Balancing of Asymmetrical Rhomboid Mechanism of External Heat Source Engine

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***Abstract*—Balancing of asymmetrical rhomboid mechanism with forked crank which is used in engines with external heat sources is considered. The main equations for correcting masses (counterweights) and their coordinates calculations are given. The conditions of full static balancing of rhomboid mechanism with forked crank are obtained.**

***Index Terms*— external combustion engine, asymmetrical rhomboid mechanism, balancing, correcting masses.**

I. Introduction

Your goal is to simulate the usual appearance of papers in the. We are requesting that you follow these guidelines as closely as possible.

Engines with external sources of heat, also known as external combustion engines, which works with Stirling thermo dynamical cycle have a wide usage with rhomboid mechanisms [1]. That mechanisms are the base ones for machines with shortened thermo dynamical cycle [2-5]. Rhomboid mechanisms (fig.1) differs from ordinary crank mechanisms by existence of right and left closed kinematical chain and two pins: working and displacing. Pins chambers connected with each other through cooler and heat source. Synchronizing gearing allows to eliminate skewness of working and displacing pins. Rhomboid mechanism may be symmetrical or asymmetrical with forked cranks or conrods [2, 6, 7].

While mechanisms links moves with accelerations force loading of machines basement consists dynamical part. When machine works is steady regime they changes cyclically, forcing periodical loads and causing vibrations of basement. For exclusion or reducing this harmful impact of dynamical loads on engines body, this parts of load should me reduced to zero level, or their amplitudes should be limited in allowable range. Solution of such a problem – balancing of mechanism – is necessary for engine longevity and stable working. Addition of correcting masses in mechanism may lead to zeroing out projections of each links principal vector of inertia forces on each coordinate axis. This means that mechanism will be fully statically balanced.

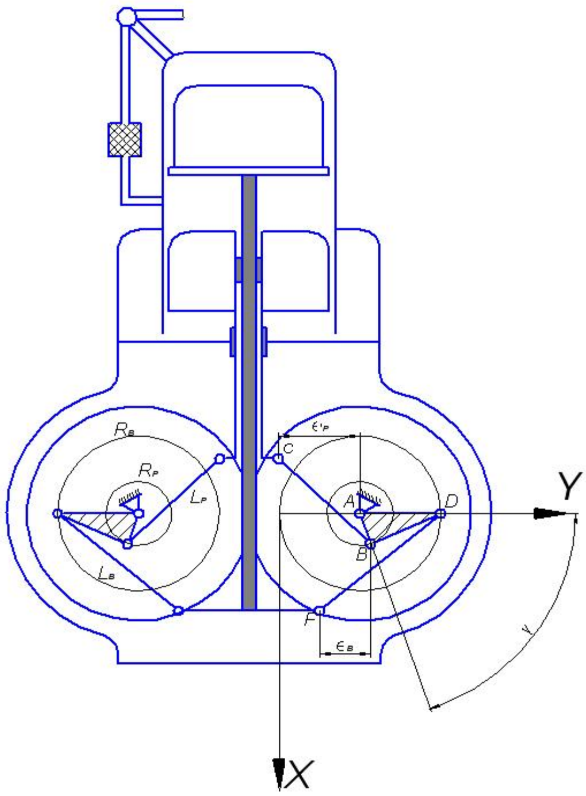

Figure 1. Generalized scheme of Stirling engine with forked crank.

It is needed to define necessary coordinates of counterweights and their masses. It may be obtained by usage of substitution mass methods, based on replacement of agile links masses by two o three equivalent masses.

Symmetry of rhomboid mechanism relatively pins axis means that the principal moment of inertia forces on OY axis are equal to zero. Projection of the principal moment of inertia forces on OX axis still not equal to zero. (Fig. 2) For mechanism with forked crank solution of dynamic reactions balancing problem is made by following method.

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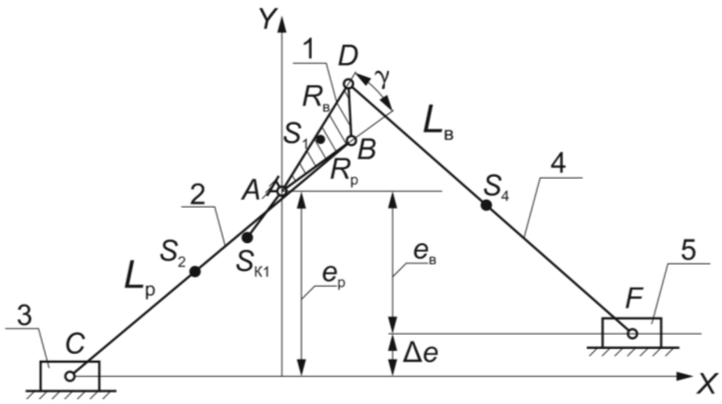

Figure 2. Kinematical chain of rhomboid mechanism of Stirling engine with forked crank: 1 – forked crank, 2 – working pin's conrod, 3 – working pin, 4 – displacing pin's conrod, 5 – displacing pin; S_1, S_2, S_3 – mass centers of links with masses m_1, m_2, m_3; R_d and R_w – cranks parts lengths for working and displacing groups, L_d и L_w – lengths of displacing and working conrods, e_d и e_w – eccentricity of pins, γ – crank angle

II. Pointed Mass Method

Distributed masses of mechanisms links replaces by concentrated mass, located in the centers of rotational kinematic pairs. These masses are selected to satisfy the laws of constancy of masses and mass centers location.

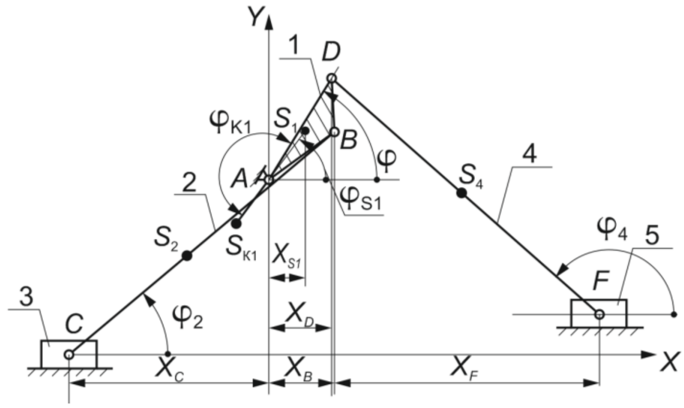

Figure 3. Calculation scheme for pointed masses

Calculation scheme of mechanism (Fig. 3) is described by pointed masses with relative abscises in parts of *RP*. i.e. while working pins crank length is equal to 1 (*R*=*RP*=1):

$$X_B = \sin\varphi, \tag{4}$$

$$X_D = a_R \cdot \sin(\varphi - \gamma) \tag{5}$$

$$X_C = \sin\varphi - \frac{a_R \cdot a_L}{\lambda_B}\cos\varphi_2 \tag{6}$$

$$X_F = a_R\left[\sin(\varphi-\gamma) - \frac{1}{\lambda_B}\cos\varphi_4\right] \tag{7}$$

$$X_{S_1} = a_{AS_1} \cdot \sin(\varphi - \varphi_{S1}) \tag{8}$$

$$X_{K_1} = a_{AK_1} \cdot \sin(\varphi - \varphi_{SK_1}) \tag{9}$$

where $a_i = \frac{l_i}{R}$ - length of i^{th} line segment, λ_B – relation between lengths of displacing conrod to the same crank, while angles φ_2, φ_4 calculates using equations:

$$\sin\varphi_2 = \frac{\lambda_B}{a_L}\left(\frac{\cos\varphi}{a_R} + \frac{k_B}{a_e}\right) \tag{10}$$

$$\sin\varphi_4 = \lambda_B\left[\cos(\varphi-\gamma) + k_B\right] \tag{11}$$

where k_B – relative lengths of displacing conrod.

As coordinates (4) – (9) are periodical functions $f_1(\varphi) = \frac{a_L}{\lambda_B}\cos\varphi_2$; $f_2(\varphi) = \frac{\cos\varphi_4}{\lambda_B}$ from φ angle with period of 2π, they may be expanded in a Fourier series with φ as a variable. Functions are even, so coefficient of expansion without sine function are equal to:

$$A_n = \frac{1}{\pi}\int_0^{2\pi}\left(f_1(\varphi)\cdot\cos n\varphi\right)\mathrm{d}n, \qquad B_n = \frac{1}{\pi}\int_0^{2\pi}\left(f_2(\varphi)\cdot\cos n\varphi\right)\mathrm{d}n \tag{12}$$

In this case

$$\cos\varphi_2 = \frac{\lambda_B}{a_L}\sum_{n=0}^{\infty}A_n\cos n\varphi = \frac{\lambda_B}{a_L}(A_0 + A_1\cos\varphi + A_2\cos 2\varphi + A_3\cos_3\varphi + A_4\cos 4\varphi + ...) \tag{13}$$

$$\cos\varphi_4 = \frac{\lambda_B}{a_L}\sum_{n=0}^{\infty} B_n \cos n\varphi = \frac{\lambda_B}{a_L}(B_0 + B_1\cos\varphi + \\ + B_2\cos 2\varphi + B_3\cos_3\varphi + B_4\cos 4\varphi + ...) \quad (14)$$

Expressing the coordinates (5), (7) through (12), (13), gains (14,15)

$$X_C = \sin\varphi - a_R\sum_{n=0}^{\infty} A_n \cos n\varphi \quad (15)$$

$$X_F = a_R\left[\sin(\varphi-\gamma) - \sum_{n=0}^{\infty} B_n \cos n\varphi\right] \quad (16)$$

III. Correcting Masses Calculation

To calculate projection of forces on OX axis it is needed to differentiate equations (4), (8), (9), (14) – (15) by time twice and multiple them on masses with opposite sign. This forces equation with R = 1 (for general case of machine movement $\omega = d\varphi/dt = \omega(t)$) take the following form:

$$\Phi_{S_B} = -m_B\left(-\omega^2\cdot\sin\varphi - \varepsilon\cdot\cos\varphi\right) \quad (17)$$

$$\Phi_{S_D} = -a_R m_D\left(-\omega^2\cdot\sin(\varphi-\gamma) - \varepsilon\cdot\cos(\varphi-\gamma)\right) \quad (18)$$

$$\Phi_{S_C} = -m_C\left[\omega^2\left(-\sin\varphi + a_R\sum_{n=0}^{\infty} A_n\cos n\varphi\right) + \\ + \varepsilon\left(-\cos\varphi + a_R\sum_{n=1}^{\infty} A_n\cos n\varphi\right)\right] \quad (19)$$

$$\Phi_{S_F} = -a_R m_F\left[\omega^2\left(-\sin(\varphi-\gamma) + \sum_{n=0}^{\infty} n^2 A_n\cos n\varphi\right) + \\ + \varepsilon\left(-\cos(\varphi-\gamma) + \sum_{n=1}^{\infty} nB_n\cos n\varphi\right)\right] \quad (20)$$

$$\Phi_{S_1} = -a_{AS_1} m_1\left(-\omega^2\cdot\sin(\varphi-\varphi_{S1}) - \varepsilon\cdot\cos(\varphi-\varphi_{S_1})\right) \quad (21)$$

$$\Phi_{SK_1} = -a_{AK_1} m_1\left(-\omega^2\cdot\sin(\varphi-\varphi_{SK_1}) - \varepsilon\cdot\cos(\varphi-\varphi_{SK_1})\right) \quad (22)$$

and others in the same sequence.

The sum of second and higher orders harmonics may be presented in following form:

$$a_R\omega^2\left(m_C\sum_{n=2}^{\infty} n^2 A_n\cos n\varphi + m_F\sum_{n=2}^{\infty} n^2 B_n\cos n\varphi\right) + \\ + a_R\varepsilon\left(m_C\sum_{n=2}^{\infty} nA_n\sin n\varphi + m_F\sum_{n=2}^{\infty} nB_n\sin n\varphi\right) = 0 \quad (23)$$

Solving this equation, gain:

$$\frac{\sum_{n=2}^{\infty} n^2 B_n\cos n\varphi}{\sum_{n=2}^{\infty} nA_n\sin n\varphi} = \frac{B_n}{A_n} = -\frac{m_C}{m_F} = -C, \quad (24)$$

$$\frac{\sum_{n=2}^{\infty} nB_n\sin n\varphi}{\sum_{n=2}^{\infty} nA_n\sin n\varphi} = \frac{B_n}{A_n} = -\frac{m_C}{m_F} = -C. \quad (25)$$

To find the connection of C parameter with engines geometry, formulae (10), expressed relative $\cos\varphi$, with considering $\frac{Ca_2}{\lambda_B}\cos\varphi = -\frac{1}{\lambda_B}\cos\varphi_4 + C_1\cos\varphi + C_0$, is reduced to the form

$$\cos\varphi_4 = -a_L\left(C\cos\varphi_2 - C_1 a_R\sin\varphi_2\right) + \\ + \lambda_B C_0 - \frac{C_1\lambda_B a_R k_B}{a_e} \quad (26)$$

In this equation coefficients with $\cos\varphi 2$ and $\sin\varphi 2$ considered as values of sine and cosine functions of auxiliary function θ:

$$\frac{C}{\rho} = \cos\theta,\ \frac{C_1 a_R}{\rho} = \sin\theta, \\ \rho = \sqrt{C^2 + C_1^2 a_R^2} \quad (27)$$

Equation (25) reduced to the form:

$$\cos\varphi_4 = -z\cos\left(\varphi_2 + \theta\right) + z_1, \quad (28)$$

$$\text{where } z = a_L\rho,\ z_1 = \lambda_B\left(C_0 - \frac{C_1 a_R k_B}{a_e}\right) \quad (29)$$

Solving (10) and (11) simultaneously, gains

$$\sin\varphi = \frac{\sin\varphi_4 - \lambda_B k_B}{\lambda_B\sin\gamma} - \left(\frac{a_L a_R\sin\varphi_2 - a_R\lambda_B k_B/a_e}{\lambda_B}\right)\text{ctg}\gamma \quad (30)$$

The expression enclosed in parentheses is denoted by z_2 and is expressed through $\cos\varphi$. Using the basic trigonometric identity, expression (29) can be reduced to the form:

$$\frac{\sin^2\varphi_4 - 2\lambda_B k_B\sin\varphi_4 + \lambda_B^2 k_B^2}{\lambda_B^2\sin^2\gamma} - \\ - \frac{2z_2(\sin\varphi_4 - \lambda_B k_B)\cot\gamma}{\lambda_B\sin\gamma} + \\ + z_2^2\cot^2\gamma + z_2^2 = 1 \quad (31)$$

Having replaced and introducing new notation, we arrive at the expression:

$$\sin\varphi_4 = \left(\frac{1 - z_3^2}{\lambda_B^2\sin^2\gamma} + z_5 - 1\right)\frac{1}{z_4}, \quad (32)$$

where: $z_3 = \cos\varphi_4$,

$$z_4 = \frac{2(k_B + z_2\cos\gamma)}{\lambda_B\sin^2\gamma},\ z_5 = \frac{k_B^2 + 2z_2 k_B\cos\gamma + z_2^2}{\sin^2\gamma}$$

as $\sin^2\varphi_4 + \cos^2\varphi_4 = 1$, squaring and folding, one can obtain an identity

$$-z_3^4+z_3^2[2(1+z_5\lambda_B^2\sin^2\gamma-\lambda_B^2\sin^2\gamma)-z_4^2\lambda_B^4\sin^4\gamma]-$$
$$-z_5^2\lambda_B^4\sin^4\gamma+2z_5(\lambda_B^4\sin^4\gamma-\lambda_B^2\sin^2\gamma)+ \quad (33)$$
$$2\lambda_B^2\sin^2\gamma-\lambda_B^4\sin^4\gamma+z_4^2\lambda_B^4\sin^4\gamma=1$$

When z_3, z_4, z_5 are replaced by their original expressions and, by carrying out the corresponding transformations, we obtain an equivalent identity $z_6=\frac{a_L^2\cdot a_R^2}{2}+\lambda_B^2k_B^2\left(1-\frac{2a_R\cos\gamma}{a_e}+\frac{a_R^2}{a_e^2}\right)$,

$$z_7=2\lambda_Bk_B\left(1-\frac{a_R\cos\gamma}{a_e}\right),$$

$$z_8=2a_La_R\cos\gamma,$$

$$z_9=2a_La_R\lambda_Bk_B\left(\cos\gamma-\frac{a_R}{a_e}\right).$$

IV. Balancing Conditions

From the condition that the values of the amplitudes of the groups of functions $\cos^4\varphi_2$, $\sin^4\varphi_2$; $\cos^3\varphi_2$, $\sin^3\varphi_2$; $\cos^2\varphi_2$, $\sin^2\varphi_2$, $\cos\varphi_2$, $\sin\varphi_2$ is equal to zero we obtain the following expressions:

$$z=a_La_R \quad (34)$$

$$\theta=\gamma \quad (35)$$

$$\frac{a_R}{a_L}=\frac{1}{\cos\gamma},$$

$$z_1=-\lambda_Bk_B\mathrm{tg}\theta \quad (36)$$

$$\gamma=0 \quad (37)$$

$$a_La_R=1 \quad (38)$$

In which (32) is equal to 1.

It is important to note that conditions (36) - (38) are one of the necessary mechanisms for complete balancing.

Replacing in the formulas from (26) and (28) $C=\rho\cos\theta$, $z=a_L\rho$ and using the conditions (34), (35), (36) in the expressions for θ and z, we arrive at the relation

$$C=aR \quad (39)$$

Applying this relation in Eq. (23), we obtain one more necessary condition for complete balancing

$$m_C=a_Rm_F \quad (40)$$

The first-order equation of the sum of all forces has the form

$$-\omega^2\Big[\big(m_B+a_Rm_D+m_C+a_Rm_F+a_{AS_1}m_1\cos\varphi_{S_1}\big)\sin\varphi-$$
$$-a_{AS1}m_1\sin\varphi_{S_1}\cos\varphi+a_{AK_1}m_{K1}\sin\big(\varphi-\varphi_{SK1}\big)\Big]- \quad (41)$$
$$\varepsilon\Big[\big(m_B+a_Rm_D+m_C+a_Rm_F+a_{AS_1}m_1\cos\varphi_{S_1}\big)\cos\varphi-$$
$$-a_{AS1}m_1\sin\varphi_{S_1}\sin\varphi+a_{AK_1}m_{K1}\cos\big(\varphi-\varphi_{SK1}\big)\Big]$$

We introduce the following notation:

$$m_B+a_Rm_D+m_C+a_Rm_F+a_{AS1}m_1\cos\varphi_{S_1}=m\cos\theta, \quad (42)$$

$$a_{AS_1}m_1\sin\varphi_{S_1}=m\sin\theta, \quad (43)$$

$$m^2=\big(m_B+a_Rm_D+m_C+a_Rm_F+a_{AS1}m_1\cos\varphi_{S_1}\big)^2+\big(a_{AS_1}m_1\sin\varphi_{S_1}\big)^2 \quad (44)$$

Then equation (41) takes the form

$$m\sin\big(\varphi-\theta_1\big)=-a_{AK_1}m_{K_1}\sin\big(\varphi-\varphi_{SK_1}\big),$$
$$m\cos\big(\varphi-\theta_1\big)=-a_{AK_1}m_{K_1}\cos\big(\varphi-\varphi_{SK_1}\big). \quad (45)$$

Solving this system of equations, we find

$$m_{K_1}=\frac{m}{a_{AK_1}} \quad (46)$$

- the value of the correcting mass and the angular coordinate of this mass

$$\varphi_{SK_1}=\pi+\theta \quad (47)$$

where

$$\theta=\mathrm{atan}\frac{a_{AS_1}m_1\sin\varphi_{S_1}}{m_B+a_Rm_D+m_C+a_Rm_F+a_{AS_1}m_1\cos\varphi_{S_1}} \quad (48)$$

From the results obtained, it follows that (36-38), (40), (45) - (48) are the basic conditions for the complete balancing of the rhombic mechanism of the drive with the forked crank.

V. Conclusion

1. The influence of the relations of out-of-axes, crank radii, lengths of connecting rods, as well as the angle of crank development on the imbalance of the rhombic drive mechanism is determined.
2. The use of the replacement mass method makes it possible to form, in a convenient form, equations by solving the conditions necessary for the complete balancing of the mechanism.
3. The symmetry of the considered schemes of mechanisms relative to the axis of motion of the pistons eliminates the effect of inertial forces in the direction perpendicular to this axis and inertial moments.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

G.A. Timofeev has developed a mathematical model. He held a calculations for this paper.

E.O. Podchasov has analyzed data, made a conclusions and wrote the article.

Both authors accepted the paper.

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