Adaptive Approximation-Based Feedback Linearization Control for a Nonlinear Smart Thin Plate

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Abstract— **This paper proposes feedback linearization control (FBLC) based on function approximation technique (FAT) to regulate the vibrational motion of a smart thin plate considering the effect of axial stretching. The FBLC includes designing a nonlinear control law for the stabilization of the target dynamic system while the closed-loop dynamics are linear with ensured stability. The objective of the FAT is to estimate the cubic nonlinear restoring force vector using the linear parameterization of weighting and orthogonal basis function matrices. Orthogonal Chebyshev polynomials are used as strong approximators for adaptive schemes. The proposed control architecture is applied to a thin plate with a large deflection that stimulates the axial loading thus, the plate behaving nonlinearly. The governing partial differential equation for the piezo-plate system is transformed into definite ordinary differential equations (ODEs) using the Galerkin approach; hence, multi-input multi-output ODEs are obtained. Simulation experiments are performed to verify the validity of the proposed control structure.**

Index Terms—**nonlinear vibrations, feedback linearization, smart plates, piezo-patches**

I. INTRODUCTION

Much attention has been paid to applications of smart materials in active vibration control of flexible structures such as cables, beam-like structures, and plate-like structures and so on. The interesting point is that they are configurable and adaptable if external stimuli are applied. They can behave as actuators or sensors depending on the external motivator. In view of the above, they are extensively used in vibration suppression of flexible structures; however, the design of a suitable control system is still required to stabilize the structure vibration. The task of the control system could be difficult if the vibrating structure undergoes nonlinear vibrations since the conventional linear controllers cannot be useful in this case. Therefore, this work is concerned with the design of a nonlinear control strategy that is sufficient to regulate the plate motions. The PDF for the vibrating plate is transformed into multi-ODEs using the Galerkin approach, considering the effect of axial stretching resulted from large deflection behavior. For more details on the modelling of plate dynamics, see, e.g. [1-6].

There are miscellaneous control schemes for attenuation of the structure vibrations [7-13]. However, what important here is design of a control structure that can stabilize the piezo-plate system under uncertain modelling. In general, there are two categories of control strategies that deal with the control of uncertain dynamics which are adaptive control and robust control. The core of this work is focused on adaptive approximation technique that attempts to approximate the target uncertain parameter/term in terms of weighting and basis function matrices. Then the weighting coefficient matrix is updated based on Lyapunov theory (see [14-20] and the references therein for more details). Besides, one of the powerful tools to deal with nonlinear dynamics is feedback linearization control. It selects a nonlinear control law for controlling the dynamic system while the closed-loop dynamics is linear with guaranteed stability.

As a result, this paper suggests FAT-based FBLC for vibration suppression of a nonlinear plate with piezo-patches. One limitation of FBLC is that it requires calculation of inverse mass matrix and modal acceleration that complicates the control task. Therefore, we assume here that the mass matrix is known, since it can easily be measured for plate structure, while the coupled nonlinearity resulted from bending-axial stretching effect is unknown. The nonlinear restoring force vector is estimated using the linear parameterization of weighting and basis function matrices. Orthogonal Chebyshev polynomials are used as strong approximators for adaptive schemes. Simulation experiments were performed using MATLAB/SIMULINK package to verify the proposed control architecture.

The remainder of the paper is organized as follows. Section 2 introduces nonlinear dynamic modelling of the smart plate while the control architecture is presented in

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Sect. 3. Simulation results and discussion are described in Sect. 4. Section 5 concludes.

II. Nonlinear Dynamics of Smart Plates

In this section, a detailed derivation for a piezo-electrically thin with axial stretching is presented in Fig. 1. Let us consider the following assumptions [21]:

1. The plate has a uniform thin thickness.
2. The attached piezo-patches are of neglected dynamics.
3. The coupled bending-axial loading is considered.

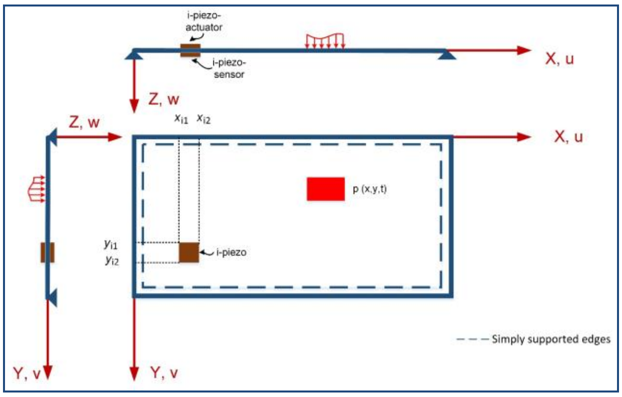

Figure 1. A simply supported thin plate with attached piezo-transducers.

Based on [1, 22, 23], the PDF for the target smart plate can be expressed as

$$D\nabla^4 w = \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} - m\frac{\partial^2 w}{\partial t^2} - p + \frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2} \quad (1a)$$

$$\frac{\nabla^4 F}{Eh} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \quad (1b)$$

with, $\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}, D = \frac{Eh^3}{12(1-v^2)}$

where $w(x,y,t)$ is the transverse deflection for the target plate, F is the Airy's stress function, m is the mass density of the plate, h refers to the plate thickness, p is the external transverse load per unit area, $M_{p(.)}$ is the piezo-actuator external moment per unit length, D is the flexural rigidity, E is the Young's modulus, and v is the Poisson's ratio. Using the Galerkin technique to expand w and F

$$w(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\psi_{mn}(x,y)q_{mn}(t) \quad (2a)$$

$$F(x,y,t) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\phi_{rs}(x,y)a_{rs}(t) \quad (2b)$$

Substituting (2) into (1), multiplying (1a) and (1b) by ψ_{ij} and ϕ_{tu} respectively and integrating the equations across the area of the plate to get

$$M_{mnij}\ddot{q}_{mn} + \sum_{m,n} q_{mn}K_{mnij} + p_{ij} - \sum_{r,s,m,n} a_{rs}q_{mn}L_{rsmnij} = \int_0^a\int_0^b \left(\frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2}\right)\psi_{ij}(x,y)dxdy \quad (3a)$$

$$\sum_{r,s} a_{rs}A_{rstu} = \sum_{m,n,k,l} q_{kl}q_{mn}B_{mnkltu} \quad (3b)$$

where $\sum_{i,j,k}(.) = \sum_i\sum_j\sum_k(.)$,

$M_{mnij} = \iint m\psi_{mn}\psi_{ij}dxdy = m_{ij}$ (Orthogonality property)

$M_{mnij} = \iint \nabla^4\psi_{mn}\psi_{ij}dxdy = k_{ij}$ (Orthogonality property)

$p_{ij} = \iint p\psi_{ij}dxdx$, $A_{rstu} = \iint \frac{\nabla^4\phi_{rs}}{Eh}\phi_{tu}dxdy$,

$$B_{mnkltu} = \iint\left(\frac{\partial^2\psi_{mn}}{\partial x\partial y}\frac{\partial^2\psi_{kl}}{\partial x\partial y} - \frac{\partial^2\psi_{mn}}{\partial x^2}\frac{\partial^2\psi_{kl}}{\partial y^2}\right)\phi_{tu}dxdy$$

$$L_{rsmnij} = \iint\left(\frac{\partial^2\phi_{rs}}{\partial y^2}\frac{\partial^2\psi_{mn}}{\partial x^2} + \frac{\partial^2\phi_{rs}}{\partial x^2}\frac{\partial^2\psi_{mn}}{\partial y^2} - 2\frac{\partial^2\phi_{rs}}{\partial x\partial y}\frac{\partial^2\psi_{mn}}{\partial x\partial y}\right)\psi_{ij}dxdy$$

Substituting (3b) into (3a) leads to

$$m_{ij}\ddot{q}_{ij} + k_{ij}q_{ij} + \sigma_{ij}(\phi_{(.)},\psi_{(.)},q_{(.)}q_{(.)}q_{(.)}) + p_{ij} = \int_0^a\int_0^b\left(\frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2}\right)\psi_{ij}(x,y)dxdy \quad (4)$$

Equation (4) includes a linearized stiffness term represented by the second term while the nonlinear restoring force is represented by σ_{ij}. Let us reformulate the right-hand term associated with piezo-moments assuming piezoelectric charge constants in x and y directions, thus

$$M_{px} = M_{py} = \zeta\big(H(x-x_1) - H(x-x_2)\big)\big(H(y-y_1) - H(y-y_2)\big)v_a(t) \quad (5)$$

where ζ is a constant, $H(.)$ is a Heaviside unit step function and $v_a(t)$ is the piezo-actuator voltage. Taking the second derivatives for M_{px} and M_{py} with further manipulations to obtain

$$\frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2} = \sum_{k=1}^{N_a}\alpha_k(x,y)v_{ak}(t) \quad (6)$$

with

$$\alpha_k(x,y) = \zeta_k\left[\left(\frac{\partial\delta(x-x_{1k})}{\partial x} - \frac{\partial\delta(x-x_{2k})}{\partial x}\right)\big(H(y-y_{1k}) - H(y-y_{2k})\big) + \left(\frac{\partial\delta(y-y_{1k})}{\partial x} - \frac{\partial\delta(y-y_{2k})}{\partial x}\right)\big(H(x-x_{1k}) - H(x-x_{2k})\big)\right]$$

and N_a refers to the number of piezo-actuators used that is assumed equal to the number of piezo-sensors (i.e., collocated piezo-transducers). As a result, the right-hand side of (4) becomes

$$\int_0^a\int_0^b\left(\frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2}\right)\psi_{ij}(x,y)dxdy = \sum_{k=1}^{N_a}\left(\int_0^a\int_0^b\alpha_k(x,y)\psi_{ij}(x,y)dxdy\right)v_{ak}(t) \quad (7a)$$

Using Dirac-Delta function property

$$\int_{-\infty}^{\infty} \frac{d^n \delta(x-x_0)}{dx^n} \theta(x) dx = (-1)^n \left. \frac{d^n(\theta(x))}{dx^n} \right|_{x=x_0} \tag{7b}$$

Then (7a) becomes

$$\sum_{k=1}^{N_a} \left(\int_0^a \int_0^b \alpha_k(x,y) \psi_{ij}(x,y) dx dy v_a(t) \right) = \sum_{k=1}^{N_a} \zeta_k \mu_{ijk}(x,y) v_{ak}(t) \tag{8}$$

where

$$\mu_{ijk} = \int_{y_{1k}}^{y_{2k}} \frac{\partial \psi_{ij}(x_{2k},y)}{\partial x} dy - \int_{y_{1k}}^{y_{2k}} \frac{\partial \psi_{ij}(x_{1k},y)}{\partial x} dy + \int_{x_{1k}}^{x_{2k}} \frac{\partial \psi_{ij}(x,y_{2k})}{\partial y} dx - \int_{x_{1k}}^{x_{2k}} \frac{\partial \psi_{ij}(x,y_{1k})}{\partial y} dx$$

Equation (4) can be re-written as

$$m_{ij}\ddot{q}_{ij} + k_{ij}q_{ij} + \sigma_{ij} + d_{ij} = u_{ij}, \qquad i = 1,2,\ldots,N \\ \text{and } j = 1,2,\ldots,M \tag{9}$$

where

$$d_{ij}(x,y,t) = \int_0^a \int_0^b p(x,y,t) \psi_{ij}(x,y) dx dy ,$$

$$u_{ij}(t) = \sum_{k=1}^{N_a} \zeta_k \mu_{ijk} v_{ak}(t)$$

The above equation can be represented in matrix form as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \boldsymbol{\sigma} + \mathbf{d} = \mathbf{u} \tag{10}$$

where

$$\mathbf{M} = \begin{bmatrix} m_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & m_{NM} \end{bmatrix}, \mathbf{K} = \begin{bmatrix} k_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k_{NM} \end{bmatrix}, \boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} \\ \vdots \\ \sigma_{NM} \end{bmatrix}, \mathbf{d} = \begin{bmatrix} d_{11} \\ \vdots \\ d_{NM} \end{bmatrix}, \mathbf{u} = \begin{bmatrix} u_{11} \\ \vdots \\ u_{NM} \end{bmatrix}$$

At this stage, it is suitable to consider the damping effect where a viscous damping term is used

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \boldsymbol{\sigma} + \mathbf{d} = \mathbf{u} \tag{11a}$$

with

$$\mathbf{C} = \begin{bmatrix} c_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_{NM} \end{bmatrix}$$

For control purpose, (11a) can be reformulated as

$$\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\eta}(\mathbf{q},\dot{\mathbf{q}},t) = \mathbf{u} \tag{11b}$$

where, $\boldsymbol{\eta}(\mathbf{q},\dot{\mathbf{q}},t) = \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \boldsymbol{\sigma} + \mathbf{d}$ (11c)

Remark 1. For detailed description of modal dynamics of a simply supported plate with cubic nonlinearities, it is recommended to follow [22,23].

Remark 2. For clarity, we will represent the dynamic matrices of (11) in terms of the number of the mode shapes (l). Therefore, the dimension of the dynamic matrices can be represented as

$$\mathbf{M} = \begin{bmatrix} m_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & m_l \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_l \end{bmatrix}, \mathbf{K} = \begin{bmatrix} k_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k_l \end{bmatrix}, \boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_l \end{bmatrix}, \mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_l \end{bmatrix}, \mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_l \end{bmatrix}, l = N \times M$$

III. Control Architecture

As aforementioned, the FBLC strategy attempts to design a nonlinear control law such that the closed-loop dynamics are linear, and hence the intuitive controller can be selected as [24,25]

$$\mathbf{u} = \hat{\mathbf{M}}\left[\ddot{\mathbf{q}}_d - \mathbf{K}_d(\dot{\mathbf{q}} - \dot{\mathbf{q}}_d) - \mathbf{K}_p(\mathbf{q} - \mathbf{q}_d)\right] + \hat{\boldsymbol{\eta}} - \boldsymbol{\kappa}\,\mathrm{sgn}(\mathbf{B}^T\mathbf{P}^T\mathbf{x}) \tag{12a}$$

where the symbol $(\hat{\cdot})$ refers to the estimation, $\mathbf{q}_d$ is the desired value for the modal vector, $\mathbf{K}_d \in R^{l \times l}$ and $\mathbf{K}_b \in R^{l \times l}$ are both diagonal positive definite matrices, $\boldsymbol{\kappa} \in R^{l \times l}$ is a robust sliding gain, $\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_l \end{bmatrix} \in R^{2l \times l}$,

$\mathbf{x} = \begin{bmatrix} \mathbf{e}^T & \dot{\mathbf{e}}^T \end{bmatrix}^T \in R^{2l}$, $\mathbf{P} = \mathbf{P}^T \in R^{2l \times 2l}$ is a symmetric positive definite matrix satisfying the Lyapunov equation

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q} \tag{12b}$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_l \\ -\mathbf{K}_p & -\mathbf{K}_d \end{bmatrix} \in R^{2l \times 2l}$$

and $\mathbf{Q} = \mathbf{Q}^T \in R^{2l \times 2l}$ is also a symmetric positive definite matrix. Substituting (12a) into (11) results in the following closed-loop dynamics

$$\ddot{\mathbf{e}} + \mathbf{K}_d\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e} + \boldsymbol{\kappa}\,\mathrm{sgn}(\mathbf{B}^T\mathbf{P}^T\mathbf{x}) = -\hat{\mathbf{M}}^{-1}(\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\boldsymbol{\eta}}) + \boldsymbol{\varepsilon}, \mathbf{e} = \mathbf{q} - \mathbf{q}_d \tag{13}$$

where $(\tilde{\cdot}) = (.) - (\hat{\cdot})$ and $\boldsymbol{\varepsilon} \in R^l$ represents the modelling/approximation error. Equation (13) is basically a linear closed-loop dynamics (if it is without the robust sliding term), however, due to the presence of the robust term $\boldsymbol{\kappa}\,\mathrm{sgn}(\mathbf{B}^T\mathbf{P}^T\mathbf{x})$ the system is no longer linear. Using the FAT, the mass and nonlinear matrices/vectors can be represented as

$$\mathbf{M} = \mathbf{W}_M^T \varphi_M + \boldsymbol{\varepsilon}_M \tag{14a}$$

$$\boldsymbol{\eta} = \mathbf{W}_\eta^T \varphi_\eta + \boldsymbol{\varepsilon}_\eta \tag{14b}$$

where $\mathbf{W}_M \in R^{l\beta \times l}$, and $\mathbf{W}_\eta \in R^{l\beta \times l}$ are weighting matrices, $\varphi_M \in R^{l\beta \times l}$ and $\varphi_\eta \in R^{l\beta}$ are matrices of basis function, with β referring to the number of basis function terms. Using the same set of basis functions, the corresponding estimates can be represented as

$$\hat{\mathbf{M}} = \hat{\mathbf{W}}_M^T \varphi_M \tag{15a}$$

$$\hat{\boldsymbol{\eta}} = \hat{\mathbf{W}}_\eta^T \varphi_\eta \tag{15b}$$

Thus, the controller (12a) becomes

$$\mathbf{u} = \hat{\mathbf{W}}_M^T \varphi_M \left[\ddot{\mathbf{q}}_d - \mathbf{K}_d(\dot{\mathbf{q}} - \dot{\mathbf{q}}_d) - \mathbf{K}_p(\mathbf{q} - \mathbf{q}_d) + \hat{\mathbf{W}}_\eta^T \varphi_\eta - \boldsymbol{\kappa}\,\mathrm{sgn}(\mathbf{B}^T\mathbf{P}^T\mathbf{x})\right] \tag{16}$$

And the closed-loop dynamics (13) are

$$\ddot{\mathbf{e}}+\mathbf{K}_d\dot{\mathbf{e}}+\mathbf{K}_p\mathbf{e}+\boldsymbol{\kappa}\operatorname{sgn}(\mathbf{B}^T\mathbf{P}^T\mathbf{x})=-\hat{\mathbf{M}}^{-1}(\tilde{\mathbf{W}}_M^T\varphi_M\ddot{\mathbf{q}}+\tilde{\mathbf{W}}_\eta^T\varphi_\eta)+\boldsymbol{\varepsilon} \quad (17)$$

Representing (17) in a state space form

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}-\mathbf{B}\left[\hat{\mathbf{M}}^{-1}\left(\tilde{\mathbf{W}}_M^T\varphi_M\ddot{\mathbf{q}}+\tilde{\mathbf{W}}_\eta^T\varphi_\eta\right)-\boldsymbol{\varepsilon}+\boldsymbol{\kappa}\operatorname{sgn}(\mathbf{B}^T\mathbf{P}^T\mathbf{x})\right] \quad (18)$$

Let us select the following update laws

$$\dot{\hat{\mathbf{W}}}_M=-\boldsymbol{\Phi}_M\varphi_M\ddot{\mathbf{q}}(\mathbf{x}^T\mathbf{P}\mathbf{B}\hat{\mathbf{M}}^{-1}) \quad (19a)$$

$$\dot{\hat{\mathbf{W}}}_\eta=-\boldsymbol{\Phi}_\eta\varphi_\eta(\mathbf{x}^T\mathbf{P}\mathbf{B}\hat{\mathbf{M}}^{-1}) \quad (19b)$$

where $\boldsymbol{\Phi}_{(.)}\in R^{l\beta\times l\beta}$ is the adaptation matrix.

Theorem 1. The dynamics of the vibrating plate modelled in (11) with the control law, closed-loop dynamics and the associated updating laws described in (16)-(19) are stable in the sense of the Lyapunov theory.

Proof.

Let us select the following Lyapunov-like function along the closed-loop dynamics (18)

$$V=\frac{1}{2}\mathbf{x}^T\mathbf{P}\mathbf{x}+\frac{1}{2}tr\left(\tilde{\mathbf{W}}_M^T\boldsymbol{\Phi}_M^{-1}\tilde{\mathbf{W}}_M+\tilde{\mathbf{W}}_\eta^T\boldsymbol{\Phi}_\eta^{-1}\tilde{\mathbf{W}}_\eta\right) \quad (20)$$

Taking the time-derivative of (20) and substituting (18) leads to

$$\dot{V}=-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}-\mathbf{x}^T\mathbf{P}\left[\mathbf{B}\hat{\mathbf{M}}^{-1}\left(\tilde{\mathbf{W}}_M^T\varphi_M\ddot{\mathbf{q}}+\tilde{\mathbf{W}}_\eta^T\varphi_\eta\right)-\mathbf{B}\boldsymbol{\varepsilon}+\mathbf{B}\boldsymbol{\kappa}\operatorname{sgn}(\mathbf{B}^T\mathbf{P}^T\mathbf{x})\right]- tr\left(\tilde{\mathbf{W}}_M^T\boldsymbol{\Phi}_M^{-1}\dot{\hat{\mathbf{W}}}_M\right)-tr\left(\tilde{\mathbf{W}}_\eta^T\boldsymbol{\Phi}_\eta^{-1}\dot{\hat{\mathbf{W}}}_\eta\right) \quad (21)$$

Equation (21) can be re-written as

$$\dot{V}=-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}-\mathbf{x}^T\mathbf{P}\left[\mathbf{B}\hat{\mathbf{M}}^{-1}\left(\tilde{\mathbf{W}}_M^T\varphi_M\ddot{\mathbf{q}}+\tilde{\mathbf{W}}_\eta^T\varphi_\eta\right)-\mathbf{B}\boldsymbol{\varepsilon}+\mathbf{B}\boldsymbol{\kappa}\operatorname{sgn}(\mathbf{B}^T\mathbf{P}^T\mathbf{x})\right]- tr\left(\tilde{\mathbf{W}}_M^T\boldsymbol{\Phi}_M^{-1}\dot{\hat{\mathbf{W}}}_M\right)-tr\left(\tilde{\mathbf{W}}_\eta^T\boldsymbol{\Phi}_\eta^{-1}\dot{\hat{\mathbf{W}}}_\eta\right) \quad (22)$$

Substituting (19) into above equation to get

$$\dot{V}=-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}-\mathbf{x}^T\mathbf{P}\mathbf{B}(-\boldsymbol{\varepsilon}+\boldsymbol{\kappa}\operatorname{sgn}(\mathbf{B}^T\mathbf{P}^T\mathbf{x}))=-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}+\boldsymbol{\chi}^T\boldsymbol{\varepsilon}-\sum_i\kappa_i|\chi_i| \quad (23)$$

where $\boldsymbol{\chi}=\mathbf{B}^T\mathbf{P}^T\mathbf{x}$. Selecting the components κ_i such that

$$\kappa_i\geq|\varepsilon_i|+\delta_i \quad (24)$$

where δ_i is a positive constant and hence (23) becomes

$$\dot{V}=-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}-\sum_1^l\delta_i|\chi_i|) \quad (25)$$

Equation (25) is stable in the sense of Lyapunov theory.

Remark 3. A good way for selecting the positive definite matrices **P** and **Q** in (12b) is choosing **Q** first then solving (12b) to determine **P** for a given **A**. If the produced **P** is positive definite, then the system stability is ensured.

IV. Results and Discussions

In this section, a simply supported beam with collocated 2-piezo-patches is simulated using MATLAB/SIMULINK package. To motivate the vibration of the target plate, an impulse concentrated force of (5 N pulse) with period of (10 s) and pulse width of (1 s) is applied at the geometric center of the plate surface. For control purpose, the first two mode shapes are considered for experimental implementation since the high modal amplitudes occur at the low region of frequency response. Recalling Eq. (11a), the plate dynamics include a nonlinear cubic stiffness term that makes dynamic response complicated. The plate dynamics are similar to Duffing's equation such that the nonlinear restoring force is a function to the modal displacement. In addition, jump phenomenon occurs clearly in the frequency response of the nonlinear plate structure, see [26] for more details. Table I shows the physical parameters and control gains used in simulation experiments.

TABLE I. Physical Parameters and Feedback Gains Used in Simulation Experiments

Plate	$a=400mm,\ b=350mm,\ h=3mm,$ $E=210GPa,\ \rho=7800\,kg/m^3,\ \nu=0.3$, The damping constants for the first two modes are selected as $c_1=0.0068, c_2=0.028.$
Piezo-materials*	$a_p=25mm,\ b_p=25mm,\ h_p=0.25mm,$ $E=6\times10^{10}\,N/m^2$.
Feedback gains**	$\mathbf{Q}=\mathbf{I}_{2l\times2l},\ \mathbf{K}_p=200\mathbf{I}_{l\times l},\ \mathbf{K}_d=50\mathbf{I}_{l\times l},$ $\boldsymbol{\Phi}_\eta=20\mathbf{I}_{l\beta\times l\beta},\ \boldsymbol{\kappa}=\mathbf{0},\ l=2,\ \beta=15.$

* For determining ζ, the reader can follow the work of [7].

** For finding the positive definite matrix **P**, please recall Remark 3.

For control implementation, FAT is used as a basis for control formulation with orthogonal Chebyschev polynomials as approximators. Fifteen terms of basis functions are used in approximation schemes. The inverse mass matrix is assumed known and this can ease the task of controller. The cubic nonlinearities are estimated using the FAT scheme. The modelling error is neglected in simulation experiments, so the sliding term is also neglected. Fig. 2 and 3 show the response of modal amplitudes and input voltages respectively. The results show that the proposed controller can regulate the target plate system despite of the accompanied uncertainty. It should be noted that the number of input voltages is equal to the number of mode shapes. If the number of the mode shapes is not equal to the number of input voltages, then Pseudo-inverse matrix should be used to determine the input voltages for piezo-actuators.

V. Conclusions

This work designs FAT-based FBLC for nonlinear thin plate structures with piezo-patches. The FBLC is a powerful tool for control and regulation of nonlinear dynamic systems. However, FAT plays important role in compensating for the highly coupled cubic nonlinearities resulted from axial stretching of the vibrating plate. One of the limitations of the FBLC is its requirement for calculation of inverse mass matrix and therefore it is assumed known to resolve the computational problems. Future work is required to deal with the following points:

1. A comprehensive study to compare between the nonlinear control strategies for regulation of motion of plate-like structures.
2. Extend the work to include vibration of shell structures with fluid interactions.
3. Spill-over should be dealt carefully in the control structure, which is lost in the current work.

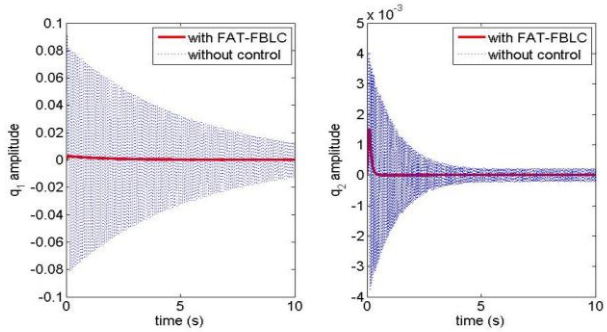

Figure 2. Modal amplitude response.

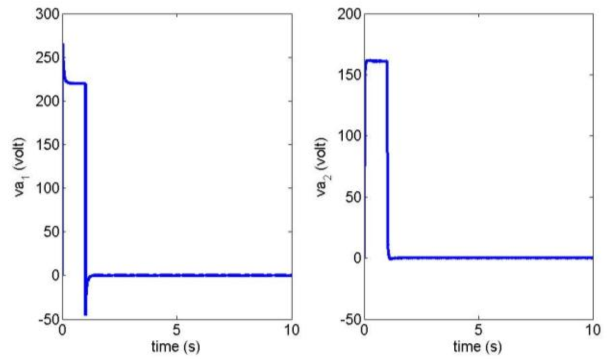

Figure 3. Input control piezo-voltages.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Conceptualization, methodology, resources and investigation were performed by A.H. Kaleel and H.F.N. Al-Shuka. Paper writing was conducted by O.H. Hussein

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