

# Estimation of Vehicle Dynamics States Using Luenberger Observer

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**Abstract**— Information about vehicle dynamics states is indispensable for modern dynamics control system on a vehicle today. Instead of using the expensive physical sensor to measure directly the states of a vehicle, this paper proposes a “virtual sensor” which bases on the dynamical model of vehicle and an observation algorithm. An observer based on Luenberger method is developed in Matlab/Simulink. To make the dynamics model consistent with a virtual vehicle in CarSim, the model parameters are identified at a pre-defined velocity using Parameter Identification toolbox of Simulink. The whole system is constructed by connecting Matlab/Simulink to CarSim to simulate a complete real system. The simulation results show the good performance of the observer when the estimated values are well converged to real values given by CarSim. The precision of results depends on what extent the velocity of the vehicle whether it is near the velocity of parameters identification or not.

**Index Terms**—Vehicle dynamics, Luenberger observer, estimation, identification, simulation.

## I. INTRODUCTION

Automatic control systems, such as anti-lock braking system (ABS) and electronic stability control system (ESC) play an important role in the modern vehicle today. To perform efficiently, these controllers require the exact information about vehicle dynamics states which come from the measurement system on a vehicle. However, equipping a commercial vehicle with expensive sensors leads to an increase in the cost for production, maintenance and as the results reducing the competence of the product. A solution for this problem is using the “software sensors” which based on the dynamics model of an object and the observer algorithms to estimate the real value of needed grandeur. This technique has been applied to vehicle control applications. In Tanoury’s work [1], high gain observer and second-order sliding mode (SOSM) for estimating rolling resistance force and detecting a sudden decrease of tire pressure had been developed. A quarter-car model had been used and its observability has been proved numerically by an evaluated determinant value of the transformation matrix during the simulation process. By comparing mean errors between real and estimated value, the author concluded

that sliding mode observer provides better results than high gain one thanks to its robustness properties. However, estimating the rolling resistance coefficient during real driving condition has not yet been implemented.

The second order observer of a mechanical system has been presented in [2] is not only for states estimation but also for parameters identification and unknown input reconstruction. In [3], second order SMO is developed to estimate lateral force by using a bicycle model. However, this method does not allow to estimate the separately lateral force of each tire. In [4], the authors try to estimate individual lateral tire force thanks to the distribution of the vertical load of each tire. This estimation still exists significant errors between simulation and estimated value due to the nonlinear of lateral forces. In [5], the second order SMO in [6] has been applied to estimate wheel rotation speed from wheel angular measurement. There are also other researches applying SMO to vehicle dynamics. In addition, the Kalman filter is also widely used to estimate the vehicle states and parameters [7]–[9]. This filter combining with SMO observer is used to estimate the side slide angle of the vehicle, traction force and cornering stiffness of tires. In [10], the authors used a system of observers, including Kalman filter and Luenberger observers to estimate the mass, roll and pitch angles of a vehicle. Reaction forces at wheels are also estimated. Experimental results show that there is an error of about 10% between measured and estimated values.

In this paper, we use Luenberger observer to estimate the lateral acceleration of the vehicle due to the important role of this grandeur in vehicle dynamics control. The rest of this paper is organized as follows. Section II describes the dynamics model of a vehicle. CarSim and virtual vehicle with double-lane change test are presented in Section III. Section IV outlines the identification of model parameters. Section V focuses on designing a Luenberger observer using pole placement method. Finally, Section IV provides conclusions and future work.

## II. DYNAMICS MODEL OF VEHICLE

Vehicle motions are very complicated in actual conditions. Therefore, to facilitate the research, in this paper, we use a flat model of a rigid bicycle-vehicle having forward, lateral, yaw, and roll motions presented in Fig. 1. The model of a roll-able rigid vehicle is more

exact and more effective compared to the rigid bicycle-vehicle planar model.

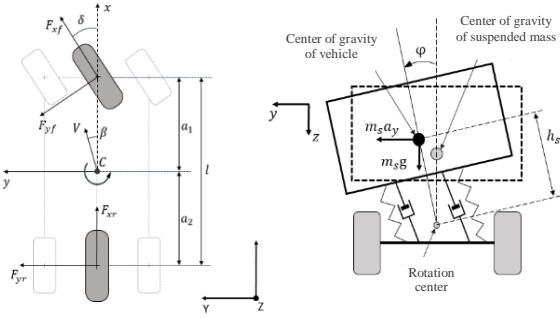


Figure 1. Rollable bicycle vehicle model.

The dynamic equations of vehicle motion can be determined as the following equations

$$\begin{cases} \dot{v}_y = \frac{C_\beta}{m} \beta + \left(\frac{C_r}{m} - v_x\right) r + \frac{C_\phi}{m} \phi + \frac{C_p}{m} p + \frac{C_\delta}{m} \delta \\ \dot{\omega}_z = \frac{D_\beta}{I_z} \beta + \frac{D_r}{I_z} r + \frac{D_\phi}{I_z} \phi + \frac{D_p}{I_z} p + \frac{D_\delta}{I_z} \delta \\ \dot{\phi} = p = 0\beta + 0r + 0\phi + 1p + 0\delta \\ \dot{\omega}_x = \frac{E_\beta}{I_x} \beta + \frac{E_r}{I_x} r + \frac{E_\phi}{I_x} \phi + \frac{E_p}{I_x} p + \frac{E_\delta}{I_x} \delta \end{cases} \quad (1)$$

The formulas of symbols in system (1) are given below and described in Table 1.

$$\begin{aligned} C_\beta &= (-C_{\alpha_f} - C_{\alpha_r}) \\ C_r &= \left(\frac{a_2 C_{\alpha_r} - a_1 C_{\alpha_f}}{v_x}\right) \\ C_\phi &= (C_{\alpha_f} C_{\delta\phi_f} + C_{\alpha_r} C_{\delta\phi_r} - C_{\phi_f} - C_{\phi_r}) \\ C_p &= \left(\frac{C_{\alpha_f} C_{\beta_f} + C_{\alpha_r} C_{\beta_r}}{v_x}\right) \\ C_\delta &= C_{\alpha_f} \\ D_\beta &= -a_1 C_{\alpha_f} + a_2 C_{\alpha_r} \\ D_r &= \left(\frac{-a_1^2 C_{\alpha_f} - a_2^2 C_{\alpha_r}}{v_x}\right) \\ D_\phi &= \left(a_2 (C_{\phi_r} - C_{\alpha_r} C_{\delta\phi_r}) - a_1 (C_{\phi_f} - C_{\alpha_f} C_{\delta\phi_f})\right) \\ D_p &= \left(\frac{a_1 C_{\alpha_f} C_{\beta_f} - a_2 C_{\alpha_r} C_{\beta_r}}{v_x}\right) \\ D_\delta &= a_1 C_{\alpha_f} \\ E_\beta &= \frac{m_s h_s (-C_{\alpha_f} - C_{\alpha_r})}{m} \\ E_r &= \frac{m_s h_s (a_2 C_{\alpha_r} - a_1 C_{\alpha_f})}{m v_x} \\ E_p &= \left(\frac{m_s h_s (C_{\alpha_f} C_{\beta_f} + C_{\alpha_r} C_{\beta_r})}{m v_x} - C_\phi\right) \\ E_\delta &= \frac{m_s h_s C_{\alpha_f}}{m} \end{aligned}$$

TABLE I. LIST OF SYMBOLS

Symbol	Definition	Unit
$\beta$	Side slip angle of vehicle	rad
$\delta$	Steering angle	rad
$\delta_f$	Front Steering angle	rad
$\delta_r$	Rear Steering angle	rad
$\psi$	Yaw angle	rad
$\dot{\psi}$	Yaw velocity	rad/s
$l$	Base length of vehicle	m
$a_1$	Distance from front wheel to CoG	m
$a_2$	Distance from rear wheel to CoG	m
$B$	Base width of vehicle	m
$m$	Vehicle mass	kg
$C_{\alpha_f}$	Cornering stiffness at front wheel	N/rad
$C_{\alpha_r}$	Cornering stiffness at rear wheel	N/rad
$C_{\delta\phi_f}$	Front roll-steering coefficient	
$C_{\delta\phi_r}$	Rear roll-steering coefficient	
$C_{\beta_f}$	Front tire roll rate coefficient	
$C_{\beta_r}$	Rear tire roll rate coefficient	
$C_{\phi_f}$	Front tire camber thrust coefficient	
$C_{\phi_r}$	Rear tire camber thrust coefficient	
$k_\phi$	Roll stiffness	Nm/rad
$C_\phi$	Roll damping	Nms/rad
$I_x$	Moment of inertial around x axis	kg.m <sup>2</sup>
$I_z$	Moment of inertial around z axis	kg.m <sup>2</sup>
$v$	Motion velocity of vehicle	m/s
$v_x$	Longitudinal velocity	m/s
$v_y$	Lateral velocity	m/s
$a_y$	Lateral acceleration	m/s <sup>2</sup>
$\omega_x$	roll rate	rad/s
$\omega_z$	Yaw rate	rad/s
$\phi$	Roll angle	rad
$u$	Steering angle	rad

System (1) can be described in state space as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (2)$$

where  $x = [v_y, p, \phi, r]^T$  is state vector,  $u = \delta$  is input vector,  $y = (a_y, p, r)$  is output vector with assumption that lateral acceleration, roll rate and yaw rate are measurable.

$$A = \begin{bmatrix} \frac{C_\beta}{m} & \frac{C_r}{m} - v_x & \frac{C_\phi}{m} & \frac{C_p}{m} \\ \frac{D_\beta}{I_z} & \frac{D_r}{I_z} & \frac{D_\phi}{I_z} & \frac{D_p}{I_z} \\ 0 & 0 & 0 & 1 \\ \frac{E_\beta}{I_x} & \frac{E_r}{I_x} & \frac{E_\phi}{I_x} & \frac{E_p}{I_x} \end{bmatrix} \text{ is state matrix}$$

$$B = \begin{bmatrix} \frac{C_\delta}{m} \\ \frac{D_\delta}{I_z} \\ 0 \\ \frac{E_\delta}{I_x} \end{bmatrix} \text{ is input matrix; } C = \begin{bmatrix} \frac{C_\beta}{m} & \frac{C_r}{m} & \frac{C_\phi}{m} & \frac{C_p}{m} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ is output}$$

matrix and  $D = \begin{bmatrix} \frac{C_\delta}{m} \\ 0 \\ 0 \end{bmatrix}$  is feedforward matrix.

The system (2) is the linear invariant time at a predefined longitudinal velocity of the vehicle. It means that the model of the system varies with the change of vehicle speed. Therefore, to evaluate the performance of

this model, it is necessary to fix the longitudinal velocity of the vehicle.

### III. CARSIM AND VIRTUAL VEHICLE

CarSim has been developed by Mechanical Simulation Corporation (an American company) since 1996. By using multibody dynamics modeling, this software allows to recreating the behavior of passenger vehicles and light-duty trucks. One advantage of CarSim is its module structure including many sub-models of each component of vehicles. This allows users to modify and test a vehicle in many different configurations. In addition, it integrates inside many types of test including the road surface, ISO lane change test.

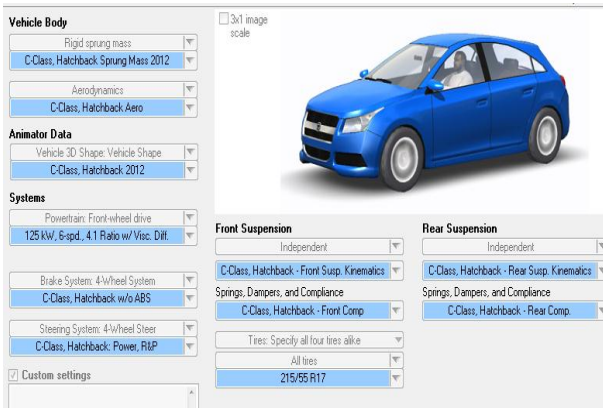


Figure 2. Reference vehicle in CarSim.

Fig. 2 shows the vehicle using in this study, and Table II presents the parameters of the reference vehicle.

TABLE II. PREDEFINED PARAMETERS OF REFERENCE VEHICLE

Parameters	Value	Parameters	Value
$m$	1412	$I_x$	536.6
$m_s$	1270	$a_1$	1.015
$I_z$	1536.7	$a_2$	1.8950

Because CarSim uses different models, the parameters of the model (2) should be identified so that this model can be valid to a real vehicle. This step will be presented in the following section.

### IV. VEHICLE PARAMETER IDENTIFICATION

Simulink parameter identification is a useful tool to determinate the parameters of a model. Its principle is simple: both reference and research model are excited by the same input signals. Then the parameters of the research model will be changed by a recursive algorithm so that the outputs of the research model converge to ones of the reference model. In this case, the input signal is the steering angle  $\delta$  during the double lane change test and the outputs are lateral acceleration and velocity, yaw and roll rate. The longitudinal velocity is fixed at 65 km/h. Fig. 3 shows the main interface of this tool during the initial configuration process.

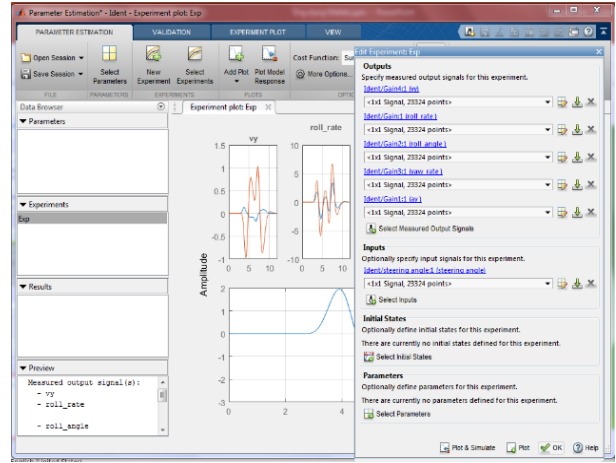


Figure 3. The interface of Simulink parameter identification tool.

TABLE III. INITIAL AND IDENTIFIED VALUES OF VEHICLE PARAMETERS

Symbol	Initial value	Identified value
$C_{af}$	2 x 28648	150000
$C_{ar}$	2 x 26356	47222
$C_{\delta\phi f}$	0.01	-0.063
$C_{\delta\phi r}$	0	0.7679
$C_{\beta f}$	0.01	-1.2293
$C_{\beta r}$	0.01	1.2247
$C_{\phi f}$	2	1.99
$C_{\phi r}$	1	0.99
$h$	0.54	0.5
$k$	50000	94169
$c$	2000	17802

Figs. 4 and 5 show the comparisons between the performance of the reference vehicle and the model (2) with initial values of parameters are given in the second column of Table III. The results show that there are significant errors between the two models. Therefore, the identification process is necessary.

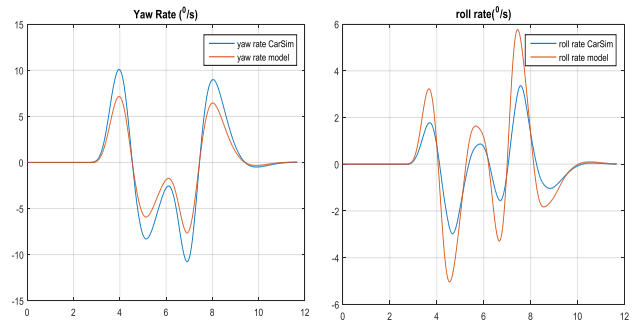


Figure 4. Roll and yaw rate of reference and model vehicles.

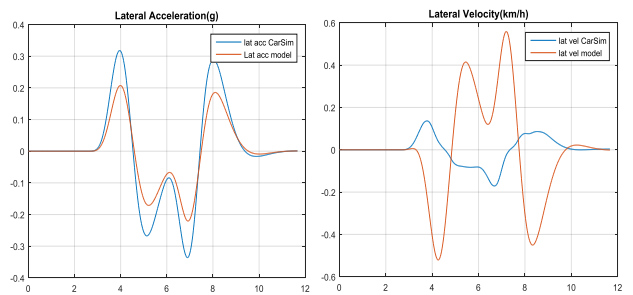


Figure 5. Lateral acceleration and velocity of reference and model vehicles.

Figs. 6 and 7 show the comparisons between the performance of the reference vehicle and the model after identification. The results reveal that the outputs of the model now match well with ones of the reference vehicle. Up to now the model of vehicle is complete and the next step is to design observer.

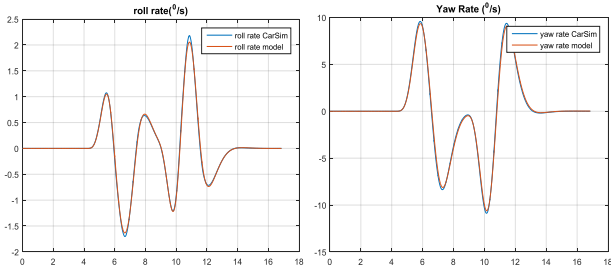


Figure 6. Roll and yaw rate of reference and model vehicles after identification parameters.

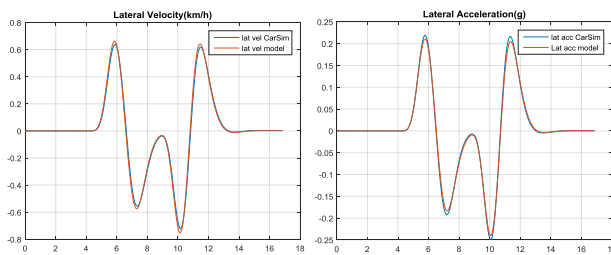


Figure 7. Lateral velocity and acceleration of reference and model vehicles after identification parameters.

V. OBSERVER DESIGN AND SIMULATION RESULTS

The system now is linear invariant time and therefore a linear Luenberger observer will be used. Firstly, the observability of the system will be discussed. The observability matrix is built from A and C matrix as  $Ob = [C, AC, A^2C, \dots, A^{n-1}C]$  It can be tested using Matlab that rank of observability matrix, in this case, is 4 (full rank) and therefore the system is observable. The structure of the observer is presented in (3) where the main part of the system is conserved and added a correction term  $L(y - \hat{y})$  which makes system convergence. Fig. 8 presents the structure of the observer where the input of the observer is input  $u$  and output  $y$  of the system.

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \\ y = C\hat{x} + Du \end{cases} \quad (3)$$

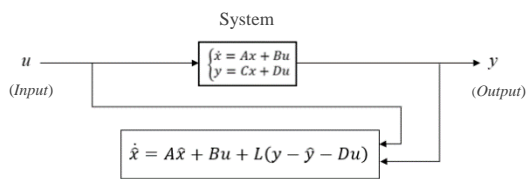


Figure 8. Structure of observer

It can be seen that the error dynamics of observer and model can be described as (4).

$$\begin{aligned} \dot{x} - \dot{\hat{x}} &= A(x - \hat{x}) - L(Cx - C\hat{x}) \\ &= (A - LC)(x - \hat{x}) \end{aligned} \quad (4)$$

The problem now is to determine the value of L to stabilize the system (4). This can be done by using pole placement method where the pole of the error dynamics system can be chosen arbitrarily. In most of the cases, the recommended values of observer poles are 5 times the poles of the original system so that the estimated values can converge rapidly to real ones. Matrix L was determined by using Matlab, the final structure of the system is presented in Fig. 9.

$$L = \begin{bmatrix} 1.3137 & -18.2159 & -63.1412 \\ -0.1084 & 28.4993 & 3.2035 \\ 0.5995 & -3.8569 & -11.3383 \\ -1.9779 & 3.5376 & 58.5043 \end{bmatrix} \quad (5)$$

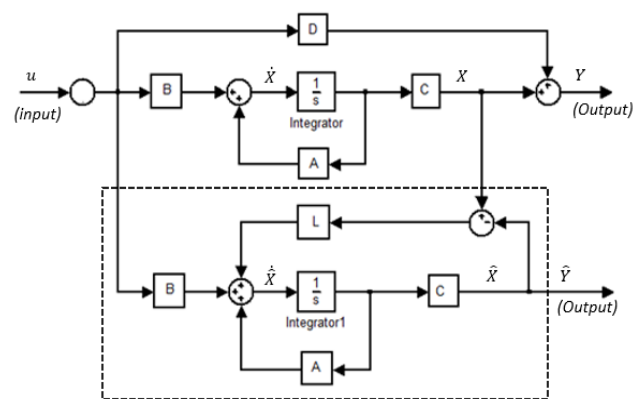


Figure 9. Structure of Luenberger state observer.

Figs. 10 and 11 compare the results of reference and estimated lateral velocity and roll angle at longitudinal velocity 35 km/h. The result reveals that lateral velocity is very well estimated with a small error. The roll angle is also well estimated with the only small error at the end of the experiment.

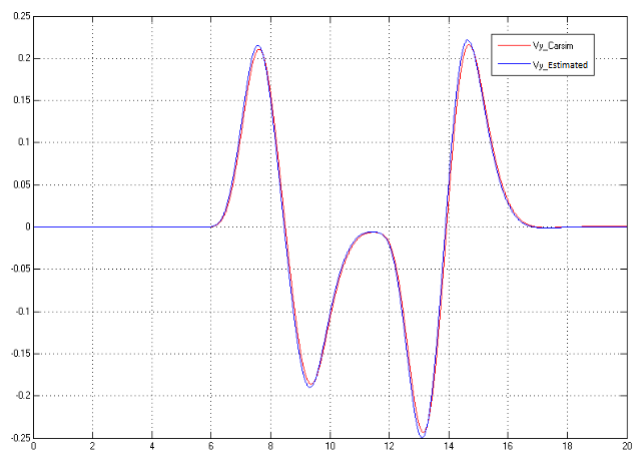


Figure 10. Estimation of lateral velocity (at a longitudinal velocity of 35 km/h).

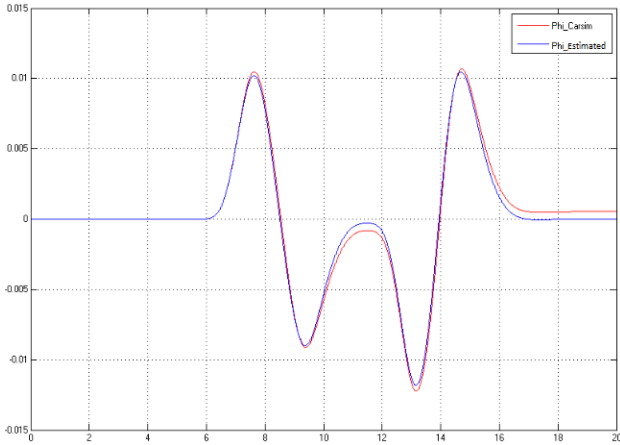


Figure 11. Estimation of roll angle (at a longitudinal velocity of 35 km/h).

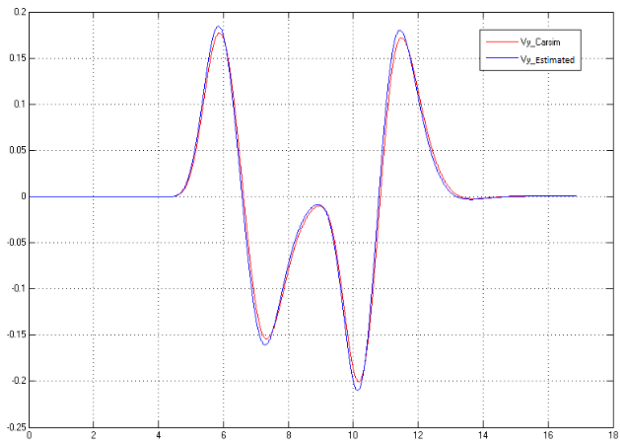


Figure 12. Estimation of lateral velocity (at a longitudinal velocity of 45 km/h).

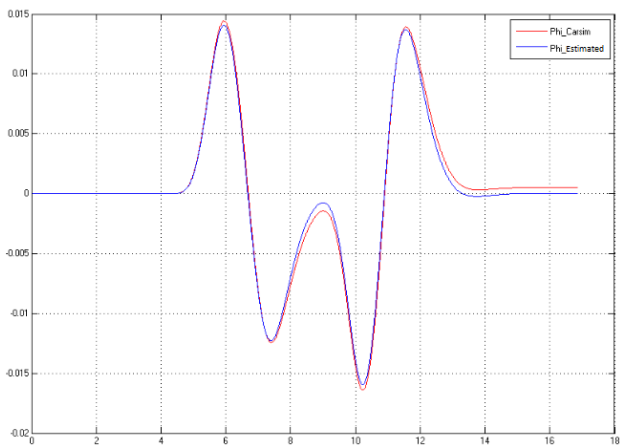


Figure 13. Estimation of roll angle (at a longitudinal velocity of 45 km/h).

The similar results are obtained at a longitudinal velocity of 45 km/h which are shown in Figs 12 and 13. The results reveal that the velocity in the lateral direction is estimated accurately. There is only a small error of estimation of roll angle at about time instant 9s. This can be explained that at that time, the vehicle is recovering to the straight path from curvature path and therefore the response of roll angle may be more complex than a linear invariant model and it causes that error.

However, when the longitudinal velocity increase, the estimation results have different behaviours. Fig. 14 and shows the estimation of lateral velocity during a double lane change test at 65 km/h. The results reveal that the estimation which is good, but there are some moments when the errors become more important than the case 35 km/h and 45 km/h. This can be explained that at this velocity, the dynamics of the vehicle have a change in the sign of the side slip angle during the second cornering of double lane change test and that creates the abnormal behaviours. However, there is no influence on the estimation of the roll angle as shown in Fig. 15 where the result is still perfect.

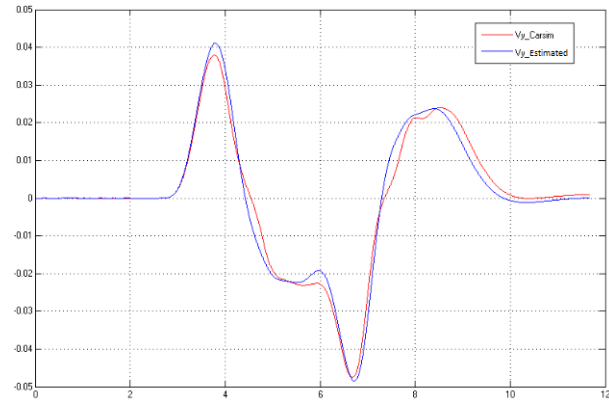


Figure 14. Estimation of lateral velocity (at a longitudinal velocity of 65 km/h).

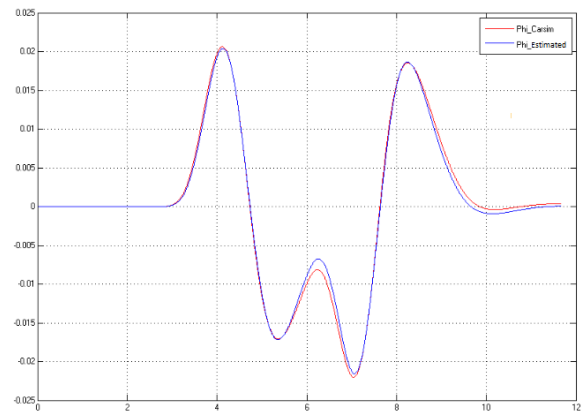


Figure 15. Estimation of roll angle (at a longitudinal velocity of 65 km/h).

Increasing longitudinal velocity to 110 km/h, the estimation results of lateral velocity in Fig. 16 shows clearly the errors in comparisons with the lower longitudinal velocity cases. This phenomenon happens not only to lateral velocity, but also in lateral acceleration in Fig. 17 and the roll angle in Figure 18. These errors can be explained by the modeling errors of our linear invariant time model. This model is created by identifying the parameters of the vehicle at 65 km/h. But in the case the velocity is near two times by this point, the precision of this model is reduced significantly. One other important factor is that when maneuvering double lane change at 110 km/h the lateral acceleration is excess 0.4g, the threshold of the linear model and therefore the linear model is not valid in this case.

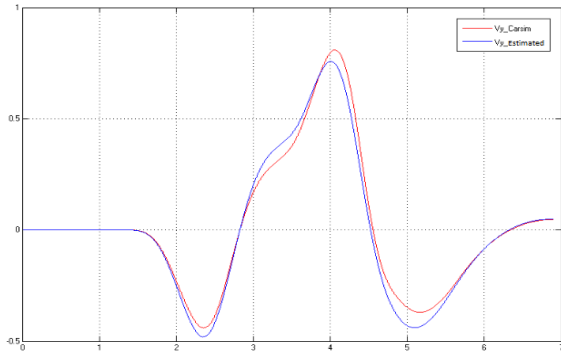


Figure 16. Estimation of lateral velocity (at a longitudinal velocity of 110 km/h).

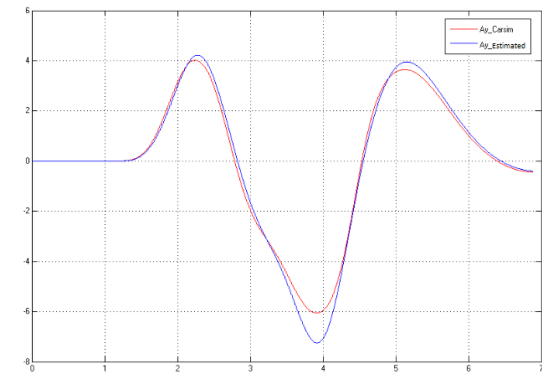


Figure 17. Estimation of lateral acceleration (at a longitudinal velocity of 110 km/h).

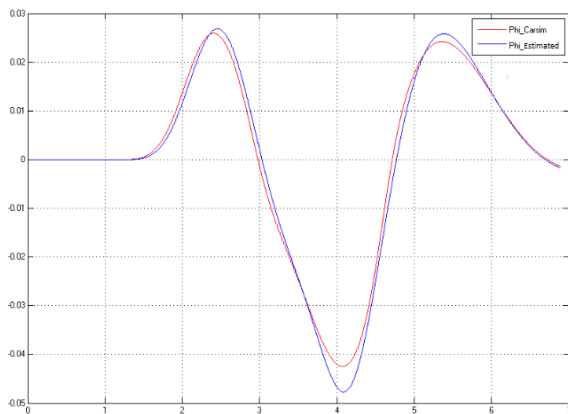


Figure 18. Estimation of roll angle (at a longitudinal velocity of 110 km/h).

## VI. CONCLUSION

This paper presents an approach for estimating vehicle dynamical states using a Luenberger observer. Firstly, the dynamics model is developed and then the model parameters are identified using an identification toolbox of Matlab/Simulink. Finally, the Luenberger observer is designed by the pole placement method. The simulation results show that the estimated values are consistent with the data from the CarSim simulator in a wide range of longitudinal velocity. However, when longitudinal speed increase, the errors become considerable. Future work will concentrate on developing a suitable model to cover problems of varying longitudinal speed.

## ACKNOWLEDGMENT

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