A Study of Tension Distribution for Control of Planar Cable Driven Parallel Robot Using Quadratic Programming Algorithm

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***Abstract*—The kinematic problem of Cable Driven Parallel Robots is not like that of other conventional robots due to the transmission mechanisms are the cables that can only exert the pulling force on the moving frame because the cable cannot be used to push. Therefore, the robot can only perform the task if the tensions in all cables are positive. The calculation of the kinematics and control for these types of robots as well as calculating the tension distribution to ensure the equilibrium for the moving frame at the nodes have to be done simultaneously. This paper described analysis and experiments of a planar cable driven parallel robot, in which, the inverse kinematics of the robot is solved based on a cable tension distribution model with a quadratic cost function, the values of tensions of cables are preferred to follow the mean value of the cable tension limit. A force feedback controller is also designed to control the robot based on calculated data, then the experimental results are analyzed to evaluate the effectiveness of this model.**

***Index Terms*—planar cable robot, tension distribution, static kinematics**

I. INTRODUCTION

Cable Driven Parallel Robots (CDPRs) have been researched and developed in many applications due to their advantages in structure and workspace [1-2]. CDPRs are a kind of parallel structure robot with cables are used as the driving mechanism. Each cable is stored and distributed by a cable winch and connected to the moving frame or end effector after passing through the diverter pulleys. The cable is lighter in weight and smaller in volume than the rigid link of a traditional or parallel robot, which enables the development of very long drive cables without the need for an overly large cable distribution mechanism. With this structure, the moving frame of cable robot has high acceleration and speed, large workspace and be widely applied in the fields of agriculture, construction,…[3- 4].

Kinematic analysis of the CDPRs is not like other types of parallel robots because the cables can only pull a force to the end effector and they cannot push. Therefore, the robot only performs the task if the tensions in all cables is positive [5-7]. In contrast to the large operating space, the mass and elasticity of the cable will cause sagging [5]. The workspace CDPRs is defined as the area in space where the moving frame reaches equilibrium status under the action of external forces and positive cable tensions. The workspace and performance of CDPRs can be expanded and improved by adding drive cables, but this will complicate the kinematics and dynamics problem and increase the cost of the controller and expand the mechanical structure. When the number of driven cables is larger than the number of degrees of freedom (D.O.F) of end effector, it is necessary to find the cable tension systems satisfying the constraint conditions, which affects the design of the controller and the application of the cable robot. Hassan Bayani et al. [6] developed a planar CDPR with camera position feedback controller several controllers (sliding, adaptive sliding) used to design the controller for the robot, the experimental results are compared to evaluate the response of each controller, the pseudo-inverse matrix is used to calculate the cable tension distribution solution, the calculation results can be used as reference for the design controllers for other cable robot configurations. Javad Bolboli [7] et al. analyzed the operation space based on the stiffness matrix of the planar cable robot structure, the study showed the relationship between the internal force and the stiffness of the structure, which can be used as a standard when determining the tension distribution of other cable-driven robot configurations, in order to ensure the robot's rigidity in the operating space.

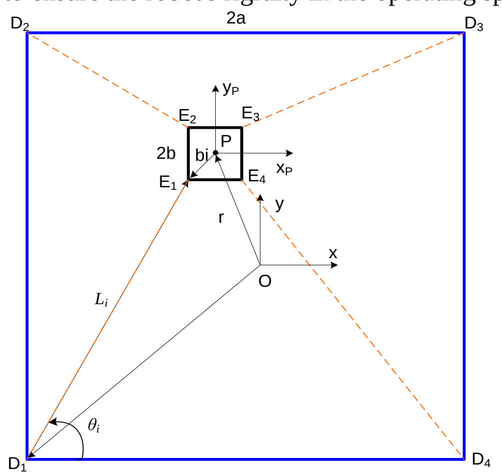


Figure 1. Kinematic structure of Planar CDPR

Manuscript received October 1, 2021; revised May 17, 2022.

doi: 10.18178/ijmerr.11.7.479-485

Min-Cheol Kim et al. [8] developed a sensor-based capsule endoscope monitoring device based on the motion of a planar CDPR. A permanent magnet is placed in the capsule endoscope, the position of the capsule endoscope is determined based on measurements of the semi-static magnetic field of the HALL effect sensor array and the forward kinematics problem of the planar cable robot. Some other studies [9][10] also used linear programming and quadratic programming to calculate the distribution of cable tension. Most of the above studies have not specifically mentioned the design of the trajectory, the calculation of the joint trajectory and the tension for the control experiment, or the monitoring of the tension value when controlling the working head according to the points. given button. In order to evaluate the response and feasibility of designing and controlling a planar cable robot, this study presents a kinematic model of a planar CDPR driven by 4 cables and builds a kinematic simulation program for that model. Based on the quadratic programming algorithm for determining the combinations of tension forces, it is possible to determine the distribution of cable tension according to the joint trajectory, avoiding large changes of cable tension during the operation, otherwise, the tension values of cables are taken close to the average value to avoid cable stagnation when the tension is small or the motor overload when the tension is high due to the response of the controller. A cable tension feedback controller is designed to control the CDPR to execute the design trajectories to evaluate the accuracy of the calculated model.

II. Kinematics of Planar Cable Robot

Planar CDPRs consist of a moving frame driven by m cables distributed from cable dispensing mechanisms. The analyzed configuration in this paper includes four cables, which generate two degrees of freedom shown in Fig. 1. Each cable is connected to the fixed frame at D_i points and to the moving frame at E_i points. The length of cable i is denoted by L_i and the cable angle to the x_i axis is θ_i $(i = 1, 2, ..., m)$.

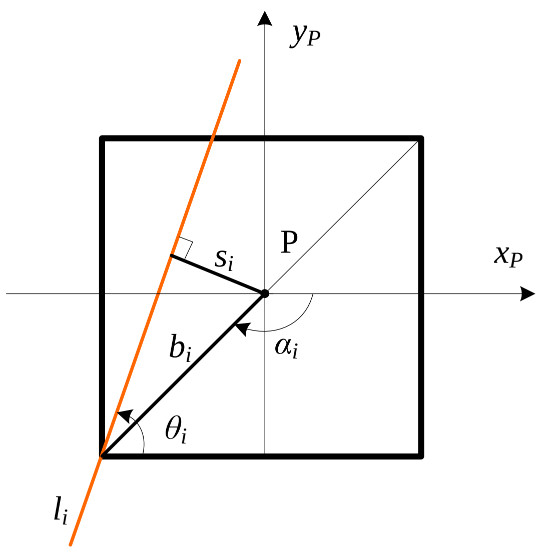


Figure 2. Diagram showing the relationship between parameters

The inverse position kinematics assumes as follows: known position P to determine the cable lengths L_i. From the position of end-effector P and each vertex D_i of the base link, we have the below equation.

$$L_i = \sqrt{\left(E_{ix} - D_{ix}\right)^2 + \left(E_{iy} - D_{iy}\right)^2} \quad i = 1, ..., m \tag{1}$$

According to the kinetic structure in Fig. 1, we have like as.

$$\begin{bmatrix} E_1 & E_2 & E_3 & E_4 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & P_x \\ 0 & 1 & P_y \end{bmatrix} \begin{bmatrix} -b & -b & b & b \\ -b & b & b & -b \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} D_1 & D_2 & D_3 & D_4 \end{bmatrix}^T = \begin{bmatrix} -a & -a & a & a \\ -a & a & a & -a \end{bmatrix} \tag{3}$$

Cable tension is the force on the cable starting from the winch to end-effector. In moving, the robot can be considered to be in equilibrium status due to slow motion. However, the effect of cable sag needs to be taken into account because of the resulting length of the cable (in the case of a long cable). The balancing problem of a CDPRs is to determine the orientation and position of the end-effector when the length of each cable is known, and the position and direction that can keep the robot in a state balance. The problem of robot kinematics is not enough to control the CDPRs because the cable cannot provide thrust but only pull forces. Cable tension greatly affects the determination of the workspace. From the structural diagram in Fig. 1, the total forces and moments acting on the end effector of CDPRs can be represented by following system of equations:

$$\sum F = \begin{bmatrix} F_x \\ F_y \end{bmatrix} + \sum_{\mathbf{i=1}}^{m} \boldsymbol{\tau}_\mathbf{i} \tag{4}$$

$$\sum M = M_z + \sum_{\mathbf{i=1}}^{m} s_i \boldsymbol{\tau}_\mathbf{i} \tag{5}$$

With s_i being the actuation distance of the moment caused by the cable tension i on P (center of end effector, from the diagrams of Fig. 1 and Fig. 2, we can obtain.

$$s_i = b_i \sin(\alpha_i - \theta_i) \tag{6}$$

$$\cos\theta_i = \frac{l_{xi}}{\left\| l_i \right\|}, \sin\theta_i = \frac{l_{yi}}{\left\| l_i \right\|}, \qquad i = 1, ...m \tag{7}$$

Where F and M are vectors of the external forces and the external torques acting on the end effector, respectively (in this case, zero); τ_i is tension force of cable l_i; b_i is the distance from attaching points B_i to the center of end effector P. Form the direction of driven cable, the unit vector u_i and tension vector τ_i can be obtained.

$$\boldsymbol{\tau}_\mathbf{i} = \tau_i \times \mathbf{u}_\mathbf{i} \tag{8}$$

$$\mathbf{u_i} = \frac{L_i}{\|L_i\|} = \begin{bmatrix} \frac{L_{xi}}{\|L_i\|} \\ \frac{L_{yi}}{\|L_i\|} \end{bmatrix} = \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix} \quad (9)$$

The equilibrium equation can be described as follow.

$$\begin{bmatrix} \mathrm{u}_1 & \cdots & \mathrm{u}_\mathrm{m} \\ \mathrm{s}_1 & \cdots & \mathrm{s}_\mathrm{m} \end{bmatrix} \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_m \end{bmatrix} + \begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = 0 \quad (10)$$

By defining

$$\mathbf{A} = \begin{bmatrix} \mathrm{u}_1 & \cdots & \mathrm{u}_\mathrm{m} \\ s_1 & \cdots & s_\mathrm{m} \end{bmatrix} \quad (11)$$

$$\mathbf{w}_\mathrm{p} = \begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} \quad (12)$$

Equation (10) can be written like below equation.

$$\mathbf{A\tau} + \mathbf{w}_\mathrm{p} = \mathbf{0} \quad (13)$$

where $\tau = [\ \tau_1\ \tau_2\ \ldots\ \tau_m]^T$ is a cable tension vector; $\mathbf{A^T}$ is a structure matrix of the cable robot *(n x m);* $\mathbf{w_p}$ is a vector of the external force act on the moving frame.

Therefore, a tension distribution is calculated for satisfying the system of equations (13) with the specific constraints depending on the design of the system [9-11] in a given pose of the moving frame. In this case, the static equilibrium equation and the positive tension constraints as well as the magnitude limit of the cable tension ensure that the cable does not sag nor exceed the load of the motor of cable distribution mechanism. System constraint is shown in below equation.

$$0 < \tau_{\min} \le \boldsymbol{\tau} \le \tau_{\max} \quad (14)$$

where τ_{min} is lower limit and τ_{max} is the upper limit of cable tensions.

There are many methods of tension distribution for cable robots to calculate tension of cables like as linear programming, pseudo-inverse matrix, and quadratic programming. Linear programming [9] is a method to search the best (most optimal) solution from an infinite number of solutions decided in a mathematical model with requirements are expressed in terms of equations in linear relationship, where the cost function value is the sum of the tension minimized. The advantage of this method is saving energy, the optimal solution is the one that satisfies the stated objectives of the problem depending on the constraints. The disadvantage of this method is that some of the tension tends to reach the boundary value, the tension values have a large change at the trajectory transition points, causing the actuator to be overloaded or causing slack on the driven cables, thereby reducing the accuracy of the system. The second alternative is to use a pseudo-inverse matrix [6] with the goal of finding continuous string tensions along a changing trajectory of joint variable, this method can cause some cases with points located near the spatial boundary workspace, the cable tension may be negative. The quadratic programming algorithm was used to calculate the sets of cable tensions with the criterion that the value of the cable tensions is continuous and the preferred value clings to the average value of the cable tension limits [10]. The model of the problem of minimizing the relative tension is shown as follows.

$$\min \sum_{1}^{m} (\tau_i - \overline{\tau}_i)^2 \qquad i = 1, \ldots, m \quad (15)$$

$$\overline{\tau}_i = \frac{\tau_{i\max} + \tau_{i\min}}{2} \quad (16)$$

where $\overline{\tau_i}$ is the average of i^{th} cables tension with following constrains.

$$\mathbf{A\tau} = -\mathbf{w_p} \quad (17)$$

$$0 < \tau_{\min} \le \boldsymbol{\tau} \le \tau_{\max} \quad (18)$$

Therefore, the objective function (15) is a quadratic form of first-order constraints, the sequential quadratic programming algorithm [11][12] can be used to solve this issue. The results show that the cable tension follows the average values and the corresponding starting trajectory, which is convenient for feedback control of the driven cable, avoiding the condition of the cable being slack or the tension being too large to cause motor overload. Calculation results of cable tension along with moving trajectories and joint trajectories will be performed in the next section.

III. Simulations, Experiments and Evaluations

A prototype of planar cable robot was used to carry out several experiments, then experimental results also compared simulating ones to evaluate robot's performances. Fig. 3 shows the designed and developed planar cable robot with cable distribution mechanism, which is used to conduct the experiment. The structure of robot consists of 4 cable distributors indexed (1) put below to drive end-effector (3) moving on the horizontal plane through the pulleys (4) mounted on the frame (2) and the cable (6) with the force-feedback control (5). The cable distributor is driven by the motor (8) through the coupling attached to the winding roller (7) which also rotates. When the roller (7) rotates, the toothed belt drive transmits the motion that rotates the lead screw. Lead screw mechanism makes nut (9) reciprocating with a specified pitch to distribute the driven cable. A cable distribution mechanism is placed on the nut (9), which helps the wire to be fixed at the specified position. The lead screw mechanism helps to keep the wire parallel to the roller in a defined step. The cable after being distributed by the roller and the lead screw, is guided through the pulley (10) placed on the loadcell (11), this loadcell is mounted on the cable distribution mechanism frame (12) to measure the cable tension, combined with the roller placed on the robot frame to ensure the fixed direction of the tension on the loadcell.

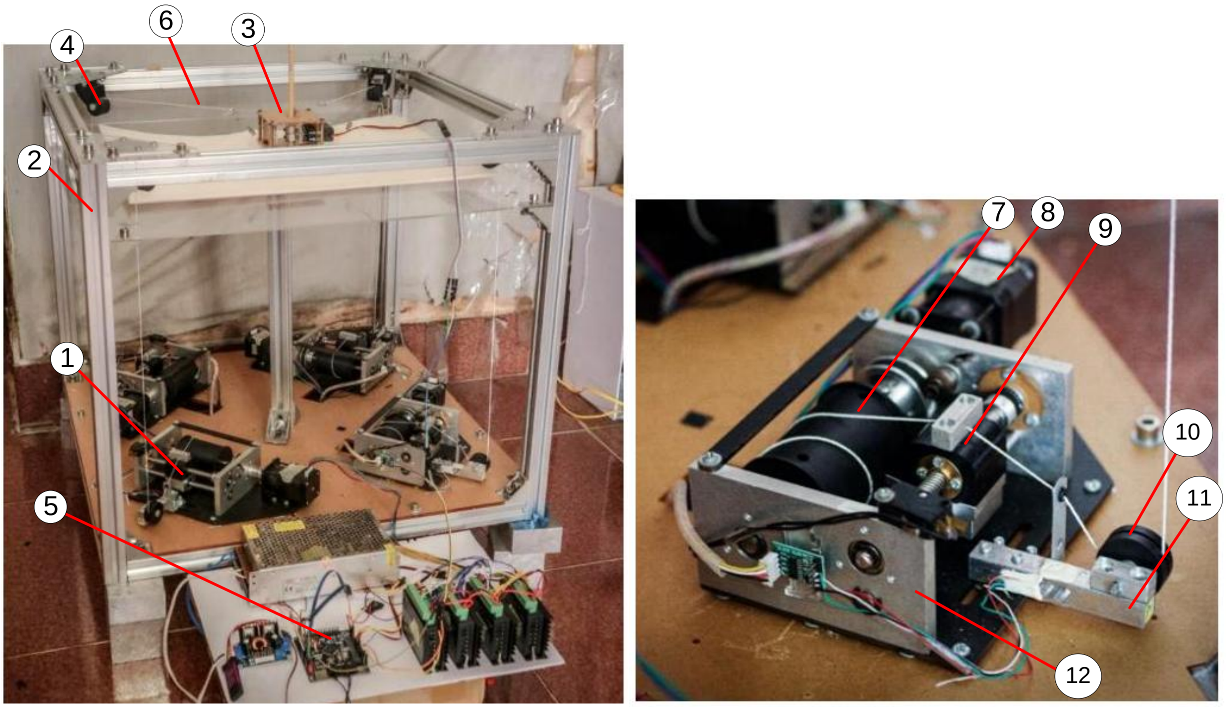
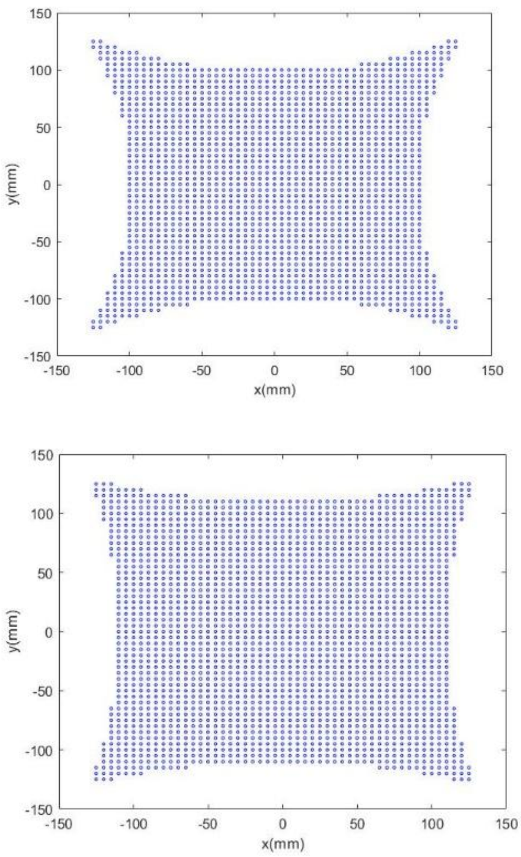


Figure 3. Planar cable robot (left) and cable distribution mechanism (right).

TABLE I. SPECIFICATION OF PLANAR CABLE ROBOT.

a	175mm
b	45mm
τ_{min}	50N
τ_{max}	200N

With the configuration and specifications as shown in Table I, a null space of structure matrix analysis algorithm is applied to determine the static workspace of the planar CDPR with linear constraints [13]. Static workspace is defined as the set of moving frame positions where at least one combination of cable tension exists satisfying the equilibrium equation (13) with the constraints of cable tension limit and different external forces, the workspace in this case is defined with different tension limits with external force vector is zero. After comparison between Fig. 4(a) and Fig. 4(b), the static workspace corresponding to the force limit cable tension of $[\tau_{min}, \tau_{max}]$ in range *[50N 200N]* is smaller than corresponding to a larger cable tension $[\tau_{min}, \tau_{max}]$ in range *[50N 300N]*. This result shows that the size of robot's workspace increases when the limits of tensions are extended, so it is possible to expand the workspace by increasing the motor power, which is also an important property as a design criterion and calculate the robot structure as well as choose the appropriate actuator power. The structure of the controller for cable robot is shown as the diagram in Fig. 5. There are 3 layers in this controller, the first layer is the main controller designed to calculate the value of joints, velocity and tension distribution of cables from the motion trajectory of MP through synthesis of kinematics and force balance problems.



(a)

(b)

Figure 4. Static workspace of planar CDPR with different limits of cable tension $\tau_{min} = 50N$, $\tau_{max} = 200N$, and $\tau_{min} = 50N$, $\tau_{max} = 300N$

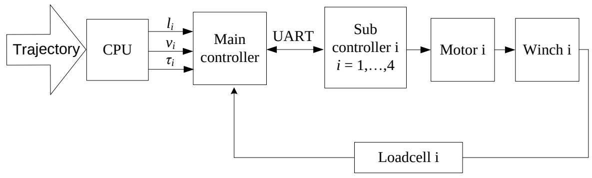


Figure 5. Diagram of cable robot control system

The second layer is the sub-controller or the position controller, which receives signals from the main controller and is responsible for outputting control signals to the servos through the drivers. The third layer controls the position, velocity and cable tension of the motor that drives the cable distributor through signals received from the sub-controller and feedback from the corresponding coded values. To evaluate the suitability of the computational model, the basic trajectories (triangle, rectangle, ...) will be designed and tested on this robot model, with the workspace being the XY plane. The robot will perform trajectories through the pen holder on the moving frame. The trajectory obtained during the experiment will be compared with the designed trajectory to measure errors in order to evaluate the model and the robot's performance. Fig. 6 shows the simulation results of rectangle trajectory (a) and experimental results (b) with dimension 55mmx70mm. The experimental results on the robot model shown in Fig. 7 show that the robot's trajectory has a continuous profile, follows the given trajectory with an error of less than 2mm, the trajectory has no fracture phenomenon due to slack phenomenon of cable during movement, the results of cable tension measured at the nodes show good response of the controller.

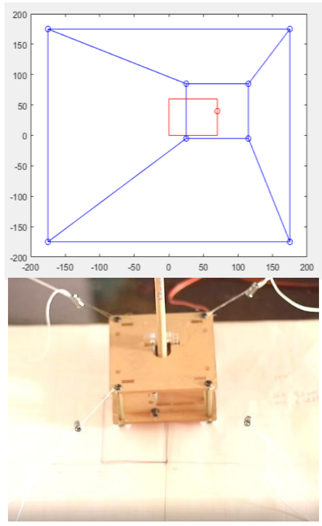


Figure 6. Simulated trajectory (a) and experimental trajectory (b) of the rectangle

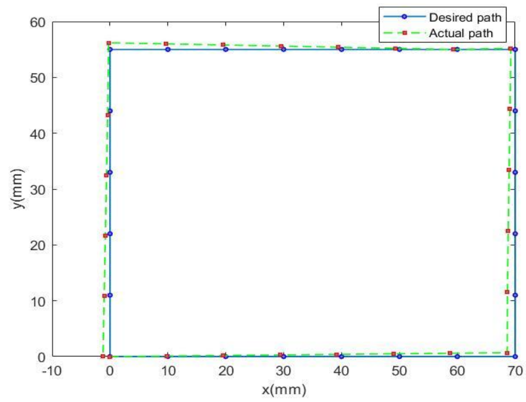


Figure 7. Rectangular path according to design and experiment

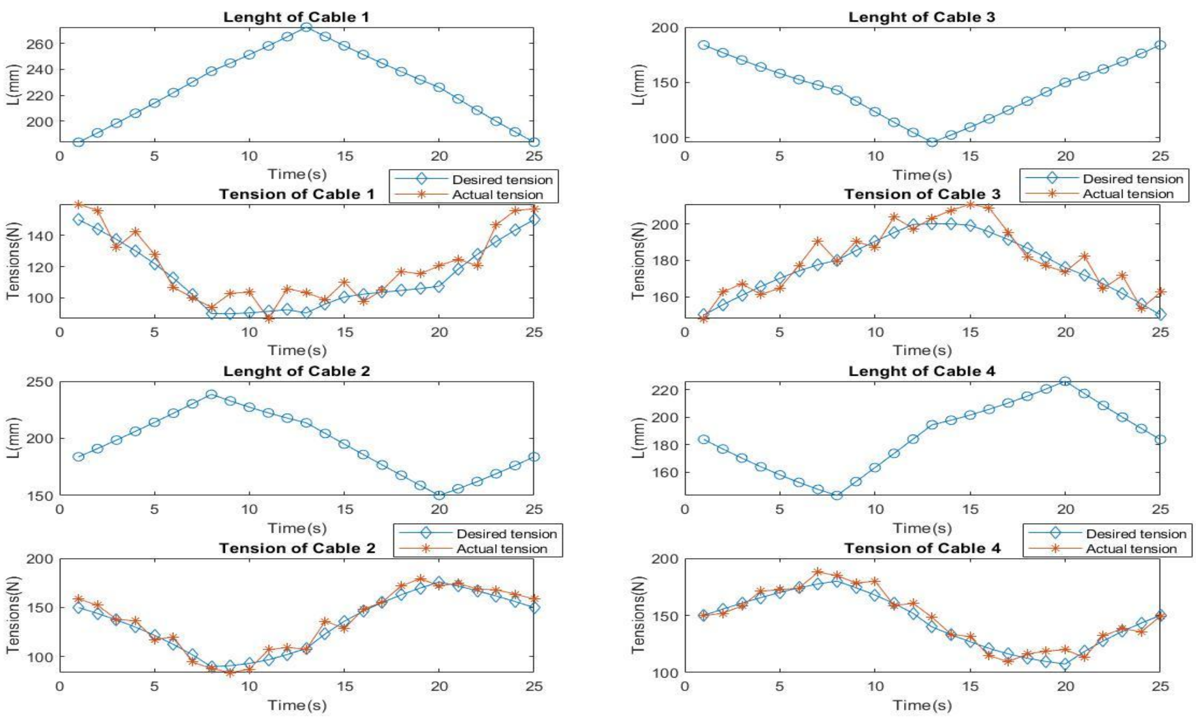


Figure 8. Diagram of joint trajectories and tension of cables according to calculations and experiments when interpolating rectangle

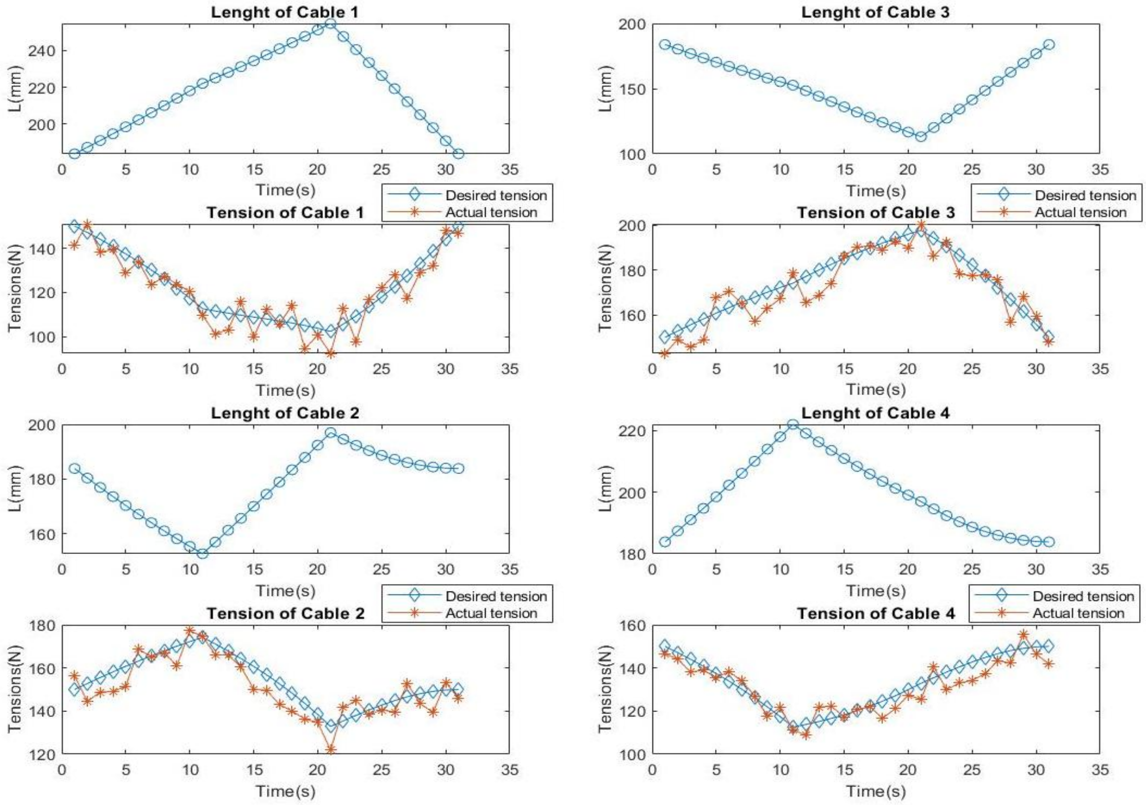


Figure 9. Diagram of joint trajectories and tension of cables according to calculations and experiments when interpolating triangle

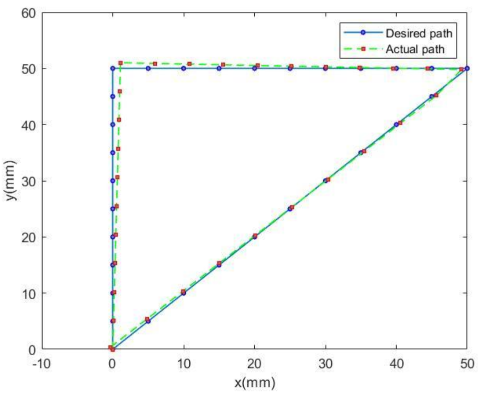


Figure 10. Triangle path according to design and experiment

The corresponding joint trajectory response for rectangle path is showed in Fig. 8 with the calculated and experimental cable tension values, this graph shows that the tension values satisfy the equilibrium equation (13), these cable tensions change continuously, following a trajectory that matches values close to the mean within the range of cable tension. This is very convenient for control based on cable tension, limiting cable slack when the control cable tension is less than the lower limit of cable tension or overloading the motor when the tension exceeds the upper limit due to the response of the controller. Similar to the above experimental results, Fig. 9 and Fig. 10 also are simulating results in joint space, cable tension for triangle-perimeter path and experimental results. The values of cable tensions corresponding to the joint trajectories have a continuous change, therefore there is no sudden change in cable tension, causing difficulties for the control process. The cable tension values are always within the tension range based on the equilibrium equation. When the MP moves to a point, the 4 cables are stretched reaching positive cable tensions, and the cable tensions also eliminate at nodes of the moving-path. Through the simulating results based on the sequential quadratic programming method, it shows that the cable tension satisfies the constraints of the balanced equation and has values that change relatively continuously according to the nodes, thereby showing that the results of the problem have a suitable response for the tension feedback controller. The tension errors at the nodal points are set at suitable thresholds for the trajectory to move continuously.

IV. Conclusion

This paper presents a complete design, calculation and experimental procedure for a planar cable robot with 3 D.O.F, driven by 4 inelastic cables. In which, the calculation of the kinematic problem with the number of D.O.F less than the number of driven cables is performed based on the nonlinear programming method. The cable robot configuration and the null space analysis of the structure matrix method was used to determined static workspace of the CDPRs, the cable tension distribution is selected based on the given trajectories determined and the sequential quadratic programming algorithm with a quadratic objective function designed for the purpose of determining the value of tension systems along the joint trajectory calculate and follow the average value of the upper and the lower limit of the cables tensions, a tension feedback controller is designed for a prototype planar CDPR with a tension feedback unit placed on a cable distributor, the experimental results show the agreement of the design model with the experimental strain response and trajectories according to the given trajectories. This model can be used to design and control redundant CDPRs of larger sizes with different configurations. The future work of this research direction is to apply a model for large sizes redundant CDPRs that take into account the influence of cable sags with different cabling configurations and degrees of freedom.

ACKNOWLEDGEMENTS

This work belongs to the project grant No: B2021-SPK-05. funded by Ministry of Education and Training, and hosted by Ho Chi Minh City University of Technology and Education, Vietnam.

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