Robust Control Law Using H-infinity for Wheeled Inverted Pendulum Systems

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***Abstract*—Many authors have utilized Lyapunov's direct technique to design robust adaptive controllers. In recent years, the work of applying H? theory to control wheeled inverted pendulums is a topic of much concern due to its underactuated, external-disturbance-prone and nonlinear model. In this paper, we propose a new control method by applying the H-infinity and Backstepping technique based on Lyapunov's direct technique to stabilize tracking error for converging to arbitrary ball of origin. Under disturbances belonging L_2-space, the simulation results of WIP demonstrate the effectiveness of the proposed controller.**

***Index Terms*—** $H\infty$ **control, Backstepping design, Wheeled Inverted Pendulum, Linear Matrix Inequalities**

I. INTRODUCTION

The wheeled inverted pendulum described in [2] includes a pair of identical wheels, a chassis, wheel actuators, an inverted pendulum and a motion control unit, within which the pair of wheels and the inverted pendulum are supported by chassis. The wheel actuators rotate the wheels with respect to the chassis, and they are controlled by the motion control unit to move the vehicle and to stabilize the inverted pendulum.

The dynamic model and several control methods are presented in [3]. In this paper, we use a dynamic model of WIP built via the Newton Euler method [3]. This model is separated into two subsystems: the subsystems describe the rotation of Cart and straight motion of WIP, which is similar to Cart–Pole model. Previous authors [4] [5] approached the control of inverted pendulum based on energy function, but the disadvantage of their approach is that disturbances impacting considered system is ignored and pendulum always fluctuate around origin. Olfati-Saber [6] proposed coordinate transformation to change the Cart–Pole model to strict forward system to apply nest saturation method. This transformation will not be effective when some parameters are uncertainties or disturbances appear. Therefore, Do [2] used a similar transformation as that in [6] and combined with the nest saturation method, disturbance observer to steady error to converge to origin asymptotically with assumption of zero straight accelerator of Cart. Researchers in [7] applied properties of nonholomic system and backstepping technique, but their technique's drawback is in the assumption of satisfying state constraints. The instantaneous switching of control input is proposed in [8]; nevertheless, the position of Cart therein was not stabile at the desired point. Separation technique for WMRs has been mentioned in [10], and authors absolutely used Lyapunov's technique to find the control scheme.

In this paper, we apply the H_∞ method in [1] for rotation motion and an additional controller has been proposed for the straight motion subsystem. The first controller has been proposed based on Lyapunov's direct technique guarantee that tilt angle and angular velocity error converge to neighborhood of origin. In this region, we give some additional components for the second controller using H_∞ theory to ensure that all state variables lie in a small arbitrary ball of origin. Finally, to avoid an instantaneous change in control torque, we propose virtual control input that is designed using the backstepping technique [9].

II. DYNAMIC MODEL

TABLE I. PARAMETER OF WIP

Parameter	Symbol
Distance between two wheels	D m
Radius of wheel	R m
Moment of inertia of the wheel about the $y-axis$	J_ω $kg.m^2$
Moment of inertia of the chassis and pendulum about $z-axis$	J_p $kg.m^2$
Moment of inertia of the chassis about the $y-axis$	J_M $kg.m^2$
Moment of inertia of heading angle pendulum about the $z-axis$	J_θ $kg.m^2$
Mass of pendulum	m kg
Mass of chassis	M kg
Mass of wheel	M_ω kg
Gravitational acceleration	g m/s^2
Distance between the central point of the pendulum and chassis	l m

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TABLE II. VARIABLE TYPE OF WIP

Heading angle of pendulum	θ rad
Tilt angle of pendulum	ϕ rad
Torque control in left and right wheel	τ_L, τ_R N.m
Position of chassis	x m
Disturbances impacting on two wheels	d_L, d_R N

By applying Newton – Euler Approach in [1], the dynamic model of WIP is:

$$\ddot{\theta} = \frac{D}{2RJ_\theta}(\tau_L - \tau_R) + \frac{D}{2J_\theta}(d_L - d_R) \quad (1)$$

$$\begin{aligned} & ml\cos(\phi)\ddot{\phi} + \left[M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega\right)\right]\ddot{x} \\ & = ml\dot{\phi}^2\sin(\phi) + \frac{\tau_L + \tau_R}{R} + d_L + d_R \end{aligned} \quad (2)$$

$$\left(ml^2 + J_M\right)\ddot{\phi} + ml\cos(\phi)\ddot{x} = mgl\sin(\phi) \quad (3)$$

where:

$$J_\theta = J_p + D^2\left(M_\omega + \frac{J_\omega}{R^2}\right) \quad (4)$$

Assumption 1: The external disturbances impacting two wheels belong to the $L_\infty - space$

$$|d_R| < d_{R\max}; |d_L| < d_{L\max} \quad (5)$$

$$\left[M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega\right)\right]\left(ml^2 + J_\omega\right) - m^2l^2 > 0 \quad (6)$$

Control objective: The heading angle, position and their derivatives track their desired value and tilt angle converge to zero.

$\theta_d, \dot{\theta}_d, x_d$ and $\dot{x}_d$ are desired heading angle, heading angular velocity, position and velocity, respectively. The proposed controller needs to satisfy (7) and (8).

$$|\theta - \theta_d| \to 0,\ \phi \to 0, |x - x_d| \to 0 \quad (7)$$

$$\begin{aligned} & |\dot{\theta} - \dot{\theta}_d| \to 0,\ \dot{\phi} \to 0, |\dot{x} - \dot{x}_d| \to 0 \\ & if\ d_L = 0,\ d_R = 0 \end{aligned} \quad (8)$$

Moreover, uniform bounded in tracking error if

$$|d_L| < d_{L\max}, |d_R| < d_{R\max} \quad (9)$$

III. PROPOSED METHOD

The above equations are separated into two subsystems: θ system (12) and x, ϕ system (13,14).

A. Control Design for θ System

The control input in (12) and the error of heading angle are set in (15)

$$\begin{aligned} \tau_1 &= u_1 + \frac{2J_\theta}{D}\ddot{\theta}_{ref} \\ \theta_e &= \theta - \theta_{ref} \end{aligned} \quad (15)$$

The subsystem (12) is written as follows:

$$\ddot{\theta}_e = \frac{D}{2J_\theta}u_1 + \frac{D}{2J_\theta}d_\theta \quad (16)$$

Considering subsystem (16) as follows:

$$\begin{cases} \dot{\upsilon} = A\upsilon + B_1u_1 + B_2d_\theta \\ y = H\upsilon \end{cases} \quad (17)$$

$$\upsilon = \left[\theta_e, \dot{\theta}_e\right]^T, A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ \dfrac{D}{2J_\theta} \end{bmatrix}, H > 0$$

Theorem 1: By selecting positive scalar γ, there exists a matrix $P_1 = P_1^T > 0$ satisfying inequality (18) – H_∞ [1]. The control input (19) ensures (20) to guarantee tracking error to converge to origin and to obtain its robust stability.

$$A^TP_1 + P_1A + P_1\left(\frac{1}{\gamma^2}B_2B_2^T - B_1B_1^T\right)P_1 + H^TH \le 0 \quad (18)$$

$$u_1 = -B_1^TP_1\upsilon \quad (19)$$

$$\int_0^T\left(\|y\|^2 + \|u_1\|^2\right)dt \le \gamma^2\int_0^T\|d_\theta\|^2\,dt \quad (20)$$

Remark 1: Let $P_1^{-1} = T$, γ be appropriately selected. The existence of solution in (28) satisfies H_∞ [1]. The system will be applied input (19) for stabilization.

Multiplying P^{-1} in left and right side of (18)

$$P_1^{-1}A^T + AP_1^{-1} + \left(\frac{1}{\gamma^2}B_2B_2^T - B_1B_1^T\right) + \left(HP_1^{-1}\right)^T HP_1^{-1} \le 0 \quad (27)$$

Schur's complement is applying in (27) to be equivalent to (28)

$$\begin{bmatrix} (AT)^T + AT + \dfrac{1}{\gamma^2}B_2B_2^T - B_1B_1^T & (HT)^T \\ HT & -I \end{bmatrix} \le 0 \quad (28)$$

B. Control design x, ϕ system

Subsystem (13) and (14) is equivalent to (30) and (31) by using local feedback linearization (29):

$$\tau_2 = -ml\dot{\phi}^2 \sin(\phi) + \frac{m^2 g l^2}{ml^2 + J_M} cos(\phi)\sin(\phi) + \left(M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega\right) - \frac{m^2 l^2 \cos^2(\phi)}{ml^2 + J_M} \right)(u + \ddot{x}_d) \quad (29)$$

$$\ddot{\phi} = -a\cos(\phi)u + ag\sin(\phi) + a\cos(\phi)(\Delta - \ddot{x}_d) \quad (30)$$

$$\ddot{x}_e = u + \Delta \quad (31)$$

where:

$$a = \frac{ml^2}{ml^2 + J_M}$$
$$\Delta = \frac{d}{M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega\right) - \frac{m^2 l^2 \cos^2(\phi)}{ml^2 + J_M}} \quad (32)$$

The desired position and velocity of WIP are:

$$x_e = x - x_d ; \dot{x}_e = \dot{x} - \dot{x}_d$$

Remark 2: It is difficult to directly design a H$_\infty$ controller for subsystems (30) (31) because of underactuated property and interconnect control input in both two equations. We propose the Lyapunov direct method for $\phi, \dot{\phi}$ to lead to given attractor where the linearization model can be applied. The proposed H$_\infty$ controller guarantees that $x, \dot{x}$ track simultaneously to the desired value and that $\phi, \dot{\phi}$ are bounded by neighborhood origin. Because the proposed method includes two sub-controllers, the switching must be employed in control system.

Assumption 2: External disturbances Δ , desired accelerator $\ddot{x}_d$ and parameters satisfy

$$|\Delta| < \Delta_{\max} , |\ddot{x}_d| < \ddot{x}_{d\max} \quad (33)$$

$$\left[M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega\right)\right](ml^2 + J_\omega) - m^2 l^2 > 0 \quad (34)$$

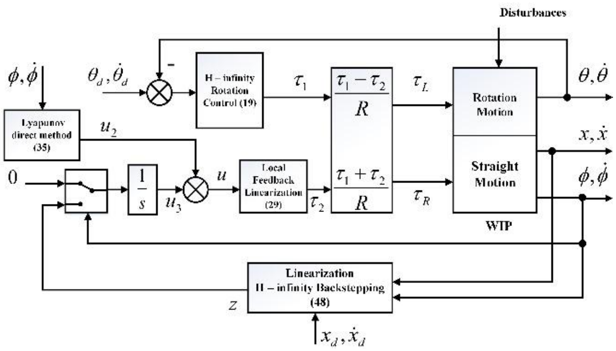

Figure 1. Control Structure for WIP.

Theorem 2: Considering subsystem (30) under (33) and (34). By applying control torque (35) found by Lyapunov's direct method, two variables $\phi, \dot{\phi}$ converge to the arbitrary attractor $\Omega = \left\{ \Omega \in R^2 \middle| |\dot{\phi}| \le \varepsilon_1, |\phi| \le \varepsilon_2 \right\}$ by adjusting controller coefficients k_1, k_2 largely.

$$u_2(\dot{\phi}, \phi) = \frac{k_1\dot{\phi} + k_2\phi + g\sin\phi}{\cos\phi} \quad (35)$$

Remark 3: From *Remark 2* and controller (48), the applied switching may cause instantaneous change of control input, leading to unreality. Therefore, a virtual input in (49) is proposed to reduce this problem significantly. The new linear state space model with virtual input is considered as:

$$\begin{cases} \dot{\eta} = F\eta + Gu_3 + K(\Delta - \ddot{x}_d) \\ \dot{u}_3 = z \end{cases} \quad (49)$$

Theorem 3: The proposed virtual control input (49) will be designed and based on backstepping following (50) to satisfy *Remark 3*

$$z = -\kappa(u_3 - \bar{u}) - \frac{\partial V_2}{\partial \eta} G + \frac{\partial \bar{u}}{\partial \eta}(F\eta + Gu_3) \quad (50)$$

Where $\kappa > 0$ is given, V_2 is the above Lyapunov candidate function designed by H$_\infty$ in [1] and $\bar{u}$ is control law generated from (48)

Remark 4: The proposed control input (49, 50) is the next step of work in [10].

IV. Simulation Results

Firstly, we obtain the results using controller in [10] as described in Figs. 2 and 3. We continue to implement the proposed controller (49, 50) based on the scenario simulation as follows: The heading angle and tilt angle deviated from their balanced position and position of chassis is arbitrary. The objective of work is to guarantee that $x, \dot{x}, \theta, \dot{\theta}$ track given desired strategies $x_d, \dot{x}_d, \theta_d, \dot{\theta}_d$ with unbalanced initial state, wherein the WIP system can move to given place via state trajectory. The good performance of simulation results (Fig. 4 to Fig. 7) demonstrates the ability of the proposed control law, which executes the requirement. The parameter values and initial state in [2] are used here to simulate this system.

TABLE III. Parameter Values in Simulation

Parameter	Value
Distance between two wheels	$D = 0.15m$
Radius of wheel	$R = 0.25m$
Moment of inertia of the wheel about the $y - axis$	$J_\omega = 1.5kg.m^2$
Moment of inertia of the chassis and pendulum about $z - axis$	$J_M = 2.5kg.m^2$
Moment of inertia of the chassis about the $y - axis$	$J_\theta = 1.5kg.m^2$
Moment of inertia of heading angle pendulum about the $z - axis$	$m = 1.5kg$
Mass of pendulum	$M = 5kg$
Mass of chassis	$M_\omega = 1kg$
Mass of wheel	$g = 9.8m/s^2$
Gravitational acceleration	$l = 1.2m$

TABLE IV. INITIAL STATE IN SIMULATION

Initial state variable	Value
$\theta(0)$	$0.5rad$
$\dot{\theta}(0)$	$0rad/s$
$x(0)$	$-1.5m$
$\dot{x}(0)$	$0m/s$
$\phi(0)$	$0.8rad$
$\dot{\phi}(0)$	$0.2\,rad/s$

TABLE V. PARAMETER OF CONTROLLERS

Parameter	Value
γ	1.5
H	$diag([100,1])$
u_1	$-[884.43\ \ 477.63][\theta_e, \dot{\theta}_e]^T$
β	100
ε_1	0.3
ε_2	1
λ_1	2.78
λ_2	0.25
k_1	48.41
k_2	39.27
C	$diag([0.1, 0.1, 1.2, 0.1])$
γ_1	100
u_3	$[98.49, 31.99, 3.44, 10.65]\eta$

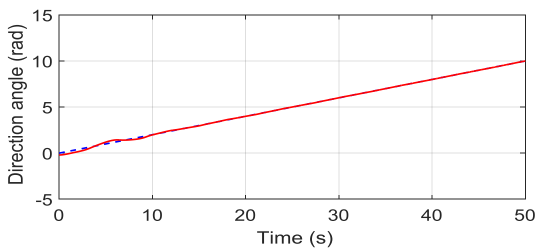

Figure 2. The behaviour of direction angle

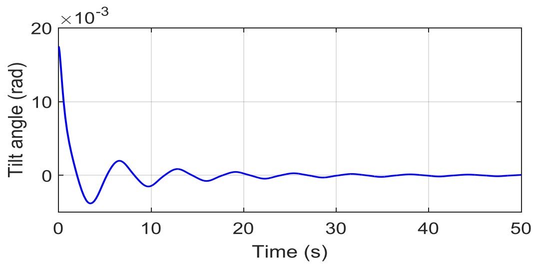

Figure 3. The behaviour of tilt angle

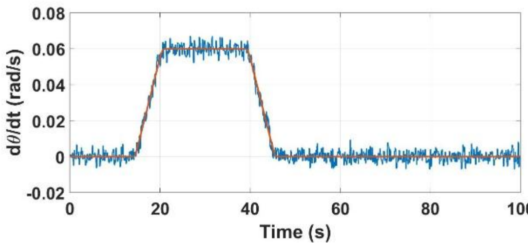

Figure 4. Heading angular velocity.

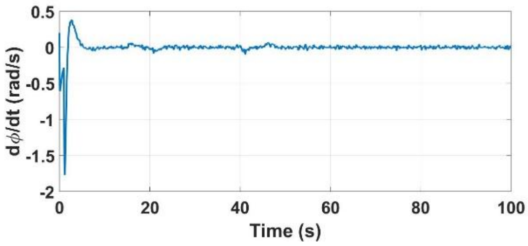

Figure 5. Tilt angular velocity.

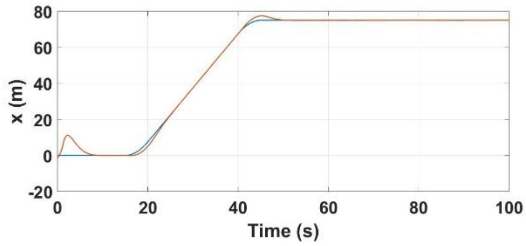

Figure 6. Position of chassis.

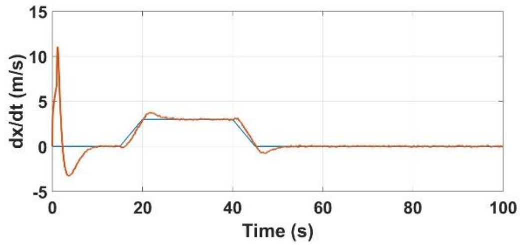

Figure 7. Velocity of chassis.

This system is able to self-balance and straight velocity, heading angle track reference signal under disturbances. Control forces is continuous. H-infinity control is also able to track zero of tilt angle better than robust control.

V. CONCLUSION

The proposed approach applied in this paper is that of separating dynamic model into two subsystems, including rotation and straight to design two controller. Heading angle tracks its reference when controller is designed by H_∞. Tilt angle and position reach their balanced point by combining Lyapunov's direct method controller and H_∞ controller in order to avoid instantaneous change of controllers, the virtual input is applied. The simulation results in previous chapter verify performance of the proposed method.

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