A Study on the Median Run Length Performance of the Run Sum S Control Chart

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***Abstract*—Control charts play a very important role in Statistical Process Control. Run sum S control chart is sensitive in detecting small to moderate shifts. It is an excellent alternative to Shewhart control chart. The performance of the run sum S control chart based on median run length (MRL) performance is proposed in this study. The Statistical Analysis System (SAS) program was used to calculate the in-control ARL and in-control MRL for the nine run sum S chart schemes with different sample sizes, magnitude of shift in the process standard deviation, and the in-control run lengths. The findings show that the MRL measure provides better explanation than the ARL criterion. Moreover, the MRL performance of the run sum S chart schemes is substantially affected by the sample sizes, magnitude of shift in the process standard deviation, and the in-control run lengths.**

***Index Terms*—run sum S chart, average run length (ARL), median run length (MRL), process standard deviation**

I. INTRODUCTION

Control charts are very useful tools in monitoring the quality of products. The basic control chart is the Shewhart-type chart. Since it is not sensitive in detecting small to moderate shifts, many alternatives were introduced, such as cumulative sum (CUSUM) chart, exponential weighted moving average (EWMA) chart and the run sum control chart. Champ and Rigdon compared the average run length (ARL) performances among the Shewhart $\bar{X}$ control charts with supplementary runs rules, the CUSUM control chart, the EWMA control chart and the run sum $\bar{X}$ control chart [1]. Results from the comparison indicate that the run sum $\bar{X}$ charts with proper regions are more competitive in monitoring the mean shifts in a process than other type of charts.

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The run sum $\bar{X}$ control chart is a simple and powerful control charting procedure to monitor the process mean. It was proposed by [2] and studied further by [3]. Since then, numerous research studies have been done. For example, [4] proposed using the zone control chart procedure in the range chart. [5] proposed a fast initial response (FIR) feature for the run sum R control chart which is more sensitive in detecting small shifts in process dispersion. [6] proposed a run sum Hotelling's χ^2 control chart and their findings showed that the chart has better ARL performance than other χ^2 control charts with runs rules. [7] introduced the run sum *t* control chart which is more robust than the run sum $\bar{X}$ control chart and more sensitive than the other *t* type charts.

Since monitoring the process variance is as important as monitoring its mean. [8] researched on monitoring the variability of a process based on ARL performance using the run sum *S* control charts which could improve the poor performance of the two-sided Shewhart *S* control chart in detecting small to moderate shifts in the process standard deviation [8].

The ARL which is a traditional performance indicator of control schemes has its average number of samples plotted on a statistical control chart before an out-of-control signal is detected. However, the interpretation merely based on the ARL could trigger a false alarm as the in-control run length distribution of the run sum *S* control chart is highly skewed. Therefore, median run length (MRL) is proposed as an alternative performance measurement in this study. MRL is the 50th percentage point of the run length distribution. It denotes the median number of samples drawn on a control chart until it issues an out-of-control signal.

In order to overcome the weakness of the ARL as the sole measurement of the performance of a control chart, researchers suggested using more reliable measurements. For example, [9] introduced the percentage points of the run length distribution measurement. [10] presented an optimal design of a multivariate exponentially weighted moving average (MEWMA) control chart based on both

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ARL and MRL performances. [11] suggested the application of five percentiles: 5th, 25th, 50th, 75th and 95th for the Shewhart control chart. An optimal design of the EWMA *t* chart based on MRL was proposed by [12] which extended and complemented the research of [13]. [14] proposed two optimal designs of double sampling $\overline{X}$ chart based on MRL performance [14]. For other research studies on MRL, refer to [15-19]. Since the run sum *S* control chart based on MRL performance has not been study yet, this paper fills the gap.

The organization of this paper is as follows: In Section II, the run sum *S* control chart is outlined. Section III analyses the performance of the run sum S chart based on the MRL. Last but not least, conclusions are drawn in Section IV.

II. Run Sum S Control Chart

The *S* control chart is the most basic chart used in detecting the process standard deviation shift. The sample mean and sample standard deviation for the chart are defined as

$$\bar{x}_i = \frac{1}{n}\sum_{i=1}^{n} x_{ij} \,, \tag{1}$$

and

$$S_i = \sqrt{\frac{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}{n-1}} \,, \tag{2}$$

respectively, where x_{ij} denotes the j^{th} sample observation at time $i = 1, 2, \ldots, n$.

The parameters based on probability limits which include the UCL, CL and LCL of the two-sided *S* control chart with false alarm rate α are given by

$$UCL = \sigma_0 \sqrt{\frac{\chi^2_{\alpha/2;\, n-1}}{n-1}} \,, \tag{3}$$

$$CL = \sigma_0 \sqrt{\frac{\chi^2_{0.5;\, n-1}}{n-1}} \,, \tag{4}$$

and

$$LCL = \sigma_0 \sqrt{\frac{\chi^2_{1-\alpha/2;\, n-1}}{n-1}} \,, \tag{5}$$

where $\chi^2_{\alpha/2;\, n-1}$ denotes a Chi-square distribution with $(n-1)$ degree of freedom. Note that the median line is used as the center line of the chart.

According to [8], the probability of the out-of-control ARL of two-sided *S* control chart is

$$p = 1 - F_{n-1}\left(\frac{(n-1)UCL^2}{(\lambda\sigma_0)^2}\right) + F_{n-1}\left(\frac{(n-1)LCL^2}{(\lambda\sigma_0)^2}\right) \tag{6}$$

where $F_{n-1}(\cdot)$ is the cumulative distribution function (CDF) of the Chi-square distribution with the degree of freedom $(n-1)$. The symbol σ_0 represents the nominal standard deviation of a quality characteristic $x_{i,\,j}$. Then $\sigma_1 = \lambda\sigma_0$ ($\lambda > 0$) denotes the out-of-control standard deviation where λ is the magnitude of the process standard deviation shift while the mean remains at its nominal value μ_0. When $\lambda = 1$, the process is in-control; when $0 < \lambda < 1$, the standard deviation decreased, indicating the process has improved; when $\lambda > 1$, the standard deviation increased, indicating the process has deteriorated [8].

The run sum *S* control chart is proposed to overcome the weakness of the basic *S* control chart, i.e. the poor sensitivity in detecting small to moderate shifts. It is a very effective control charting procedure in detecting shift in the process variance. In this paper, the run sum *S* chart is divided into two distinct regions consisting of above and below the center line. Scores are assigned into the regions by the practitioners and accumulated. The initial cumulative score usually begins at 0. The control chart will issue an out-of-control signal when the cumulative score surpasses the predefined triggering value. If a plotted sample value falls on the other side of the center line, the cumulative scoring starts anew.

Generally, the two-sided run sum *S* control chart is based on the CS_i statistic, i.e.

$$CS_i = \max\{CSU_i, CSL_i\} \,, \tag{7}$$

where

$$CSU_i = \begin{cases} CSU_{i-1} + a_{j+1}, & \text{if } UCL_j \le S_i < UCL_{j+1} \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

and

$$CSL_i = \begin{cases} CSL_{i-1} - a_{j+1}, & \text{if } LCL_{j+1} \le S_i < LCL_j \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

for $i = 1, 2, \ldots, n$ and $j = 0, 1, \ldots, k$. The starting values of CSU_0 and CSL_0 are both 0 if there is no FIR procedure. When the value of CS_i reaches or exceeds the critical value K , the two-sided run sum *S* control chart issues an out-of-control signal and assignable cause(s) may occur at that time.

According to [8], three control limits on each side of the center line are used and it is known as the zone control chart. Table I gives the probabilities and scores associated with the two-sided run sum control chart with four regions on each side of the center line.

TABLE I. The Scores and Probabilities Associated with the Two-Sided Run Sum S Control Chart With 4 Regions on Each Side of The Center Line

Region	Interval	Probability	Score
1	$[\mathrm{UCL}_3, +\infty)$	p_1	α_4
2	$[\mathrm{UCL}_2, \mathrm{UCL}_3)$	p_2	α_3
3	$[\mathrm{UCL}_1, \mathrm{UCL}_2)$	p_3	α_2
4	$[\mathrm{CL}, \mathrm{UCL}_1)$	p_4	α_1
5	$[\mathrm{LCL}_1, \mathrm{CL})$	p_5	$-\alpha_1$
6	$[\mathrm{LCL}_2, \mathrm{LCL}_1)$	p_6	$-\alpha_2$
7	$[\mathrm{LCL}_3, \mathrm{LCL}_2)$	p_7	$-\alpha_3$
8	$[0, \mathrm{LCL}_3)$	P_8	$-\alpha_4$

The upper and lower control limits [2,6,18] for the proposed two-sided run sum *S* control chart are

$$UCL_k = \sigma_0 \sqrt{\frac{\chi^2_{n-1;\,\Phi(kC)}}{n-1}} \ , \tag{10}$$

and

$$LCL_k = \sigma_0 \sqrt{\frac{\chi^2_{n-1;\,1-\Phi(kC)}}{n-1}} \ , \tag{11}$$

respectively, where $\Phi(\cdot)$ is the CDF of the standard normal distribution and C is the value selected to create the specified in-control ARL and MRL values. UCL_k and LCL_k denote the three distinct control limits above and below the center line where *k* = 1, 2, 3. Note that the determination of UCL_k and LCL_k of $RS_K\,(a_1, a_2, a_3, a_4)$ chart only depends on the C value. Hence, the design procedure of the run sum *S* control chart is given as

1. Determine the sample size *n*, the critical value *K* and the score vector (a_1, a_2, a_3, a_4).
2. Set an initial value for the in-control ARL (ARL_0), such as $\text{ARL}_0 = 200$, 370 and 500.
3. Calculate the unique value *C* and the control limits UCL_k and LCL_k ($k = 1, 2, 3$) to obtain the ARL_0 of 200.
4. Issue an out-of-control signal when the cumulative scoring CS_i is greater than the critical value *K*.

Simulation in Steps 1 to 4 is done repeatedly for 50,000 times to compute the out-of-control ARL (ARL_1). The same process is repeated for $\text{MRL}_0 = 200$, 370 and 500.

III. Analyses of Result

This study focuses on the discussion of MRL performance. MRL is an excellent alternative to ARL. The ARL and MRL values and the control limits for different sample sizes $n \in \{5, 10, 20\}$, $\text{ARL}_0 \in \{200, 370, 500\}$ and $\text{MRL}_0 \in \{200, 370, 500\}$ are obtained from Monte Carlo simulation using SAS program. The nine schemes $RS_5\ (0,1,2,3)$, $RS_4\ (0,1,2,4)$, $RS_{12}\ (0,1,6,12)$, $RS_{12}\ (0,3,4,12)$, $RS_8\ (0,2,3,8)$, $RS_{14}\ (1,2,7,14)$, $RS_{19}\ (1,3,11,19)$, $RS_{13}\ (1,2,7,13)$ and $RS_{15}\ (1,3,8,15)$ are represented by S_1 to S_9, respectively.

As mentioned in Section I, the in-control run length distribution of a control chart is highly skewed, performance measure solely on ARL could lead to misleading and confusing results. For example, the ARL_0 values are approximately equal to 200 regardless of the run sum *S* control chart schemes while the MRL_0 values which are the 50 percentile of the run lengths are around 140 (see Table II). For all the nine schemes, they indicate that 50% of the run lengths are less than 140 when the ARL_0 is equal to 200. Furthermore, the run length distribution is highly and positively skewed because the median value of the run length distribution is far less than the average value of the distribution.

TABLE II. Comparison of ARL_0 and MRL_0 Values for Nine Schemes When $n \in \{5,10,20\}$ and $\text{ARL}_0 = 200$

Schemes	***n* = 5**		***n* = 10**		***n* = 20**	
	ARL	**MRL**	**ARL**	**MRL**	**ARL**	**MRL**
S_1	199.89	140	199.83	141	200.13	140
S_2	199.87	139	200.16	139	199.86	136
S_3	199.98	139	200.08	139	200.06	135
S_4	200.10	140	200.20	140	199.99	133
S_5	200.13	140	200.20	139	200.11	139
S_6	200.00	136	200.04	139	199.95	139
S_7	199.99	137	199.90	138	199.95	137
S_8	199.72	138	199.96	139	199.95	138
S_9	199.96	138	200.08	140	199.88	138

In order to reduce the rate of false alarm for a better performing control chart, the MRL values should be closer to MRL_0. Whereas, having a small value of the MRL_1 means the control chart could detect the process shifts quickly. In this study, all of the run sum *S* control chart schemes were set with MRL_0 s of 200, 370 and 500 with process standard deviation shifts ranging from $\lambda = 0.20$ to $\lambda = 2.00$.

The MRL performances of the nine schemes with $\text{MRL}_0 = 200$ and sample size $n = 5$, 10 and 50 are given in Tables III to Table V respectively. The first column refers to the process standard deviation shift denoted by λ which varies from 0.20 to 2.00 with an increment of 0.10. For clarity, $\lambda = 0.95$ and $\lambda = 1.05$ are also given in the tables to show the effects of small shifts on the control chart.

In Table III, when the process standard deviation shifted by $\lambda = 0.50$, it means σ_0 decreased by 50% representing a process improvement. For example, S_1, S_2, S_4 and S_5 have the smallest MRL_1 value of 5 when $\lambda = 0.50$. When the process standard deviation shifted by $\lambda = 1.50$ it means σ_0 increased by 50% representing a process deterioration. For example, all the schemes have a common MRL_1 value of 6 when $\lambda = 1.50$. As the λ value moves away from 1, the MRL_1 values decrease. This is because when $\lambda = 1$, the process is functioning at nominal variability where $\sigma_1 = \sigma_0$. Thus, MRL approaches the initial in-control value, i.e. $\text{MRL}_0 = 200$. As λ moves away from 1, say $\lambda > 1$ which corresponds to the increase of the process variability or $\lambda < 1$ corresponds to decrease of the process variability, the MRL_1 values are minimized. Similar trends are observed in Tables IV and V for samples size of 10 and 20.

The boldfaced entries in Table III denote the lowest MRL_1 values among the nine schemes under a given shift of λ. For example, when $\lambda = 0.90$, the smallest value among the nine schemes is 105 which is boldfaced in Table III. The last row denotes the percentage of the best MRL_1 performance which means the smallest MRL_1 value among the nine schemes under different levels of shift. For example, scheme S_1 has 12 boldfaced entries under 20 different shifts of λ (excluded $\lambda = 1.00$), the percentage is given by $12/20 \times 100\% = 60\%$.

From Table III, for the detection of large increasing shifts ($\lambda \geq 1.60$), scheme S_3 has the best performance among all the schemes. For example, when $\lambda = 1.70$, the MRL_1 value for scheme S_3 is 3 while others are 4. For small to moderate increasing shifts ($1.00 < \lambda < 1.60$), S_1 outperforms all the other schemes because it attains the smallest MRL value for those shifts (except for $\lambda = 1.30$). When $\lambda = 1.30$, S_2 performs best with the MRL_1 value of 12.

TABLE III. MRL VALUES FOR THE NINE SCHEMES OF RUN SUM S CONTROL CHART WHEN $\text{MRL}_0 = 200$ AND $n = 5$

λ	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
0.20	**2**	**2**	**2**	3	3	**2**	**2**	**2**	**2**
0.30	3	3	**2**	4	3	3	3	4	4
0.40	**4**	**4**	**4**	**4**	**4**	5	**4**	**4**	**4**
0.50	**5**	**5**	7	**5**	**5**	6	6	6	6
0.60	**7**	**7**	12	**7**	**7**	9	9	9	8
0.70	**12**	14	27	13	13	14	15	14	13
0.80	**32**	37	80	36	36	36	38	34	33
0.90	106	117	189	118	117	114	121	109	**105**
0.95	**179**	185	228	188	187	184	187	181	**179**
1.00	200	200	200	200	200	200	200	200	200
1.05	**127**	128	132	131	130	130	129	131	132
1.10	**68**	69	76	75	73	73	71	74	73
1.20	**24**	**24**	28	28	26	26	25	27	26
1.30	13	**12**	13	14	13	13	13	14	13
1.40	**8**	**8**	**8**	9	**8**	**8**	**8**	9	**8**
1.50	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**6**
1.60	5	**4**	**4**	5	5	5	5	5	5
1.70	4	4	**3**	4	4	4	4	4	4
1.80	4	**3**	**3**	4	**3**	**3**	**3**	**3**	4
1.90	3	3	**2**	3	3	3	3	3	3
2.00	3	**2**	**2**	**2**	**2**	**2**	**2**	**2**	3
%	60%	55%	50%	25%	30%	25%	30%	25%	30%

[a] Boldfaced values indicate the smallest MRL_1 values within the λ.

[b] The last row percentages indicate the percentage of every scheme having the smallest MRL_1 values.

For large decreasing shifts, S_1 and S_2 have the same performance of MRL_1 values of 2, 3, 4, 5 given $\lambda = 0.20, 0.30, 0.40, 0.50$ respectively. The larger the process standard deviation shift, the less significant the MRL_1 values differ among the nine run sum S control chart schemes. For example, when $\lambda = 0.20$, seven of nine schemes (except for S_4 and S_5) have the same MRL_1 values of 2 and when $\lambda = 2.00$, there are also seven schemes ($S_2 - S_8$) with the same MRL_1 values of 2.

Overall performance from Table III indicates S_1 has the best performance because 60% of the MRL_1s of S_1 scheme are less than or equal to that of the other schemes, across all the λ shifts. It is followed by S_2 and S_3 schemes with 55% and 50%, respectively. Among the nine schemes, S_4, S_6 and S_8 have the worst performance because only 25% of the MRL_1s of these schemes are less than or equal to that of the other six schemes, across all the λ shifts. The efficiency in detecting shifts of S_1, S_2 and S_3 is more than or equal to two times of the three schemes. The similar result is obtained for n = 10 as presented in Table IV. However, when sample size increases to 20, different scheme's performance is observed.

Due to space constraints, the tables of $\text{MRL}_0 = 370$ and $\text{MRL}_0 = 500$ are not presented. They are available from the corresponding author upon request.

TABLE IV. MRL VALUES FOR THE NINE SCHEMES OF RUN SUM S CONTROL CHART WHEN $\text{MRL}_0 = 200$ AND $n = 10$

λ	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
0.20	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
0.30	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
0.40	**2**	**2**	**2**	3	3	**2**	**2**	**2**	**2**
0.50	3	3	**2**	4	3	3	3	3	4
0.60	**4**	**4**	**4**	**4**	**4**	5	**4**	5	**4**
0.70	**6**	**6**	9	**6**	**6**	7	7	8	7
0.80	**12**	13	23	13	13	14	14	14	13
0.90	**50**	56	107	58	56	58	60	55	51
0.95	**130**	141	187	142	140	135	140	134	**130**
1.00	200	200	200	200	200	200	200	200	200
1.05	**94**	96	113	104	101	102	102	104	100
1.10	**38**	39	51	43	42	43	42	42	41
1.20	**12**	**12**	14	13	13	13	13	13	13
1.30	**6**	**6**	7	7	7	7	7	7	7
1.40	**4**	**4**	**4**	5	5	5	5	5	5
1.50	4	**3**	**3**	4	4	**3**	**3**	4	4
1.60	3	3	**2**	3	3	3	3	3	3
1.70	3	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
1.80	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
1.90	2	2	**1**	**1**	**1**	**1**	2	2	2
2.00	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
%	60%	60%	60%	40%	40%	40%	40%	30%	40%

TABLE V. MRL VALUES FOR THE NINE SCHEMES OF RUN SUM S CONTROL CHART WHEN $\text{MRL}_0 = 200$ AND $n = 20$

λ	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
0.20	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
0.30	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
0.40	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
0.50	2	2	**1**	**1**	**1**	**1**	**1**	2	2
0.60	3	**2**	**2**	3	3	**2**	**2**	**2**	3
0.70	4	**3**	4	4	4	5	4	4	4
0.80	**6**	**6**	9	7	**6**	7	7	8	7
0.90	**23**	25	46	26	26	27	27	26	24
0.95	**80**	89	131	88	88	88	90	85	81
1.00	200	200	200	200	200	200	200	200	200
1.05	**66**	67	85	72	70	70	71	71	68
1.10	**21**	22	29	24	23	23	23	24	22
1.20	7	6	7	7	7	7	7	7	**4**
1.30	4	4	4	4	4	4	4	5	**3**
1.40	3	3	**2**	3	3	3	3	3	**2**
1.50	3	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
1.60	2	2	2	**1**	**1**	**1**	**1**	2	**1**
1.70	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1.80	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1.90	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
2.00	2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
%	25%	55%	55%	50%	55%	55%	55%	45%	60%

IV. CONCLUSION

In this paper, run sum S control charts are used to detect the shift in process standard deviation based on MRL performance. MRL is a wonderful alternative to ARL due to the high skewness of the run length distribution. For small sample sizes of 5 and 10, schemes S_1, S_2 and S_3 have better performances compared with other schemes followed by S_9. Specifically, S_1 and S_9 perform better for small to moderate shifts in the process standard deviation while schemes S_2 and S_3 outperform for large shifts.

However, the scheme's performance is different when $n = 20$. The performance is also affected by the MRL_0. Thus, we recommend the practitioners should select the proper scheme according to the magnitude of the process standard deviation shifts, sample sizes and the in-control run lengths in the production. A good direction to the future studies is to divide the run sum S charts into more regions.

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