j = 3, 4, \cdots, i-3) \end{cases},$$

$$k = \frac{1}{\sum_{s=p+1}^{n+1} \sum_{j=s-p}^{s-1} l_j}.$$

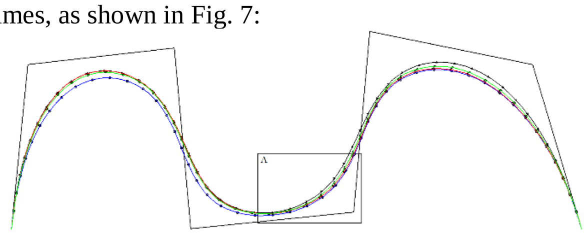
C. *Experimental Results*

The calculation results using the algorithm in the previous section are shown in Table II:

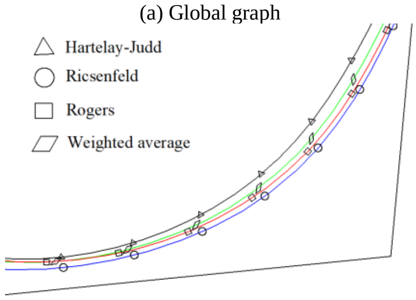
TABLE II. KNOT VECTOR RESULTS CALCULATED BY WEIGHTED AVERAGE METHOD

Knot serial number	Knot value
*u*0–*u*3	0.000000
*u*4	0.208258
*u*5	0.378222
*u*6	0.552450
*u*7	0.730399
u8–*u*11	1.000000

NURBS curves are drawn from the knot vectors obtained in Table II. To facilitate observation, all curves are still drawn together, and area a is enlarged by four times, as shown in Fig. 7:



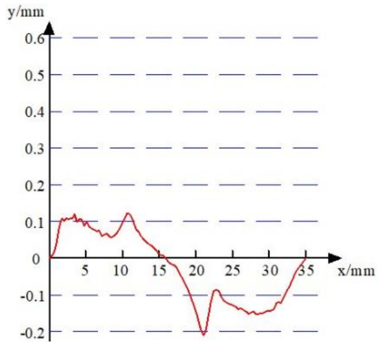
(a) Global graph



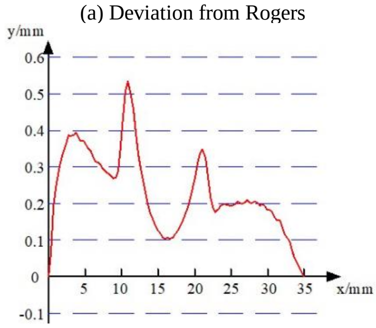
(b) Zoom in 4×

Figure 7. vector curves of four different knots

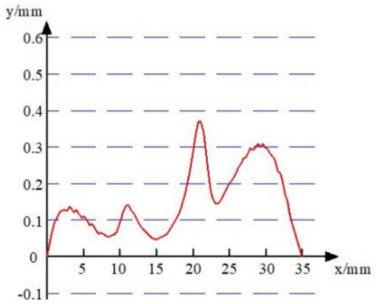
The parallelogram marks the curve obtained by the knot vector after a weighted average, and the difference between the four curves in the figure is still clearly visible. The deviation curves with Rogers, Riesenfeld and Hartley Judd are drawn successively below, as shown in Fig. 8:



(a) Deviation from Rogers



(b) Deviation from Riesenfeld



(c) Deviation from Hartley Judd

Figure 8. Deviation curve

The obtained variance is as follows:

$$s_4{}^2 = 0.015499\,,$$

$$s_5{}^2 = 0.020852\,,$$

$$s_6{}^2 = 0.034708\,.$$

From the graph, the curve obtained by this method is between the three curves, which shows that the result curve also has the characteristics of a weighted average; From the deviation curve, there is still some deviation between the curve obtained by the new method and other curves, but compared with the deviation between the above three curves, the fluctuation and peak value of the curve are reduced to a great extent; The variance results show that the deviation curve obtained by this method is slightly better than the one above, but it still maintains a moderate deviation from each curve.

D. Advantages of Weighting Algorithm

The following advantages of the weighting algorithm are shown in the diagram, error analysis and variance results:

(1) Different from the traditional method, the interpolation curve has different shapes.

(2) The curve calculated by the method described in this study is located in the middle of the curve cluster calculated by the traditional method.

(3) It is reasonable to conclude that the curve calculated by this method is generally closer to the target value.

E. Curve Fitting Results

The method described in [1] adopts four node vector calculation methods to fit the curve shown in Fig 9.

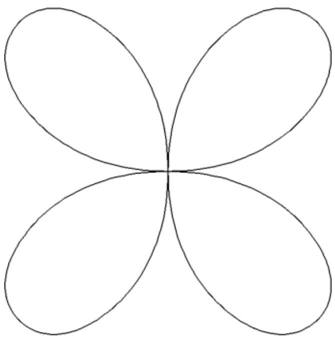


Figure 9. Target curve

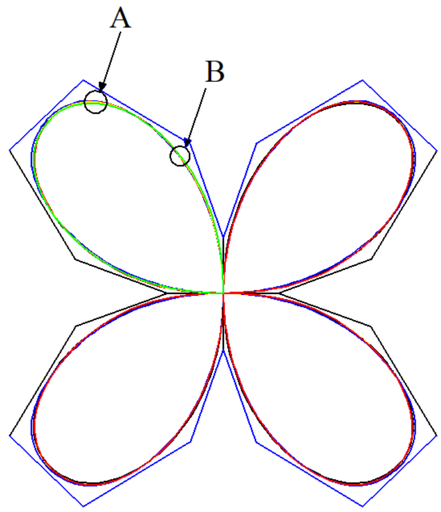
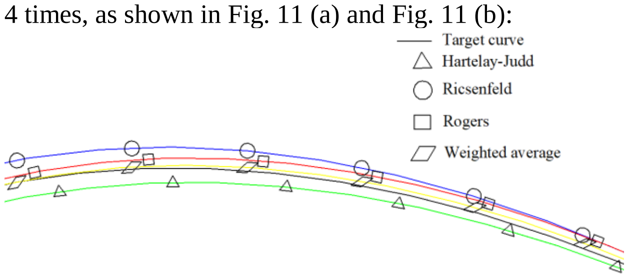
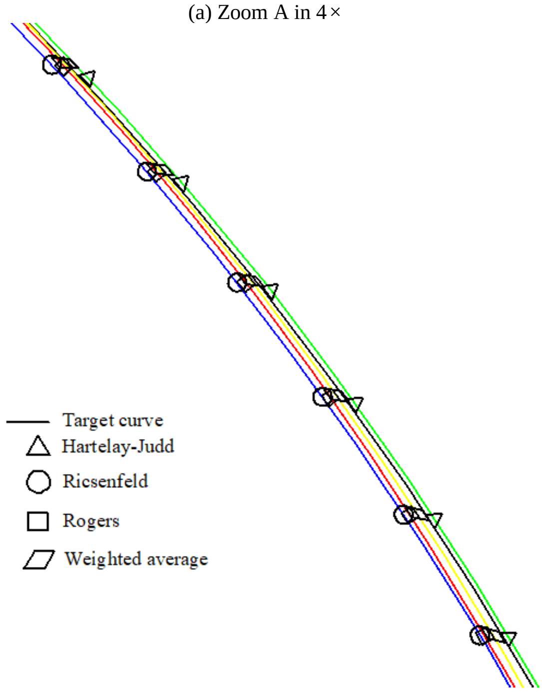


Figure 10. Fitting results

Then, enlarge the selected parts A and B in Fig. 10 by 4 times, as shown in Fig. 11 (a) and Fig. 11 (b):



(a) Zoom A in 4×



(b) Zoom B in 4×

Figure 11. Curve enlargement

The results of curve fitting show that different node vectors will get different curves under the same control points. The curve obtained by the weighting method is in the middle of the traditional curve cluster, which is closer to the target curve.

VI. Conclusion

Different knot vectors always produce more or less deviation from the calculated NURBS curve. The weighted average method used in this study combines the Rogers, Riesenfeld and Hartley Judd methods, which can be used to approximate the corresponding curve by varying the weight, making it more flexible than the existing methods. The result shows that the curve drawn by the new knot vector is in between the three curves, demonstrating that the curve used in this method has weighted characteristics and demonstrating the method's feasibility. At the same time, the problem of calculating multiple knot vectors will increase the number of calculations is resolved by parallel calculation and the integration formula. Finally, future studies will focus on how to apply this method to engineering practice.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Mr. Zheng Shuai as first author to conducte the research under the supervision of Professor Wang Xingbo (corresponding author), who is in charge of the research project; Zheng Shuai is responsible for formulating methods and procedures; Yan Jihong was responsible for the experiment, obtained the experimental results, and finally completed a manuscript. Three authors participated in the writing of the manuscript and approved the final version.

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