Design of Adaptive Fuzzy Sliding Mode Controller for Mobile Robot

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***Abstract*—A Wheeled Mobile Robot (WMR) system is one of the well-known non-holonomic systems. In this paper, an Adaptive Fuzzy Sliding Mode Controller (AFSMC) is proposed for trajectory tracking control of a non-holonomic system, which the centroid doesn't coincide to the connection center of driving wheels. First, a Sliding Mode Controller (SMC) is proposed to the convergence of WMR on the desired position, velocity and orientation trajectories. However, the SMC is still fluctuation around trajectory tracking also the system response time is slow. So the second fuzzy logic controller (FLC) is combined with SMC to improve quality of control WMR for quick response time. The results of Matlab/Simulink demonstrated the efficiency of the AFSMC proposed good working.**

***Index Terms*—sliding mode control, adaptive control, trajectory tracking, wheeled mobile robots, autonomous**

I. INTRODUCTION

Mobile robots have been used in many applications and areas such as industrial, transportation, inspection and other fields. WMRs are considered as the most widely used class of mobile robots. Mobile robots are complex and combine many technologies such as sensors, controller design, electronic components. In the process of design and development of a mobile robot, the controller is an important role because of the working ability of a mobile robot based on the controller. The possible motion tasks can be classified as follows: point to point motion, path following motion but stability and trajectory tracking control is the key issue in the control also interested in many research.

Many studies are conducted to develop an adaptive controller to control the motion of a mobile robot with disturbances and uncertainties [1], [2]. The global trajectory tracking problem has been discussed based on back-stepping [3]-[5], proposed method to controller WMR by kinematic controller is designed first so that tracking error between a real robot and a reference robot converges to zero, and secondly a torque controller is designed by using back-stepping so that the velocities of a mobile robot converge to desired velocities which are given by the kinematic controller designed at first step. The fuzzy logic controller has been found to be the most attractive. The theory of fuzzy logic systems is inspired by the remarkable human capability to operate on and reason with perception-based information. Fuzzy logic approach to control problems mimics how a person would make a decision. The main advantages of a fuzzy navigation strategy lie in the ability to extract heuristic rules from human experience. The hallmark [6] investigates the application of an adaptive neuro-fuzzy inference system (ANFIS) to path generation and obstacle avoidance for an autonomous mobile robot in a real-world environment [7]. A control structure that makes possible the integration of a kinematic controller and an adaptive fuzzy controller for trajectory tracking is developed for non-holonomic mobile robots. Sliding mode control is widely utilized to control robotics system also SMC has been considered recently to improve the performance of the nonlinear controller [8]. However, the chattering phenomenon caused by the Sign function leads to fluctuation in object high – frequency dynamics. Moreover, selecting a large value of the switching gain in the sliding mode control to ensure the effectiveness of the control system could cause severe solicitation in the control inputs and increase the chattering phenomenon.

In this paper, we proposed FLC combine with SMC to improve quality control of WMR which created error trajectory tracking by SMC. The paper is organized as follows: the modeling of WMB is shown in Section II. Section III presents SMC and AFSMC. The simulation results are indicated in Section IV. The conclusion is address in Section V.

II. STRUCTURE AND MODELING OF THE MOBILE ROBOT

A. Kinematic Model

The considered mobile robot is composed of two similar driving wheels mounted on a bar and independently controlled by two actuators, as indicated in Fig. 1. Where *R* is the wheel radius (*m*), v_R and v_L are right and left drive wheel velocities respectively (*m/s*), (*x*, *y*) present mobile robot that is defined to mid-point *A*, on the axis between the wheels, center of axis of wheels in world Cartesian coordinates in (*m*) and *2L* is axle length between the drive wheel (*m*). The orientation of the mobile robot is given by the angle $\theta\,(rad)$ between the instant linear velocities of the mobile robot body. The center of mass *C* of the robot is assumed to be on the axis of symmetry at a distance *d* from the origin *A*. The position vector state of the robot is defined as $q = (x, y, \theta)$. The linear and the angular velocities of the mobile robot

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are expressed by (1) and the kinematic model equation of the mobile robot is given by (2) as follows [9].

$$v = \frac{v_R + v_L}{2} = R\frac{(\dot{\varphi}_R + \dot{\varphi}_L)}{2}$$
$$\omega = \frac{v_R - v_L}{2L} = R\frac{(\dot{\varphi}_R - \dot{\varphi}_L)}{2} \quad (1)$$

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (2)$$

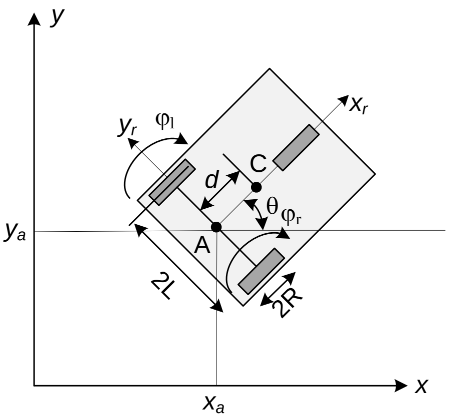

Figure 1. Structure of WMR

B. Dynamic Modeling of the WMR

Consider the following non-holonomic mobile robot that is subject to *m* constraints [10]

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - \Lambda^T(q)^{\lambda} \quad (3)$$

where q $\in R^n$ is generalized coordinates, $\tau \in R^n$ is the input vector, $\lambda \in R^m$ is the vector of constraint forces, M(q) $\in R^{nxn}$ is asymmetric and positive-definite inertia matrix, V(q, q') $\in R^{nxn}$ is the centripetal and Coriolis matrix, F(q') $\in R^n$ is the surface friction matrix, G(q)$\in R^n$ is the gravitational vector, B(q) $\in R^{rxn}$ is the input transformation matrix, $\tau_d \in R^r$ is the vector of bounded unknown disturbances including unstructured un-modeled dynamics, and Λ(q) $\in R^{mxn}$ is the matrix associated with the constraints.[9] the equation (3) can be represented as follows:

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} = B(q)\tau - \Lambda^T(q)^{\lambda} \quad (4)$$

where:

$$M(q) = \begin{bmatrix} m & 0 & -md\sin\theta & 0 & 0 \\ 0 & m & md\cos\theta & 0 & 0 \\ -md\sin\theta & md\cos\theta & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_w \end{bmatrix}$$

$$V(q,\dot{q}) = \begin{bmatrix} 0 & -md\dot{\theta}\cos\theta & 0 & 0 & 0 \\ 0 & -md\dot{\theta}\sin\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Lambda^T(q) = \begin{bmatrix} -\sin\theta & \cos\theta & \cos\theta \\ \cos\theta & \sin\theta & \sin\theta \\ 0 & L & -L \\ 0 & -R & 0 \\ 0 & 0 & -R \end{bmatrix} x \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{bmatrix}$$

Next, the system described by (4) is transformed into an alternative form which is more convenient for the purpose of control and simulation. The main aim is to eliminate the constraint term $\Lambda^T(q)\lambda$ in equation (4) since the Lagrange multipliers λ_i are unknown. This is done first by defining the reduce vector:

$$\dot{\eta} = \begin{bmatrix} \dot{\varphi}_R \\ \dot{\varphi}_L \end{bmatrix} \quad (5)$$

Next, by expressing the generalized coordinates velocities using the forward kinematic model we have

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi}_R \\ \dot{\varphi}_L \end{bmatrix} = \frac{1}{2} \begin{bmatrix} R\cos\theta & R\cos\theta \\ R\sin\theta & R\sin\theta \\ \frac{R}{L} & -\frac{R}{L} \\ 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \dot{\varphi}_R \\ \dot{\varphi}_L \end{bmatrix} \quad (6)$$

This can be written in the form:

$$\dot{q} = S(q)\eta \quad (7)$$

It can be verified that the transformation matrix S(q) is in the null space of the constraint matrix Λ(q). Therefore we have:

$$S^T(q)\Lambda^T(q) = 0 \quad (8)$$

The derivative of (7) we have:

$$\ddot{q} = S(q)\eta + S(q)\dot{\eta} \quad (9)$$

Substituting equation (7) and (9) in the main equation (4) we obtain:

$$M(q)[\dot{S}(q)\eta + S(q)\dot{\eta}] + V(q,\dot{q})[S(q)\eta] = B(q)\tau - \Lambda^T(q)\lambda \quad (10)$$

Rearranging the equation and multiplying both sides by lead to:

$$S^T(q)M(q)S(q)\dot{\eta} + S^T(q)[M(q)\dot{S}(q) + V(q,\dot{q})S(q)]\eta = S^T(q)B(q)\tau - S^T(q)\Lambda^T(q)\lambda \quad (11)$$

Now the new matrix is defined as follows:

$$\bar{M}(q) = S^T(q)M(q)S(q)$$

$$\bar{V} = S^T(q)M(q)\dot{S}(q) + S^T(q)V(q,\dot{q})S(q)$$

$$\bar{B} = S^T(q)B(q)$$

The dynamics equations are reduced form:

$$\bar{M}(q)\dot{\eta} + \bar{V}(q,\dot{q})\eta = \bar{B}(q)\tau \quad (12)$$

where:

$$\bar{M}(q) = \begin{bmatrix} I_w + \frac{R^2}{4L^2}(mL^2+I) & \frac{R^2}{4L^2}(mL^2-I) \\ I_w + \frac{R^2}{4L^2}(mL^2+I) & I_w + \frac{R^2}{4L^2}(mL^2+I) \end{bmatrix}$$

$$\bar{V}(q,\dot{q}) = \begin{bmatrix} 0 & \frac{R^2}{2L}m_c d\dot{\theta} \\ -\frac{R^2}{2L}m_c d\dot{\theta} & 0 \end{bmatrix},$$

$$\bar{B}(q) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equation (12) shows that dynamics problem of WMR is express only as a function of angular velocities (φ'_R, φ'_L) or the right and left wheels, the robot angular velocity θ' and the driving motor torques (τ_R, τ_L). The equations of (12) can be also transformed into an alternative form which is represented by linear and angular velocities *(v, w)* of WMR. Using kinematic model equation (1), it showed that equations (12) can be rearranged as follows:

$$\begin{cases} \left(m + \frac{2I_w}{R^2}\right)\dot{v} - m_c d\omega^2 = \frac{1}{R}(\tau_R + \tau_L) \\ \left(I + \frac{2L^2 I_w}{R^2}\right)\dot{\omega} - m_c d\omega v = \frac{L}{R}(\tau_R - \tau_L) \end{cases} \quad (13)$$

III. Design of Controller

A. Sliding Mode Controller

1) Problem statement

The purpose of the SMC is the design of control inputs $\tau = [\tau_R, \tau_L]^T$. which make the WMR track a feasible trajectory with bounded posture errors. The three-dimensional posture variables of the reference trajectory are defined as $q_r = [X_r, Y_r, \theta_r]$ the reference velocity and acceleration vectors are derived from q_r as $z_r = [v_r, \omega_r,]^T$ and $z'_r = [v'_r, \theta''_r,]^T$ respectively. The real world robotic systems have inherent system disturbances such as parameter uncertainties, friction, etc.

Therefore, the real dynamic equation of the WMR is considered as:

$$\bar{M}\dot{z} + \bar{V}(q,z) + \tau_d = \tau \quad (14)$$

where, the disturbance vectors, $\tau_d = [\tau_{d1}, \tau_{d2}]^T$ is defined to include the disturbance effects in the dynamical equations.

It is assumed that τ_d is bounded and satisfies the uncertainty matching condition as:

$$\tau_d = \bar{M} \times p$$

$$p = [p_1, p_2]^T, \ |p_1| \le p_{1m}, |p_2| \le p_{2m}$$

p_{1m} and p_{2m} are upper bounds of the perturbations.

2) Controller design

First, the position and orientation errors are considered as:

$$X_e = X_c - X_r$$
$$Y_e = Y_c - Y_r$$
$$\theta_e = \theta_c - \theta_r$$

To stabilize the tracking errors, the sliding surfaces are defined as [11]:

$$s_1 = \dot{X}_e + K_1 X_e \quad (15)$$

$$s_2 = \dot{Y}_e + K_2 Y_e \quad (16)$$

where K_1 and K_2 are positive constant parameters. If s_1 is asymptotically stable, X_e and X'_e converge to zero asymptotically. Because if $s_1 = 0$ then $X'_e + K_1 X_e = 0$. Therefore, if $X'_e \le 0$ then $X_e \ge 0$ and if $X'_e \ge 0$ then $X_e \le 0$. Therefore, the equilibrium state X_e X'_e is asymptotically stable. Similarly, if s_2 is asymptotically stable, Y_e and Y'_e asymptotically converge to zero. Thus, if s_1 and s_2 are stabilized, the convergence of WMR to the reference trajectory is guaranteed.

As a feedback linearization of the system, the control inputs are defined by the computed-torque method as follows [12].

$$\tau = \bar{M}\dot{z}_r + \bar{V}(q,z) + \bar{M}u_{smc} \quad (17)$$

Here $u_{smc} = [u_1, u_2]^T$ is the control law which determines error dynamics. Applying the control input (17) in the dynamic equation of WRM (14), the feedback-linearized dynamic equation is given as:

$$\dot{z} + p = \dot{z}_r + u_{smc} \quad (18)$$

Thus from (18), we have

$$\dot{v}_c + p_{1m} = \dot{v}_r + u_1 \quad (19)$$

$$\ddot{\theta}_c + p_{2m} = \ddot{\theta}_r + u_2 \tag{20}$$

The control law u_1 and u_2 which stabilizes the sliding surface s_1 and s_2 are proposed as follows.

$$u_1 = -D_1 sign(s_1) - K_1\dot{X}_e - \ddot{X}_e + \dot{v}_c - \dot{v}_r - E_1 X_e sign(s_1 X_e)$$
$$u_2 = -D_2 sign(s_2) - K_2\dot{Y}_e - \ddot{Y}_e + \ddot{\theta}_c - \ddot{\theta}_r - E_2 Y_e sign(s_2 Y_e)$$

where D_1, D_2 are bigger than p_{1m}, p_{2m} respectively and K_1, K_2, E_1, E_2 are real positive constant values. To prove the stability of s_1 and s_2 when u_1 and u_2 are applied, the Lyapunov direct method is used. Substituting u_1 and u_2 respectively in (19) and (20), X''_e and Y''_e are derived as:

$$\ddot{X}_e = -D_1 sign(s_1) - p_{1m} - K_1\dot{X}_e - E_1 X_e sign(s_1 X_e) \tag{21}$$

$$\ddot{Y}_e = -D_2 sign(s_2) - p_{2m} - K_2\dot{Y}_e - E_2 Y_e sign(s_2 Y_e) \tag{22}$$

According to Lyapunov's direct method, the following Lyapunov function is applied [11]:

$$V = \frac{1}{2}(s_1^2 + s_2^2) \tag{23}$$

The time derivative of V along the trajectory is given by:

$$\dot{V} = s_1\dot{s}_1 + s_2\dot{s} = s_1(\ddot{X}_e + K_1\dot{X}_e) + s_2(\ddot{Y}_e + K_2\dot{Y}_e) \tag{24}$$

Replacing X''_e and Y''_e from (21) and (22) in (24) result in the following negative definite function :

$$\dot{V} = -(D_1|s_1| + p_{1m}s_1 + E_1|s_1 X_e|) - (D_2|s_2| + p_{2m}s_2 + E_2|s_2 Y_e|)$$

Therefore s_1 and s_2 are asymptotically stable and the WMR converges to the reference trajectory. However, the WMR has slowly response time also fluctuation around orientation which causes by sign function.

B. Fuzzy Sliding Mode Controller

As address above the SMC has slow response time, so we propose Sugeno type fuzzy to adapt to change errors and derivate errors of WMR, the control following form:

$$u_r = -K_f u_f \tag{25}$$

where K_f is the normalizing factor. The output fuzzy variables u_f are continuously adjusted using an if-then rule base with respect to both e and e'. The configuration of AFSMC is shown in Fig. 2.

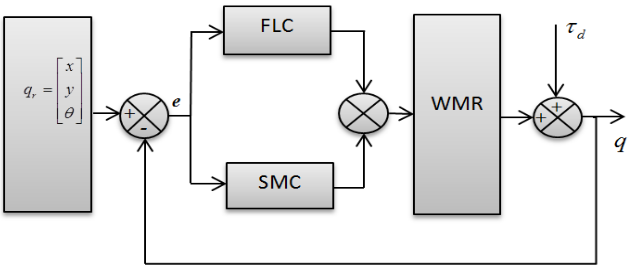

Figure 2. Structure controller of AFSMC for control WMR

By combining FLC with SMC we have FSMC law can be developed as:

$$u_{fsmc} = u_{smc} + u_r \tag{26}$$

The membership functions corresponding to the input and output fuzzy sets of e, e' and u_f are present in Fig. 3.

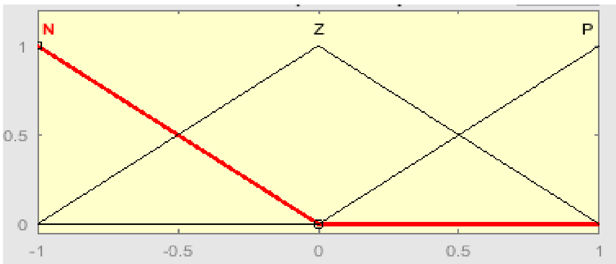

Figure 3. Membership function of inputs e, $e'$and output u_f.

In this figure, P, N, and Z stand for positive, negative and zero, respectively. Corresponding to the three membership functions for each input variable, 9 if-then rules of Table I are obtained using the expert engineering knowledge and experiences in the field of WMR.

TABLE I. RULE BASE OF FSMC

e \ $\dot{e}$	**N**	**Z**	**P**
N	P	P	Z
Z	P	Z	P
P	Z	P	P

IV. SIMULATION RESULTS

The effects of proposed methods to improve the convergence of WMR to reference position and orientation trajectories are calculated by Matlab/Simulink to implement the control structure shown in Fig. 2 using the control law given by equations (18) and (26). In all simulations, the robot starts at position (0.2, 0.1, 0) (m) and should follow a circular trajectory of reference. The center of the reference circle is at x= 0.0 (m) and y = 0.0 (m) and follows a circle having a radius of 0.15 (m) and τ_d disturbance is 0.

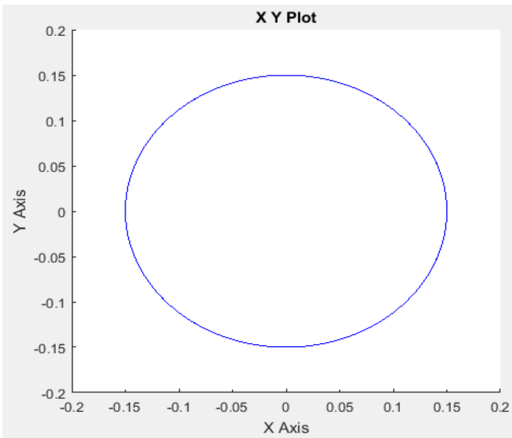

Figure 4. Reference trajectory for SMC and AFSMC

Fig. 4 indicates a reference trajectory for controller AFSMC and SMC. Fig. 5 and Fig. 6 showed the response time of AFSMC quickly and smooth than SMC follows a circle reference at approximate time 0–11*s*. Similar, the Fig. 7 and Fig. 8 demonstrated AFSMC response time better SMC for along X and Y axis.

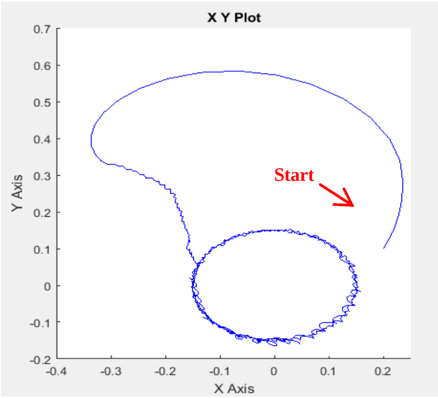

Figure 5. The actual trajectory for SMC

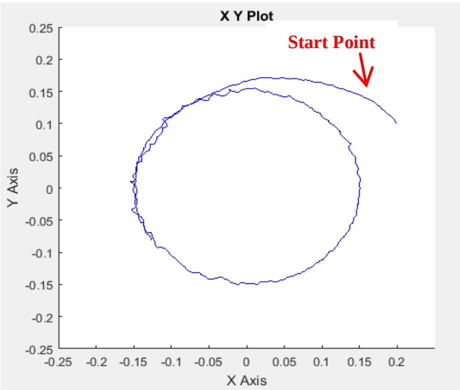

Figure 6. The actual trajectory for AFSMC

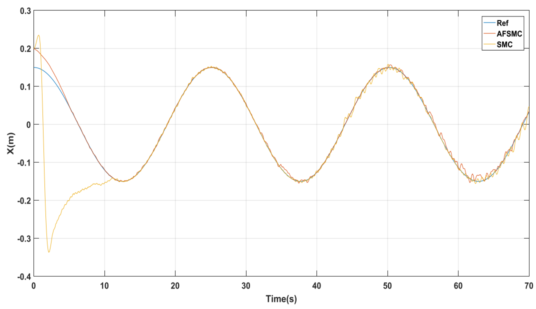

Figure 7. Trajectory tracking of WMR along the X axis

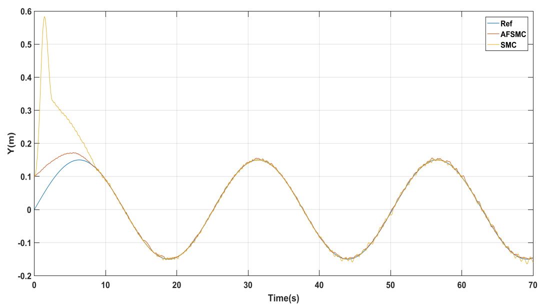

Figure 8. Trajectory tracking of WMR along the Y axis

V. Conclusion

In this paper, we proposed AFSMC based on robust SMC for control trajectory tracking WMR by adapting changing *e*, *e'*. The simulation results showed AFSMC for quick response time also stability.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Nguyen Truong Thinh, Nguyen Dao Xuan Hai, contributed to the analysis and implementation of the research, to the analysis of the results and to the writing of the manuscript. All authors discussed the results and contributed to the final manuscript. Besides, Nguyen Truong Thinh conceived the study and were in charge of overall direction and planning. Nguyen Truong Thinh is a corresponding author.

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