A Finite-Time Sliding Mode Controller Design for Flexible Joint Manipulator Systems Based on Disturbance Observer

Dao Phuong Nam, Pham Thanh Loc, Nguyen Van Huong, and Do Trong Tan
Hanoi University of Science and Technology, Hanoi, Vietnam
Email: nam.daophuong@hust.edu.vn, phamthanhhloc@gmail.com, huong.nguyenvan2102@gmail.com, trongtan@live.com

***Abstract*—Flexible joint single-link manipulator is a nonlinear system with many applications in industry such as car assembly plant, beer factory. However, this manipulator has many disadvantages including unknown parameters, external disturbances as well as holonomic constraint force. In order to overcome these challenges, many researchers employ the disturbance attenuation control scheme. It was hard to obtain the asymptotic stability in closed system due to the influence of disturbances. Moreover, the attraction region was also estimated exactly because of no knowledge of disturbance influence. This paper presents an external disturbance observer (DO) for flexible joint single-link manipulator. Moreover, the arbitrary small attraction region is obtained by using the suitable parameters. The main result of this paper is proposed based on theoretical analysis of differential equations without any traditional Lyapunov stability analysis. Furthermore, several explorations that depend on parameters are given out. Offline simulation results pointed out the high effectiveness of the proposed methods.**

***Index Terms*—disturbance observer (DO), sliding mode control (SMC), single-link manipulator, flexible joint**

I. INTRODUCTION

Almost all industrial systems are affected by external disturbances, such as manipulator control systems and robotic systems [1]. Because of removing disturbances that probably cannot be measured, we estimate them to design sliding mode control based on disturbance observer (DO) [2]. However, authors in [2] have not discussed about the arbitrary small attraction region by using the suitable parameters. In [3], the proposed controller ensures the improvement of disturbance attenuation. In contrast, the reduction of computation amount is the important task. Besides, external disturbance mentioned depends only on time. Our work uses the sliding mode control to absolutely obtain arbitrary small attraction region based on the suitable parameters. In [1], the flexible joint is a phenomenon including a loose connection between the motor and the link in the installation, or because of the material's substantial twisting properties, which results in a different angle of rotation of the motor and the arm. Soft couplings are an important accessory in pipe design that connects the parts together to ensure the stability of the operating system. In addition, the coupling also functions to significantly reduce the load, while preventing overload [4, 5, 6]. Many control schemes have been utilised based on robust adaptive control without estimating disturbance [7-11]. Moreover, we explore the influences of parameters to obtain results. The paper is organised as follows: In the second section, we focus on problem statements. In the next sections, we pay attention to design the proposed controller for single-link manipulator and explore the influences of the parameters. In the fifth section, we present the simulation results providing evidences for these theoretical analyses. The final section offers a brief conclusion.

Manuscript received June 4 2018; revised May 25 2019.

II. PROBLEM STATEMENT

Consider that a class of nonlinear system is described as follows [2]:

$$\begin{cases} \dot{x}_1 = x_2 + d_1(x,t) \\ \dot{x}_2 = x_3 + d_2(x,t) \\ . \\ . \\ . \\ \dot{x}_{n-1} = x_n + d_{n-1}(x,t) \\ \dot{x}_n = a(x) + b(x)u + d_n(x,u,t) \\ y = x_1 \end{cases} \tag{1}$$

Here, $x = \left[x_1\ x_2\ \dots\ x_n\right]^T \in \mathbb{R}^n$ is the vector of state variables, $u \in \mathbb{R}$ is the control input, and $y \in \mathbb{R}$ is the output signal. The external disturbance $d_i(x,t)\ i = 1,2,\dots,n-1$ does not depend on control input, while the disturbance $d_n(x,u,t)$ depends on $u \in \mathbb{R}$. The functions $a(x), b(x)$ are continuous functions and $b(x) \neq 0; \forall x$.

Assumption 1: Every disturbance is continuous and satisfying the inequalities:

$$\left|\frac{d^j d_i(x,t)}{dt^j}\right| \le \mu\ i = 1,2,\dots,n;\ j = 0,1,\dots,r$$

doi: 10.18178/ijmerr.8.4.619-625

The control objective is to find the control input to obtain that the output signal converges to attraction region in the presence of external disturbances.

III. External DO Based Control

Initially, we concentrate on designing the DO as described:

The system (1) is affected by disturbances $d_i(x,t)\ i=1,2,...,n-1$. We implement the observer to estimate disturbances based on the following formulas as described in [2]:

$$\begin{cases} \hat{d}_i^{(j-i)} = p_{ij} + l_{ij}x_i \\ \dot{p}_{ij} = -l_{ij}\left(x_{i+1} + \hat{d}_i\right) + \hat{d}_i^{(j)};\ j=1,2,...,(r-1); i=1,2,...,(n-1) \\ \dot{p}_{ir} = -l_{ir}\left(x_{i+1} + \hat{d}_i\right) \end{cases} \quad (2)$$

On the other hand, towards the component $d_n(x,u,t)$, the corresponding observer has been pointed out [2]:

$$\begin{cases} \hat{d}_n^{(j-1)} = p_{nj} + l_{nj}x_n \\ \dot{p}_{nj} = -l_{nj}\left(a(x) + b(x)u + \hat{d}_n\right) + \hat{d}_n^{(j)};\ j=1,2,...,(r-1) \\ \dot{p}_{nr} = -l_{nr}\left(a(x) + b(x)u + \hat{d}_n\right) \end{cases} \quad (3)$$

where p_{ij} is auxiliary variable and l_{ij} is the arbitrary positive constant ensuring that the matrix,

$$D_i = \begin{bmatrix} -l_{i1} & 1 & 0 & ... & 0 \\ -l_{i2} & 0 & 1 & ... & 0 \\ . & . & . & ... & . \\ . & . & . & ... & . \\ . & . & . & ... & . \\ -l_{i(r-1)} & 0 & 0 & ... & 1 \\ -l_{ir} & 0 & 0 & ... & 0 \end{bmatrix}$$

has all eigenvalues belonging to the left side of the complex coordinate plane.

Remark 1: We absolutely adjust the speed of observer error by selecting the eigenvalues.

Remark 2: It is necessary to ensure that the time of convergence of sliding surface is finite. The fact is described based on the following example:

We consider the system as follows:

$$\begin{cases} \dfrac{dx}{dt} = Ax + Bs \\ \dfrac{ds}{dt} = Cx + Ds \end{cases}$$

where: $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times r}, C \in \mathbb{R}^{r\times n}, D \in \mathbb{R}^{r\times r}$, s is the sliding surface. Selecting A is Hurwitz matrix and $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is not Hurwitz matrix. We obtain that although s converges to 0 in infinite time, x does not converge to 0.

In [2], the sliding surface has been selected:

$$\sigma = \sum_{i=1}^{n} c_i x_i + \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} c_{j+1}\hat{d}_i^{(j-i)}; c_n = 1$$

and the modified sliding surface:

$$\sigma^* = \sigma - \sigma(0)e^{-\alpha t}$$

where α is arbitrary positive constant.

From the estimation (2) and (3), the control input has been proposed in [2]:

$$u = \frac{-1}{b(x)}\begin{bmatrix} a(x) + \hat{d}_n + \alpha\sigma(0)e^{-\alpha t} \\ +\sum_{i=1}^{n-1} c_i\left(x_{i+1} + \hat{d}_i\right) + k_l\sigma^* + k_s sat\left(\sigma^*\right) \end{bmatrix} \quad (4)$$

$$k_l, k_s > 0$$

where $sat\left(\sigma^*\right) = \begin{cases} \operatorname{sgn}\left(\sigma^*\right) & when\ \left|\sigma^*\right| > \varepsilon \\ \dfrac{\sigma^*}{\varepsilon} & when\ \left|\sigma^*\right| \le \varepsilon \end{cases}$.

The stability analysis has been proved in [2]. We obtain that: $\|\tilde{e}_i\| \le \frac{2\|P_i E_i\|\mu_i}{\lambda_{mi}}$, where $\tilde{e}_i = \begin{bmatrix} \tilde{d}_i & \dot{\tilde{d}}_i & ... & \tilde{d}_i^{(r)} \end{bmatrix}$, Q_i is a given positive defined matrix , λ_{mi} is the smallest eigenvalue of Q_i and P_i is the solution of the equation: $D_i^T P_i + P_i D_i = -Q_i$. Let

$$\lambda_1 = \max_i \frac{2\|P_i E_i\|\mu_i}{\lambda_{mi}} \quad (5)$$

Therefore, $\|\tilde{e}_i\| \le \lambda_1$ for all i .

IV. Main Results

Firstly, we propose the analysis of parameters' efficiency on control system as follows:

- The term $\frac{-1}{b(x)}k_l\sigma^*$ guarantees the chatter reduction combined with selecting suitable value k_s described as follows:

$$\left|\sigma^*\right| \le \lambda_2 = \frac{A\mu + nB\lambda_1 - k_s}{k_l}$$

where A is defined as the sum of coefficients $d_i^{(j)}$ obtained by:

$$\dot{\sigma}^* = -k_l\left(\sigma^*\right) - k_s sat\left(\sigma^*\right) + \sum_{i=1}^{n-1}\sum_{j=i}^{n-1} c_{j+1}\left(d_i^{(j-i+1)} - \tilde{d}_i^{(j-i+1)}\right) + \sum_{i=1}^{n-1}\left[c_i + \sum_{j=1}^{n-1} c_{j+1} l_{i(j-i+1)}\right]\tilde{d}_i .$$

B is the maximum value among the coefficients $\hat{d}_i^{(j)}$.

- The selections of positive constant numbers $l_{ij}\ \left(i=1,2,...,n; j=1,2,...,n\right)$ in order to obtain the matrix D_i have all eigenvalues that are far from imaginary axis. It can be clearly seen that the higher distance would certainly increase the convergence speed of estimation errors.
- The increasing of coefficient c_i increases the convergence speed. However, the chatter of modified surface goes up respectively.

Remark 3: The above conclusions can be proved easily and they will be illustrated clearly in the first case of the next section. In [2], the authors put forward to deal with the uncertain systems by transforming $\dot{x}_n = a(x) + b(x)u + d_n(x,u,t)$ with $a(x), b(x)$ containing uncertain components and control signal u contains the external unmeasured disturbance to $\dot{x}_n = \hat{a}(x) + \hat{b}(x)\hat{u} + \hat{d}_n(x,u,t)$ with $\hat{a}(x), \hat{b}(x), \hat{u}$ that are measured signals and:

$$\hat{d}_n(x,u,t) = a(x) + b(x)u + d_n(x,u,t) - \hat{a}(x) - \hat{b}(x)\hat{u} .$$

Following this method, the authors have to know full information of the parameter of the system and it is clear that the bound region of state may be large. Therefore, we construct a new control law for the class of partly unknown systems below:

$$\begin{cases} \dot{x}_1 = x_2 + d_1(x,t) \\ \dot{x}_2 = x_3 + d_2(x,t) \\ . \\ . \\ . \\ \dot{x}_{n-1} = x_n + d_{n-1}(x,t) \\ \dot{x}_n = \theta a(x) + b(x)u + d_n(x,u,t) \\ y = x_1 \end{cases} \tag{6}$$

where the parameter θ is unknown vectors and $d_n(x,u,t)$ included uncertain parameters and disturbance of system.

Similar to (1), (2), (3), we choose:

$$\begin{cases} \sigma = \sum_{i=1}^{n} c_i x_i + \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} c_{j+1}\hat{d}_i^{(j-i)}; c_n = 1 \\ \sigma^* = \sigma - \sigma(0)e^{-\alpha t} \end{cases}$$

The control law is defined:

$$u = \frac{-1}{b(x)}\begin{bmatrix} \hat{\theta}a(x) + \hat{d}_n + \alpha\sigma(0)e^{-\alpha t} \\ +\sum_{i=1}^{n-1} c_i\left(x_{i+1} + \hat{d}_i\right) + ksat\left(\sigma^*\right) \end{bmatrix} \tag{7}$$

with the adaptive law of θ designed as follows:

$$\dot{\hat{\theta}} = \rho a(x)\sigma^* \tag{8}$$

and ρ is a positive number.

As with (5), we can prove that: $\|\tilde{e}_i\| \le \lambda_1$.

Consider Lyapunov function $V(t) = \frac{1}{2}\left(\sigma^*\right)^2 + \frac{1}{2\rho}\tilde{\theta}^2$ with the estimate error of parameter $\tilde{\theta} = \theta - \hat{\theta}$, the time derivative of V is as follows:

$$\dot{V} = \left(\sigma^*\right)\dot{\sigma}^* + \frac{1}{\rho}\left(\tilde{\theta}\right)\dot{\tilde{\theta}}$$

$$\begin{aligned} \dot{V} &= \left(\sigma^*\right)a(x)\left(\theta - \hat{\theta}\right) - k\sigma^* sat\left(\sigma^*\right) + \sum_{i=1}^{n-1}\sum_{j=i}^{n-1} c_{j+1}\hat{d}_i^{(j-i+1)}\sigma^* \\ &+ \sum_{i=1}^{n-1}\left[c_i + \sum_{j=i}^{n-1} c_{j+1}l_{i(j-i+1)}\right]\tilde{d}_i\sigma^* - \frac{1}{\rho}\left(\tilde{\theta}\right)\dot{\hat{\theta}} \end{aligned} .$$

From (8), we have:

$$\begin{aligned} \dot{V} &= -k\sigma^* sat\left(\sigma^*\right) + \sum_{i=1}^{n-1}\sum_{j=i}^{n-1} c_{j+1}\hat{d}_i^{(j-i+1)}\sigma^* \\ &+ \sum_{i=1}^{n-1}\left[c_i + \sum_{j=i}^{n-1} c_{j+1}l_{i(j-i+1)}\right]\tilde{d}_i\sigma^* \end{aligned} .$$

Since the bound of $\|\tilde{e}_i\|$ and assumption 1, there exists a finite number η such that:

$$\eta \ge \left| \sum_{i=1}^{n-1}\sum_{j=i}^{n-1} c_{j+1}\hat{d}_i^{(j-i+1)} + + \sum_{i=1}^{n-1}\left[c_i + \sum_{j=i}^{n-1} c_{j+1}l_{i(j-i+1)}\right]\tilde{d}_i \right| .$$

Therefore:

$$\dot{V} \le -k\left|\sigma^*\right| + \eta\left|\sigma^*\right| \le (-k+\eta)\sqrt{2}V^{1/2}$$

If we choose $k > \eta$, we have:

$$\frac{dV}{2V^{1/2}} \le -\frac{dt}{2\sqrt{2}(k-\eta)} \tag{9}$$

Integrating (9) over the time interval $0 \le \tau \le t$ we obtain:

$$V^{1/2}(t) - V^{1/2}(0) \le -\frac{t}{2\sqrt{2}(k-\eta)} .$$

Consequently, $V(t)$ can reach zero in a finite time t_s that is bounded by:

$$t_s \le \frac{V^{1/2}(0)}{2\sqrt{2}(k-\eta)} \tag{10}$$

V. Simulation Results

We implement simulations in the two cases:

In the first case, we consider the flexible joint manipulator being described by the following equations:

$$\begin{cases} I\ddot{q}_1 + MgL\sin(q_1) + K(q_1 - q_2) = 0 \\ J\ddot{q}_2 + K(q_2 - q_1) = u \end{cases} \tag{11}$$

where I, J are the inertia moments of joint, motor, respectively. M and L are the mass and length of link. K is the stiffness of the joint.

We give transformation from (1) into (11) by using:

$$\begin{cases} z_1 = q_1 \\ z_2 = \dot{q}_1 \\ z_3 = q_2 \\ z_4 = \dot{q}_2 \end{cases} \tag{12}$$

We obtain:
$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -\frac{MgL}{I}\sin(z_1) + \frac{K}{I}(z_3 - z_1) \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = \frac{K}{J}(z_1 - z_3) + \frac{u}{J} \end{cases} \tag{13}$$

Remark 4: The conditions of system $\dot{x} = f(x) + g(x)u$ to be transformed into the parametric strict-feedback form can be found in [12].

In order to obtain (1), we define:

$$(x_1 \quad x_2 \quad x_3 \quad x_4)^T = \left(z_1 \quad z_2 \quad \frac{Kz_3}{I} \quad \frac{Kz_4}{I}\right)^T .$$

We get the following equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 - \dfrac{MgL}{I}\sin(x_1) - \dfrac{K}{I}x_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{K}{J}x_1 - \dfrac{I}{J}x_3 + \dfrac{u}{J} \end{cases}. \quad (14)$$

However, because the manipulator is affected by external disturbance and uncertainties, therefore we obtain new dynamic equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 - \dfrac{MgL}{I}\sin(x_1) - \dfrac{K}{I}x_1 + \Delta\dfrac{MgL}{I}\sin(x_1) + \Delta\dfrac{K}{I}x_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{K}{J}x_1 - \dfrac{I}{J}x_3 + \dfrac{u}{J} + \Delta\dfrac{K}{J}x_1 + \Delta\dfrac{I}{J}x_3 + \Delta\dfrac{u}{J} + \dfrac{v}{J} \end{cases} \quad (15)$$

where $-0.1 \le \Delta \le 0.1$ is described by parameter error, v is the external disturbance of input. Define:

$$\begin{cases} d_2(x) = -\dfrac{MgL}{I}\sin(x_1) - \dfrac{K}{I}x_1 + \Delta\dfrac{MgL}{I}\sin(x_1) + \Delta\dfrac{K}{I}x_1 \\ d_4(x,t) = \Delta\dfrac{K}{J}x_1 + \Delta\dfrac{I}{J}x_3 + \Delta\dfrac{u}{J} + \dfrac{v}{J} \\ a(x) = \dfrac{K}{J}x_1 - \dfrac{I}{J}x_3 \\ b(x) = \dfrac{1}{J} \end{cases} \quad (16)$$

Based on the general design in (4), we obtain the control input as follows:

Step 1: Select the DO

We can implement the order 2 or 3 DO and the order 3 is better. Thus, we obtain the result:

$$d_2 : \begin{cases} \hat{d}_2 = p_{21} + l_{21}x_2 \\ \dot{p}_{21} = -l_{21}(x_3 + \hat{d}_2) + \hat{\dot{d}}_2 \\ \hat{\dot{d}}_2 = p_{22} + l_{22}x_2 \\ \dot{p}_{22} = -l_{22}(x_3 + \hat{d}_2) + \hat{\ddot{d}}_2 \\ \hat{\ddot{d}}_2 = p_{23} + l_{23}x_2 \\ \dot{p}_{22} = -l_{23}(x_3 + \hat{d}_2) \end{cases}$$

$$d_4 : \begin{cases} \hat{d}_4 = p_{41} + l_{41}x_4 \\ \dot{p}_{41} = -l_{41}(a(x) + b(x)u + \hat{d}_4) + \hat{\dot{d}}_4 \\ \hat{\dot{d}}_4 = p_{42} + l_{42}x_4 \\ \dot{p}_{42} = -l_{42}(a(x) + b(x)u + \hat{d}_4) + \hat{\ddot{d}}_4 \\ \hat{\ddot{d}}_4 = p_{43} + l_{43}x_4 \\ \dot{p}_{23} = -l_{23}(a(x) + b(x)u + \hat{d}_4) \end{cases}$$

Step 2: Sliding Surface and control input

The sliding surface and modified sliding surface are selected as follows:

$$\sigma = c_1x_1 + c_2x_2 + c_3x_3 + x_4 + c_3\hat{d}_2 + \hat{\dot{d}}_2$$

$$\sigma^* = c_1x_1 + c_2x_2 + c_3x_3 + x_4 + c_3\hat{d}_2 + \hat{\dot{d}}_2 - \sigma(0)e^{-\alpha t}.$$

The control input is obtained:

$$u = -\frac{1}{b(x)}\Big[a(x) + \hat{d}_4 + \alpha\sigma(0)e^{-\alpha t} + c_1(x_2 + \hat{d}_1) + c_2(x_3 + \hat{d}_2) + c_3(x_4 + \hat{d}_3) + k_l\sigma^* + k_s sat(\sigma^*)\Big].$$

We have $\left|\sigma^*\right| \le \lambda_2$

with: $\lambda_2 = \dfrac{\lambda_1(c_2 + c_3l_{21} + c_4l_{22} + 2c_3) + 2c_3\mu - k_s}{k_l}$ (17)

We absolutely obtain arbitrary small attraction region by using the suitable parameters.

Step 3: Algorithm

Find the parameters of (1)

Estimate the disturbance of input

Select $l_{21}, l_{22}, l_{23}, l_{41}, l_{42}, l_{43} > 0$ satisfy matrix $\begin{bmatrix} -l_{21} & 1 & 0 \\ -l_{22} & 0 & 1 \\ -l_{23} & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} -l_{41} & 1 & 0 \\ -l_{42} & 0 & 1 \\ -l_{43} & 0 & 0 \end{bmatrix}$ have eigenvalue belonging the left side of complex coordinate plane. Select the set $\{s_1, s_2, s_3\}$ such that each element is positive and big enough. From the equations:

$$(s+s_1)(s+s_2)(s+s_3) = s^3 + l_{21}s^2 + l_{22}s + l_{23} = s^3 + l_{41}s^2 + l_{42}s + l_{43}$$

We can choose $\{l_{21}, l_{22}, l_{23}, l_{41}, l_{42}, l_{43}\}$.

Selected $\{s_4, s_5, s_6\}$ are positive and big enough.

From the equation: $(s+s_4)(s+s_5)(s+s_6) = s^3 + c_3s^2 + c_2s + c_1$, we can choose $\{c_1, c_2, c_3\}$.

Selecting $|\Delta| < 0.1, k_s > 0$, $\alpha > 0$ big enough and $k_l > 0$ is bigger than l_{ij}, c_i.

Finally, we obtain the control law.

Simulation results based on parameters (Table I) as follows:

TABLE I. THE PARAMETER OF SYSTEM

I	J	M	G	L	K
Inertia moment of joint	Inertia moment of motor	Mass of Link	Gravity	Length of Link	Stiffness
$kg.m^2$	$kg.m^2$	kg	$m.s^{-2}$	m	N.m/rad
1	2	1	10	2	100

A. *Change the Parameter* l_{ij}

The estimation errors in d_2 and d_4 are reduced when eigenvalues of D_i matrix change. Because in [2], we have the following equation:

$$\dot{\tilde{e}}_i = D_i\tilde{e}_i + E_i d_i^{(r)} \quad (18)$$

The estimation error in d_2 and d_4 are reduced if the eigenvalues less than 0 and far from imaginary axis.

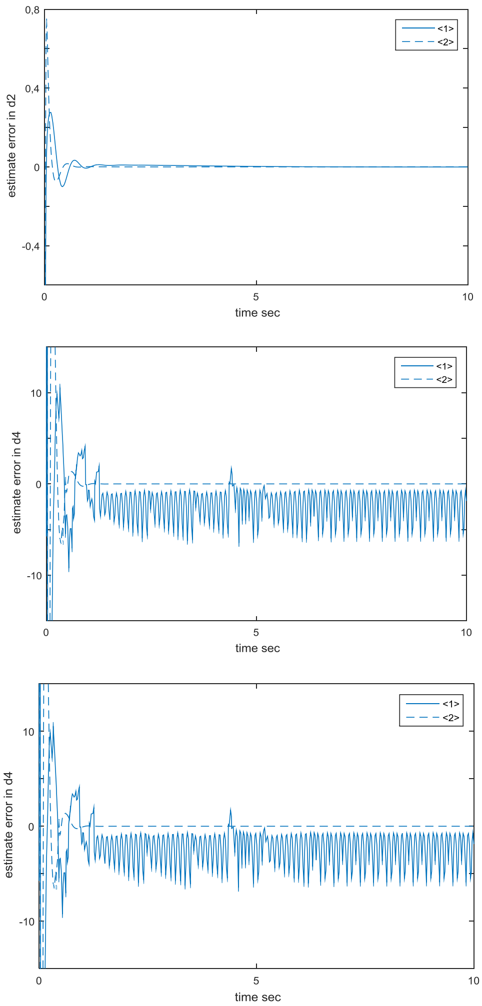

Figure 1. Result of change l_{ij}

$$<1>\ l_{21}=l_{41}=100, l_{22}=l_{42}=20$$
$$<2>\ l_{21}=l_{41}=100, l_{22}=l_{42}=1875$$

B. *Change of the Parameter c_i*

If the set of parameters c makes the polynomial's root less than 0 and far from imaginary axis, the $\tilde{d}_2, \tilde{d}_4, \sigma^*, x_1$ comes to the domain of attraction fast. The domain of attraction is less.

Since we have the following: $\exists T<\infty: \left|\sigma^*\right|<\lambda_2 \ \ \forall t>T$ (19)

Let's look at the case at the border:

$$c_1 x_1 + c_2 \dot{x}_1 + c_3 \ddot{x}_1 + \dddot{x}_1 = \pm\lambda_2 + \sigma(0)e^{-\alpha t} \qquad (20)$$

The root of the equation (20) is as follows:

$$x = \frac{\pm\lambda_2}{c_1} + Ae^{-s_1 t} + Be^{-s_2 t} + Ce^{-s_3 t} + De^{-s_4 t} + Ee^{-\alpha t} \qquad (21)$$

Therefore, in the best case (at the boundary), the output signal comes to the neighbourhood of 0 when we select parameters to $\{s_1, s_2, s_3, s_4, \alpha\}$ that are big. Subsequently, $c_1 = s_1 s_2 s_3 s_4$ is a big number.

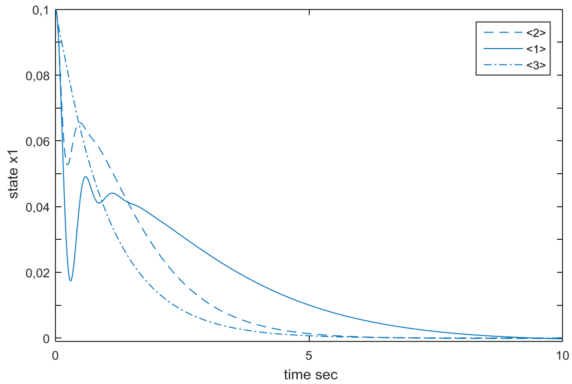

Figure 2. Result of change c_1

$$<1>\ c_1=125, c_2=75, c_3=15$$
$$<2>\ c_1=1000, c_2=300, c_3=30$$
$$<3>\ c_1=1000000, c_2=30000, c_3=300$$

C. *Change of the Parameter α*

If α becomes large, $\tilde{d}_2, \tilde{d}_4, \sigma^*, x_1$ will come fast to the attractive region.

We select $\sigma^* = \sigma - \sigma(0)e^{-\alpha t}$ to have $\sigma^*(0)=0\le\lambda_2$.

The component $e^{\alpha t}$ is selected because $\forall n: \lim_{t\to\infty}\frac{d^n e^{\alpha t}}{dt^n}=0$.

However, if α becomes large, $\frac{d\sigma^*}{dt}(0)$ increase and σ^* can be cone large when $t\approx 0$.

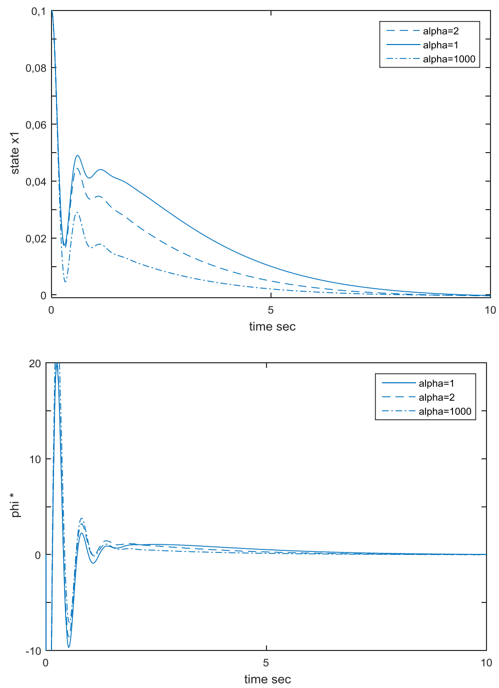

Figure 3. Result of change α

D. *Change of k_l and k_s*

Because of equation (17), we think that if k_l and k_s increase, σ^*, x_1 will come to the attractive domain fast. The domain of attraction is less.

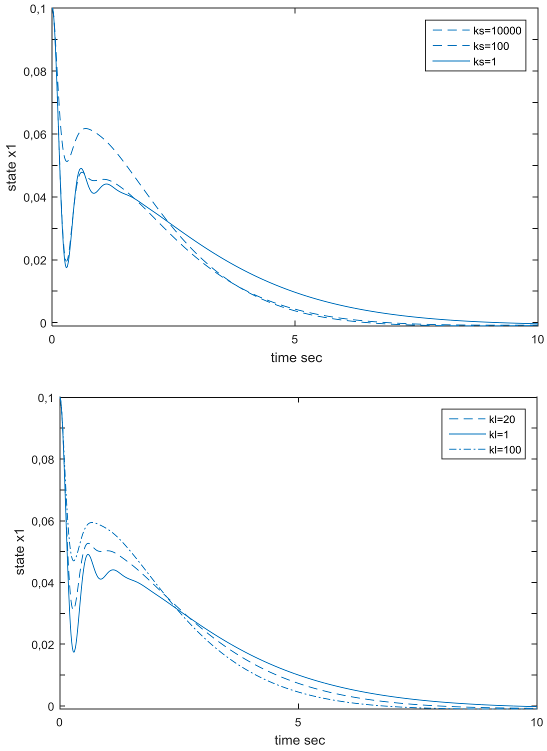

Figure 4. Result of change of k_l *and* k_s

E. The Degree of the DO

If we increase the degree of the DO, we will have better results. The estimation error in d_2 and d_4 are reduced.

We can find the difference between 2nd-DO and 3rd-DO from Fig. 5.

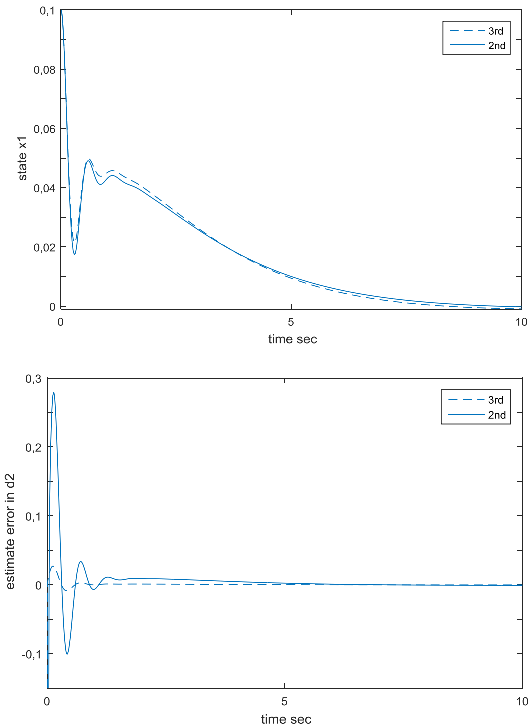

Figure 5. Comparison between 2nd-DO and 3rd-DO

In the second case, we consider the following system being the individual case of nonlinear system (1):

$$\begin{cases}\dot{x}_1 = x_2 + d_1(x,t)\\ \dot{x}_2 = \begin{bmatrix}\theta_1 & \theta_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + u + \cos(x_2)\\ y = x_1\end{cases}$$

where:

$$d_1 = x_2\sin(t) + x_1^2;\quad x(0) = \begin{bmatrix}0.1 & 0\end{bmatrix}^T;$$

$$k_l = 10;\ k_s = 0;\ c_1 = 10;\ c_2 = 1; \alpha = 1.$$

$$l_{11} = 100; l_{12} = 20;$$

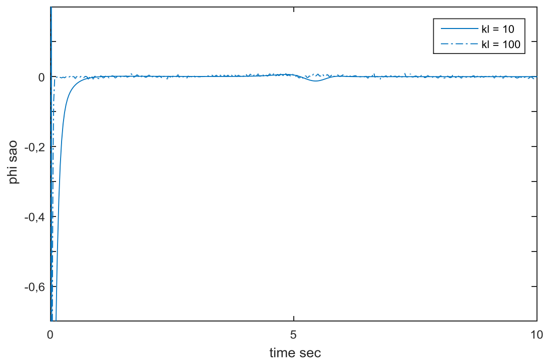

Figure 6. The dynamic of modified sliding surface

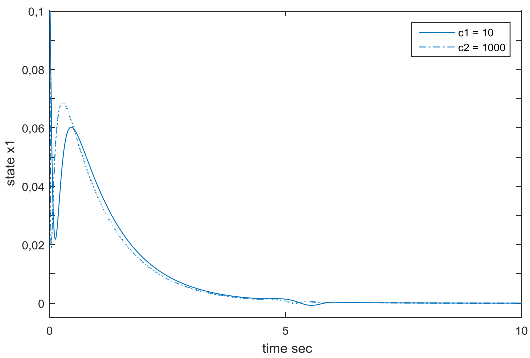

Figure 7. The behaviour of output signal corresponding to variation of c

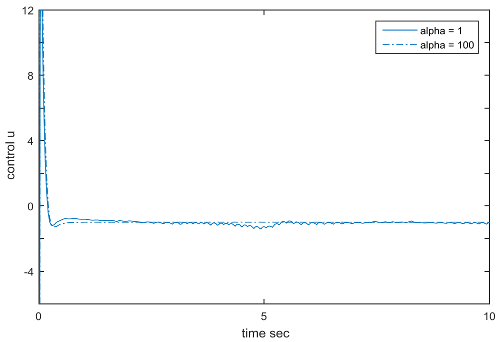

Figure 8. The behaviour of input control signal corresponding to variation of α

Simulation results show good points of sliding surface and input and output (Figs. 6, 7, 8) in our method. In Fig. 6, the first result shows that when k_l increases from 10 to 100, the border of the attraction region of modified sliding surface is smaller but the level of chatter goes up. The second result describes that when c_1 increases from 10 to 1000, the output signal converges to the attraction region faster. The last result is clearly seen that when α has a significant change from 1 to 100, the chatter of the input control will be reduced.

VI. Conclusion

This work proposed the sliding mode control law based on disturbance observer for single-link manipulator, which is a nonlinear system with unknown parameters, external disturbances as well as holonomic constraint force Thanks to the proposed example, the problem of finite time in sliding mode control was mentioned as well as the estimation of this time was pointed out by using the theoretical analysis of differential equations. Moreover, we absolutely obtain arbitrary small attraction region by using the suitable parameters. It can be noted that this estimation of attraction region enable us to obtain the better result than previous work in [11] without any traditional Lyapunov stability analysis. We propose a new method for partly unknown systems with disturbance. Furthermore, several novel estimation technique that depend on parameters are given out. Thanks to the proposed solution, the tracking problem was determined with attraction region estimation. The theory analysis and simulation results demonstrate the proposed control algorithm.

Acknowledgments

The authors gratefully acknowledge the constructive and insightful comments of reviewers to further improve the quality of this paper. This work is supported in part by the Hanoi University of Science and Technology (T2017-PC-105).

References

[1] K. Melhem and W. Wang, "Global output tracking control of flexible joint robots via factorization of the manipulator mass matrix," *IEEE Transaction on Robotics*, vol. 25, no. 2, pp. 428-437, 2009.

[2] D. Ginoya, P. D. Shendge, and S. B. Phadke, "Sliding mode /control for mismatched uncertain systems using an extended disturbance observer," *IEEE Transaction on Industrial Electronics*, vol. 61, no. 4, pp. 1983-1992, 2014.

[3] Chen, "Disturbance observer based control for nonlinear systems," *IEEE/ASME Transaction on Mechatronics*, vol. 9, no. 4, pp. 706-710, 2004.

[4] Chen *et al*, "A nonlinear disturbance observer for robotic manipulators," *IEEE Transaction on Industrial Electronics*, vol. 47, no. 4, pp. 932-938, 2000.

[5] A. Albu-Schäffer, C. Ott, and G. Hirzinger, "A Unified passivity based control framework for position, torque and impedance control of flexible joint robots," *The International Journal of Robotics Research*, vol. 26, no. 1 pp. 23 – 39 , 2007.

[6] S. E. Talole, J. P. Kolhe, and S. B. Phadke, "Extended-state-observer-based control of flexible-joint system with experimental validation," *IEEE Transaction on Industrial Electronics*, vol. 57, no. 9, pp. 1411-1419, 2010.

[7] An-Chyau Huang and Yuan-Chih Chen, "Adaptive sliding control for single-link flexible-joint robot with mismatched uncertainties," *IEEE Transaction on Control Systems Technology*, vol. 10, no. 5, pp. 770-775, 2004.

[8] Ming-Chih Chien and An-Chyau Huang, "Adaptive control for flexible-joint electrically driven robot with time-varying uncertainties," *IEEE Transaction on Industrial Electronics*, vol. 54, no. 2, pp. 1032-1038, 2007.

[9] C. A. Lightcap and S. A. Banks, "An extended Kalman filter for real-time estimation and control of a rigid-link flexible-joint manipulator," *IEEE Transaction on Control Systems Technology*, vol. 18, no. 1, pp. 91-103, 2010.

[10] Ghahramani et al., "Constrained incremental predictive controller design for a flexible joint robot," *ISA Transactions*, vol. 48, pp. 321 – 326, 2009.

[11] A. Ailon, "Asymptotic stability in a flexible-joint robot with model uncertainty and multiple time delays in feedback," *Journal of the Franklin Institute*, vol. 341, pp. 519 – 531, 2004.

[12] Marino and Tomei, "Robust stabilization of feedback linearizable time varying uncertain nonlinear system," *Automatica*, vol. 29, pp. 181-189, 1993.

PhD. Phuong Nam Dao obtained his doctorate degree in January 2013 at Hanoi University of Science and Technology (Vietnam).
Currently, he holds the position as lecturer at Hanoi University of Science and Technology, Vietnam. His research interests include control of robotic systems and robust/adaptive, optimal control.
He is author/co-author of more than 70 papers (Journals, Conferences, etc.)

Nguyen Van Huong:
Place of birth: Hai Duong, Vietnam.
Date of birth: 21/02/1995.
Nguyen Van Huong, received a B.S degree in Electronic Engineering in 2018 from the Hanoi University of Science and Technology, Vietnam. At present, he works in research lines such as robotics, electric power systems and control systems.

Pham Thanh Loc:
Place of birth: Hai Duong, Vietnam.
Date of birth: 12/08/1997.
Pham Thanh Loc, is a student at Hanoi University of Science and Technology, Vietnam and is researching robotics and control systems.