

A Maintenance Cost Optimization Approach: Application on a Mechanical Bearing System

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Abstract—In order to remain highly competitive, industrial companies found their business strategies on the quality and the cost of the product/service they deliver to their clients. Therefore, it is crucial for them to guarantee the availability and reliability of their industrial equipment through maintenance. However, while applying maintenance, industrials face a major issue: what is the optimal maintenance strategy to adopt in order to minimize the total cost of maintenance while maintaining an acceptable level of system availability?

In this paper, we answer this question by proposing an optimization approach that takes in consideration the various costs related to maintenance and integrates them in a global cost function to minimize. A critical threshold of the remaining useful life under which the system should be replaced is identified, as well as an inspection step giving the regularity with which the system should be inspected.

We then illustrate the approach with an example: a mechanical bearing system of a train motor subject to degradation and to monitoring. This example has allowed us to determine the remaining useful life threshold as well as the number of inspections that minimize the total cost of maintenance.

Index Terms— remaining useful life, weibull distribution, cost optimization, predictive maintenance, rolling bearing system.

I. INTRODUCTION

The European norm EN-13306 defines maintenance as the combination of all technical, administrative, and managerial actions during the life cycle of the system aiming to retain it, or restore it to a state in which it can perform its required function [1]. The maintenance can be corrective when it is performed after detection of failure in the system [1]. It can be preventive when it is performed at predetermined intervals intending to reduce the probability of failure or degradation of the system [1].

While corrective maintenance may imply high costs and significant system downtime [2][3][4], preventive maintenance, in the other hand, does not allow an optimal

exploitation of the system [5]. Therefore, predictive maintenance has emerged as a solution to overcome the drawbacks of corrective and preventive maintenance. Predictive maintenance is based on a regular monitoring of the system to evaluate its health state. It is usually carried out following a forecast derived from repeated analysis and evaluation of significant parameters describing the degree of degradation of the system [1][6]. One of the measures of the health state of the system is the remaining useful life "*RUL*". The "*RUL*" is defined as the expected length of time left for the system before it falls down [7].

Thanks to predictive maintenance, industrials are now able to estimate the *RUL* of the system-as one among other measures used to predict the failure time of the system-and maintain the system before it falls down [8]. One of the main challenges that industrials face nowadays is finding the optimal time to perform predictive maintenance.

The literature review provides a wide range of optimal maintenance strategies for improving system reliability, preventing system failures and reducing maintenance costs [9]. A cost model was developed in [10] taking into account finite repair, maintenance durations and costs due to testing, repair, maintenance and lost production or accidents. The objective of the maintenance optimization is to minimize the total cost rate by proper selection of two intervals: one for inspections and one for replacements [10]. In [11], the case of predictive maintenance for systems exhibiting 2-phase behavior: the phase of new condition and the phase of worn condition, was analyzed and cost-minimizing policies were developed in order to determine when monitoring should take place. A sequential imperfect preventive maintenance policy was developed in [12][13], and the optimal preventive maintenance schedule that minimizes the cost rate in the life cycle of the system or in the long run was determined in [13]. A dynamic predictive maintenance policy for complex multi-component systems was developed in [14], in order to minimize the long-term mean maintenance cost per unit time. Finally, a recent work on predictive maintenance decision-making

method based on cyber manufacturing and mission reliability state was developed in [15].

This paper tackles another aspect of maintenance cost optimization: first, we consider in this work a system subject to degradation and monitored regularly and perfectly. A predictive replacement of the system is performed once the RUL of the system is under some threshold called RUL_{lim} . A cost optimization approach is developed and tested on the mechanical bearing system to determine the threshold RUL_{lim} under which we preconize a predictive replacement of the system.

II. METHODOLOGY DESCRIPTION

The methodology of maintenance cost optimization described in this paper was already developed and published in a previous research work [8]. In this section, we give a brief overview of the methodology: section A describes the assumptions on which our work is based and section B describes the cost optimization model and the steps to follow to identify the optimal strategy for maintenance.

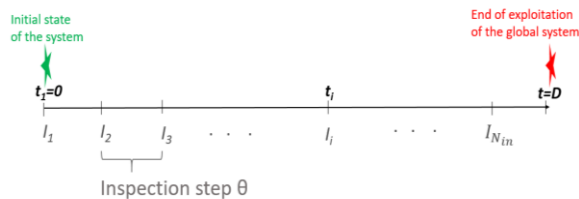
A. Assumptions

We adopted the following assumptions to develop the methodology of maintenance cost optimization described in this paper:

- The system under study is a single component.
- The system under study is part of a whole complex system, which has a duration of exploitation known beforehand, noted D .
- A perfectly reliable inspection is applied regularly on the system ("Fig.1"). The inspection gives an information on the state of health of the system. For instance, the inspection gives a real estimation of the RUL of the system. After simulations, the RUL is the expected interval of time the system is likely to operate before it requires replacement. The RUL of the system can be expressed by the following equation:

$$RUL(t) = E[T - t | T > t] = \frac{\int_t^\infty (u-t) \cdot f(u) \cdot du}{S(t)}. \quad (1)$$

With T the time of failure of the system, f the failure density function of the system and S the survival function of the system.



Key words :

I_i : inspection $n^{\circ}i$ ($i=1 \dots N_{in}$)

N_{in} : total number of inspections

D : Duration of exploitation of the global system

Figure 1. Inspection procedure.

- The inspection does not affect the system's performance.
- A first inspection is required in the early life of the system, but the health of the system is supposed not to require replacement because it is a new one ("Fig.1"). Once the system attains D , there is no use to perform inspection and the system can be replaced by a new one.
- Between inspection i and inspection $i+1$, one of these scenarios may happen :

○ Predictive maintenance scenario: the RUL of the system attains some threshold value called RUL_{lim} under which the system is considered to be deteriorated. The system is then replaced by a new one before the inspection $i+1$.

○ Non-predictive maintenance scenario: in this scenario, the system is not replaced by performing predictive maintenance. In such case, the system can fail or not: if the system fails before the next inspection $i+1$, knowing that he was operating at inspection i , the system should then be correctively replaced by a new one. The probability of occurrence of this scenario is given by $\int_{t_i, T > t_i}^{i+1} f(t) \cdot dt = \frac{\int_{t_i}^{i+1} f(t) \cdot dt}{S(t_i)}$, where t_i is the time of the i^{th} inspection. In the other hand, the system may operate normally until the next inspection $i+1$. This last scenario occurs with the complementary probability of occurrence $1 - \int_{t_i, T > t_i}^{i+1} f(t) \cdot dt$.

"Fig. 2" summarizes the different scenarios that may occur between two consecutive inspections.

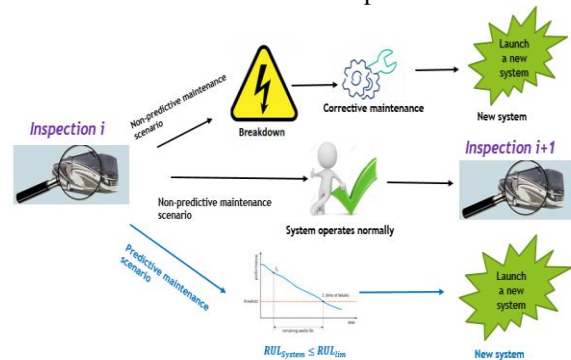


Figure 2. Maintenance scenarios

- The Weibull distribution is adopted to model the failure evolution of the system. The parameters of the Weibull distribution are updated at each inspection i .
- The durations of both predictive and corrective replacement are assumed constant and known.
- The costs of both predictive and corrective replacement, as well as the cost of inspection, are assumed constant and known.

B. Process of Maintenance Cost Optimization

In this section, we describe the different costs of maintenance (section 1) and the optimization program to understand the different steps to follow in order to identify the optimal strategy for maintenance (section 2).

1) Maintenance costs

The maintenance costs include the following costs:

- cost of predictive maintenance C_p : the cost of predictive maintenance during the time cycle D can be described by the following equation:

$$C_p = c_p \cdot \sum_{i=1}^{N_{in}-1} N_i \quad (2)$$

where c_p is the cost of a predictive replacement and N_i is a binary variable which takes 1 in case of predictive maintenance between inspection i and $i+1$ and 0 otherwise.

N_{in} refers to the total number of inspections in the time cycle D .

- cost of corrective maintenance C_c : the cost of corrective maintenance can be described by the following equation :

$$C_c = \sum_{i=1}^{N_{in}-1} c_c \cdot (1 - N_i) \int_{t_i, T > t_i}^{t_{i+1}} f(t) \cdot dt + c_c \cdot \int_{t_{N_{in}}, T > t_{N_{in}}}^D f(t) \cdot dt \quad (3)$$

where c_c is the cost of a corrective replacement.

- cost of inspection C_i : the step of inspection θ is linked to the number of inspections N_{in} per time cycle D according to the following equation :

$$N_{in} = \frac{D}{\theta} \quad (4)$$

Therefore, the cost of inspection per time cycle D can be expressed by the following equation :

$$C_i = \frac{D}{\theta} \cdot c_i \quad (5)$$

where c_i is the cost of an inspection.

- cost of operating loss C_{ol} : the cost of operating loss C_{ol} can be expressed by the following equation :

$$C_{ol} = D_p \cdot c_{dt} \cdot \sum_{i=1}^{N_{in}-1} N_i + \sum_{i=1}^{N_{in}-1} (1 - N_i) \cdot \int_{t_i, T > t_i}^{t_{i+1}} f(t) \cdot dt \cdot c_{dt} \cdot D_c + (1 - N_{in}) \cdot \int_{t_{N_{in}}, T > t_{N_{in}}}^D f(t) \cdot dt \cdot c_{dt} \cdot D_c \quad (6)$$

where D_p is the duration of a predictive replacement, D_c is the duration of a corrective replacement, and c_{dt} is the cost of system downtime per unit of time.

As expected, the first term of the equation corresponds to the cost of operating loss due to predictive maintenance and the second term corresponds to the cost of operating loss due to corrective maintenance.

Maintenance costs may also include the cost of maintenance risks. These risks as described in [8][16] can be human, environmental or financial. We refer to our previous work in [8], which describes in details the risk analysis part of our methodology.

2) Optimization program

- Input data: the input data of our optimization program are the duration parameters: D , D_p and

D_c ; the cost parameters: c_p , c_c , c_i , c_{dt} and the parameters of the Weibull distribution characterizing the failure evolution of the system.

- Decision variables: the decision variables of the optimization program are the number of inspections N_{in} and the binary variable N_i indicator of predictive maintenance in the inspection interval $[i, i+1]$.
- Objective function: the objective function that we want to minimize is the total cost of maintenance C_{tot} during the interval of time D :

$$C_{tot} = C_p + C_c + C_i + C_{ol} \quad (7)$$

- Constraints:

- o Positivity constraints: the different costs should be positive as well as the inspection step θ . The decision variable N_i should be binary for $i \in [1, N_{in}]$.

- o Constraint on the inspection process: as the system requires at least one inspection at its early life, the number of inspections N_{in} should be superior to 1.

- o Constraints on system availability: to ensure the availability of the system, the durations of both predictive and corrective replacement should be too small comparing to the inspection step θ :

$$\begin{cases} D_p \leq \varepsilon \cdot \theta \\ D_c \leq \varepsilon \cdot \theta \end{cases} \quad (8)$$

with ε a sufficiently small number.

- Flowchart of the optimization program: "Fig.3" summarizes the main steps of the methodology described in this paper :

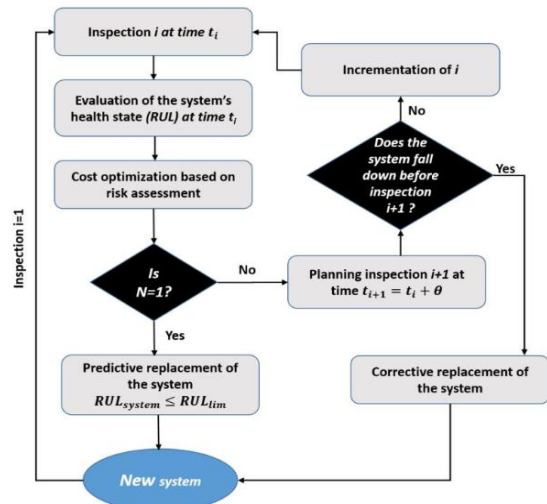


Figure 3. Flowchart of the optimization process for maintenance planning.

III. APPLICATION OF THE METHODOLOGY ON A MECHANICAL BEARING SYSTEM

A. System Description

A train is a complex system composed of several subsystems, each of these subsystems providing a set of

basic functions, all contributing to the accomplishment of the main function of the train which is carrying passengers from point A to point B ("Fig.4").

The traction system contains a bogie supporting an electric motor, which is itself a complex system ("Fig.5"). Some of the components of the electric motor are critical for the functioning of the train. In fact, according to Electric Power Research Institute and to researchers in electric machine reliability, mechanical bearings are pointed as the faultiest components [17]. This is because the mechanical bearing systems have an average probability of failure combined to a critical severity of failure: in fact, a possible failure of the bearing system is enough to stop the motor shaft from working which can cause the train to stop.

The mechanical bearing system is then a critical component of the train motor. We consider in this work that the train motor has a duration of exploitation D .

A rolling-element bearing ("Fig.6"), also known as a rolling bearing, is a type of bearing, which carries a load by placing rolling elements (such as balls or rollers) between two bearing rings called races [18]. The relative motion of the races causes the rolling elements to roll with very little rolling resistance and with little sliding. In other words, the mechanical bearing system supports the rotating elements of the motor and provide additional damping to stabilize it.

In this paper, we focus on the mechanical rolling bearing system as a critical component, necessary for the operation of the complex system: train motor. The train motor is itself a critical system of a wider complex system: the train.

"Table 1" summarizes the parameters characterizing the mechanical bearing system of the train motor. These data are realistic and are given for information.

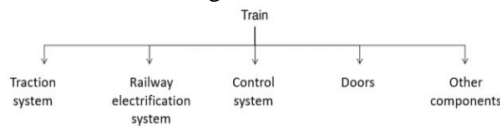


Figure 4. Train system.

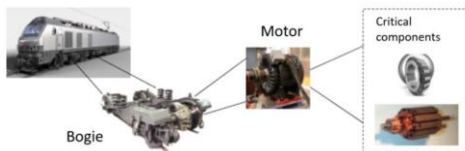


Figure 5. Critical components of the train motor.

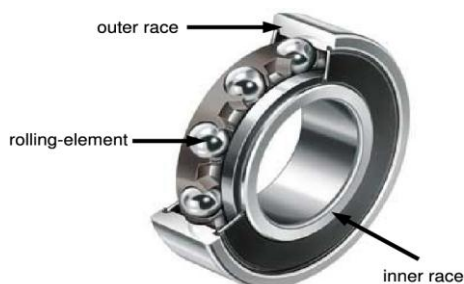


Figure 6. Mechanical rolling bearing system.

TABLE I. MAIN CHARACTERISTICS OF THE MECHANICAL BEARING SYSTEM

Parameter	Value	Unit
Duration of exploitation of the train motor D	25000	hours
Duration of a predictive replacement D_p	2	hours
Duration of a corrective replacement D_c	6	hours
Cost of a predictive replacement c_p	70	euros
Cost of a corrective replacement c_c	220	euros
Cost of an inspection c_i	20	euros
Cost of system downtime per hour c_{dt}	400	euros
Weibull scale parameter λ	19000	/
Weibull shape parameter k	2.8	/

B. Numerical Results

First, we tested if the RUL of the system converge to a some threshold RUL_{lim} that we tried to approximate. Therefore, we did not respect -a priori- the constraints (Eq.8) related to the possible number of inspections during the period of time D (section 1).

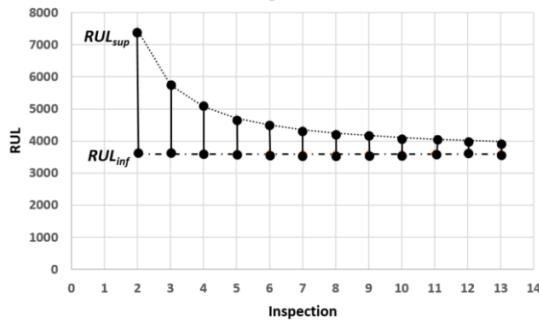
In a second step, however, we respected the possible number of inspections during D and we tried to compare the different results obtained for every number of inspection considered (section 2).

1) Evaluation of RUL_{lim}

We ran the optimization program until obtaining an approximation for the value of RUL under which the system should be predictively replaced. We followed the following steps:

- **Step 1:** for a fixed number of inspections N_{in} , we ran the optimization program to obtain the values of the decision variables N_i , $i \in \{1, \dots, N_{in}\}$. If there exist $j \in \{1, \dots, N_{in}-1\}$ where $N_j=1$, it means that, to minimize the total cost of maintenance, the system should be predictively replaced after inspection j performed at time t_j . If we note t_p the precise time where RUL_{system} is equal to RUL_{lim} , according to our optimization program, t_p is within the interval of time $[t_j, D]$. This can be translated by $RUL_{lim} \in [RUL_{system}(D), RUL_{system}(t_j)]$.
- **Step 2:** we then iterated step 1 by varying N_{in} from 2 to 13 in order to have a more precise interval for RUL_{lim} .
- **Step 3:** we verified that we did all the possible simulations: in fact, beyond 13 inspections, predictive maintenance becomes more expensive than corrective maintenance.

The results show that the best approximation for RUL_{lim} is within $[3594.8, 3977.6]$ hours. "Fig.7" gives an overview of the different intervals of RUL_{lim} per number of inspections.


Figure 7. Interval of approximation of RUL_{lim}

2) Maintenance cost evaluation respecting the constraints on the number of inspections

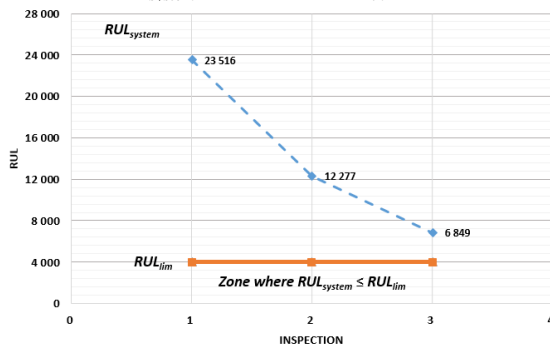
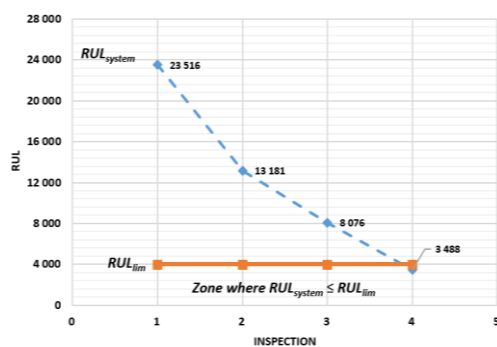
Considering the system availability constraints (Eq.8), the decision maker fixed a value of ε equal to 10^{-3} . The number of inspections N_{in} can then take the following values: $\{1, 2, 3, 4\}$. In this paper, we focused our work on the cases where $N_{in}=3$ and $N_{in}=4$ as these cases are worth to study.

- Case where $N_{in}=3$:

At each inspection i , experts were able to evaluate the Weibull parameters on the basis of real data on the health state of the system. Therefore, following (Eq.1), the evaluation of the RUL of the system at each inspection i becomes easy. "Fig.8" illustrates the variation of RUL_{system} in case where $N_{in}=3$.

- Case where $N_{in}=4$:

Similarly, the Weibull parameters were updated at each inspection i based on real data. "Fig.9" illustrates the variation of RUL_{system} in case where $N_{in}=4$.


Figure 8. Variation of RUL_{system} , case $N_{in}=3$

Figure 9. Variation of RUL_{system} , case $N_{in}=4$

C. Result Interpretation

In case where $N_{in}=3$, the evaluation of the RUL of the system at each inspection i based on real data on the health state of the system indicates that the threshold for predictive maintenance has not been achieved by the system. Therefore, predictive maintenance is not required in this case.

In case where $N_{in}=4$, the evaluation of the RUL of the system at each inspection i based on real data on the health state of the system indicates that :

- $RUL_{system} \geq RUL_{lim}$ at inspections $\{1, 2, 3\}$ meaning that the threshold for predictive maintenance has not been achieved by the system and there is no need to perform predictive maintenance in this case.
- $RUL_{system} \leq RUL_{lim}$ at inspection 4 meaning that the threshold for predictive maintenance has been achieved by the system. Therefore, the system should be predictively replaced by a new one.

This example illustrates that paying for a more inspection may save the cost of a corrective replacement. In fact, if we have stopped at 3 inspections, we would have left the system working until failure. In case where $N_{in}=4$, the 4th inspection was a sort of alarm for the decision maker to prevent him that the RUL threshold has been crossed by the system and that it was time to replace the system before failure.

Let us evaluate the total cost saved by the decision maker. Table 2 gives an overview of the different costs (in euros) involved in each case study and the total cost saved by the decision maker.

TABLE II. MAINTENANCE COSTS IN CASE WHERE $N_{in}=3$ AND $N_{in}=4$

N_{in}	C_p	C_c	C_i	C_{ol}	C_{tot}
$N_{in}=3$	0	220	60	2400	2480
$N_{in}=4$	70	0	80	140	290
Saved cost					2190

Performing 4 inspections on the system has saved us the cost of a corrective replacement and the cost of operating loss due to corrective maintenance. This saved cost is almost equivalent to 88% of the total cost paid for maintenance in case of 3 inspections.

- Some reflections to consider:

These results are obtained by assuming a constant value of RUL_{lim} . This is due to the fact that we consider a short period of exploitation D . To improve the obtained results, one possible solution is to update the RUL_{lim} as one goes along inspections. This aspect will be tackled in another research work.

If RUL_{system} falls within the interval $[RUL_{inf}, RUL_{sup}]$ hours, the choice to maintain the system before failure reveals the decision maker's attitude to risk. If the decision maker is risk averse, he will certainly choose to replace the system once $RUL_{system} \leq RUL_{sup}$. If by contrast, the decision maker is not risk averse, he will probably choose to replace the system once $RUL_{system} \leq RUL_{inf}$.

IV. CONCLUSION

In this paper, we proposed an optimization approach for maintenance, which takes as input the parameters of the failure distribution of a system and gives as output the *RUL* threshold under which the system should be predictively replaced in order to minimize the total cost of maintenance. This approach allows the decision maker to identify also the best inspection interval that minimizes the cost of maintenance while maintaining the operational availability of the system.

We applied the approach on a mechanical bearing system treated as a single component integrating a complex system (motor train) which is itself a part of another more complex system (train). The results show that to minimize the maintenance costs, it is preferable to perform 4 inspections on the system and to replace the system before failure at the fourth inspection meaning that money invested in more inspections may be saved later by avoiding us the cost of corrective maintenance which is usually expensive. However, we need to be wary of numerical values used in the example because they have a strong impact on the optimization results. Therefore, the object of our future work will be to study the influence of varying some numerical parameters (cost parameters, duration parameters ...) on the optimization results.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Author R. proposed the method and the application on a case study. Author R. wrote the paper.

Authors M. and J. validated the method and the numerical results. Authors M. and J. corrected and validated the final version of the work.

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Background: system dependability, risk analysis, complex system modeling.

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