Mixed Convection Boundary Layer Flow of Viscoelastic Nanofluid Past a Horizontal Circular Cylinder with Convective Boundary Condition

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***Abstract*—The steady mixed convection boundary layer flow of viscoelastic nanofluid past a horizontal circular cylinder taking into account the thermal convective boundary condition is investigated numerically. The nanofluid model use involves the Tiwari and Das model. The resulting system of nonlinear partial differential equations is solved numerically using an efficient implicit finite-difference scheme known as the Keller-box method. Effect of the various parameters, namely, the mixed convection parameter, the nanoparticles volume fraction, viscoelastic parameter and the conjugate parameter on the dimensionless velocity, temperature, skin friction, as well as wall temperature have been presented graphically and discussed. It is found that both skin friction and wall temperature decreases for the increase in the viscoelastic parameter. On the other hand, increasing conjugate parameter leads to the increase of the temperature and velocity profiles. For fixed nanoparticles volume fraction, as the value of the mixed convection parameter increases, the magnitude of both the skin friction coefficient and wall temperature also increases.**

***Index Terms*—Viscoelastic; nanofluid; mixed convection; horizontal circular cylinder; convective boundary condition**

I. Introduction

The concept of nanofluid was first manifested by series of research at Argonne National Laboratory and Choi [1] was the first to call the fluids with particles of nanometer dimension suspended in them as “Nano-fluids”. Nanoparticles used in nanofluid can be classified by materials. The nanoparticles are consisting of nano-sized metals, oxides, and carbon nanotubes. Thus, the study of nanofluid has become popular among researchers’ due to its various applications in many industries, engineering, and medical sciences as well such as coolants, lubricants, heat exchangers and micro-channel heat sinks.

Nowadays, the non-Newtonian nanofluid has received much considerable interest and concern by the researchers’ due to the potential of nanofluid applications in many types of industries such as petroleum drilling, manufacturing food and paper. Generally, the studies on the problems of the convective boundary layer and heat transfer focus to the problem that related to the prescribed wall temperature and heat flux. However, in 2009, the earliest idea of using the thermal convective heating boundary condition was introduced by Aziz [2] to analyze Blasius flow. After his pioneering studies, the problem of convective boundary condition with different types of geometry has been studied extensively due to its large application and demand in the engineering field. In 2011, Makinde *et al*.[3] considered the boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Then, the extended researches of convective boundary condition was investigated by Hayat *et al*. [4] [5], Grosan *et al*. [6] and Rashad *et al*.[7].

To the best of our knowledge, there is not a single article that addresses the steady mixed convection boundary layer flow of viscoelastic nanofluid with a convective boundary condition. Using a similarity approach, the governing equations are transformed into ordinary differential equations and solved numerically using a Keller-box method in FORTRAN software. The effects of relevant parameters on the dimensionless nanofluid velocity, the temperature, the nanoparticle volume fraction, as well as the skin friction coefficient and wall temperature are investigated and shown graphically and discussed. The hypothesis of this study is that nanofluid has extremely high thermal conductivities compared to the conventional liquids. Due to these properties, it has been proposed as a route for surpassing the performance of heat transfer liquids.

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II. Mathematical Formulation

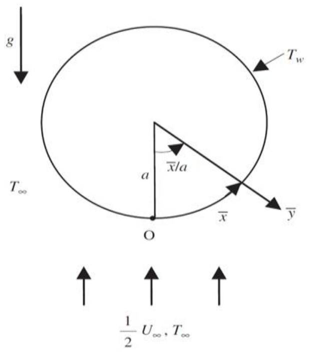

Figure 1. Physical model and coordinates system

The steady mixed convection boundary layer flow past a horizontal circular cylinder placed in a viscoelastic nanofluid is studied. Fig. 1 illustrates the geometry of the problem and the corresponding coordinates system. The surface of the horizontal circular cylinder is subjected to convective boundary condition [8].

By considering the nanofluid model proposed by Tiwari and Das [9], under the usual boundary layer and Boussinesq approximations, the basic governing equations can be written in the following form (see Merkin [10], Rashad[7]).

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right)=\rho_{nf}\bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}}+\mu_{nf}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+k_0\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)+\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right]+g\left(\rho\beta\right)_{nf}\left(T-T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$\left(\rho C_p\right)_{nf}\left[\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}\right]=k_{nf}\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

subjected to the boundary conditions

$$\bar{u}=0,\quad \bar{v}=0,\quad -k\frac{\partial T}{\partial \bar{y}}=h_s\left(T_f-T\right)\quad \text{at } \bar{y}=0,\quad \bar{x}\geq 0,$$

$$\bar{u}=\bar{u}_e\left(\bar{x}\right),\quad \frac{\partial \bar{u}}{\partial \bar{y}}=0,\quad T=T_\infty\quad \text{as } \bar{y}\to\infty,\quad \bar{x}\geq 0, \tag{4}$$

where $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates along the surface of the cylinder. The value is starting from the lower stagnation point of the cylinder. While $\bar{y}$ is the coordinate measured normal to the surface of the cylinder, $\bar{u}$ and $\bar{v}$ are the velocity components, $\bar{u}_e\left(\bar{x}\right)$ is the velocity outside the boundary layer, T is the temperature of the selected fluid, k_0 is the viscoelasticity, k is the thermal conductivity, h_f is the convective heat transfer coefficient, ρ_{nf} and μ_{nf} are the density and dynamic viscosity of nanofluid, k_{nf} is the effective thermal conductivity of the nanofluid and $\left(\rho C_p\right)_{nf}$ is the heat capacitance of nanofluid. These nanofluid constants are defined by

$$\left(\rho C_p\right)_{nf}=\left(1-\phi\right)\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_s,\quad \mu_{nf}=\frac{\mu_f}{\left(1-\phi\right)^{2.5}},\quad \rho_{nf}=\left(1-\phi\right)\rho_f+\phi\rho_s,\quad k_{nf}=k_f\frac{\left(k_s+2k_f\right)-2\phi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\phi\left(k_f-k_s\right)}. \tag{5}$$

where ϕ is the nanoparticle volume fraction of the nanofluid. The thermophysical properties of nanoparticles (Cu) and base fluid (CMC-water) are given in Table I [11].

TABLE I. Thermophisical Properties of Nanoparticles and Base Fluid

Physical Properties	$\rho\left(\text{kg } m^{-3}\right)$	$C_p\left(\text{J kg}^{-1}\text{K}^{-1}\right)$	$k\left(\text{Wm}^{-1}\text{K}^{-1}\right)$	$\beta\times10^5\left(\text{K}^{-1}\right)$
Base Fluid (CMC-water)	997.1	4179	0.613	21
Nanoparticle (Cu)	8933	385	401	1.67

Then, we introduce the following non-dimensional variables

$$x=\bar{x}/a,\quad y=\text{Re}^{1/2}\left(\bar{y}/a\right),\quad u=\bar{u}/U_\infty,\quad v=\text{Re}^{1/2}\left(\bar{v}/U_\infty\right),$$

$$u_e\left(x\right)=\bar{u}_e\left(\bar{x}\right)/U_\infty,\quad \theta=\frac{T-T_\infty}{T_f-T_\infty}, \tag{6}$$

where $\text{Re}=U_\infty a/v$ is the Reynolds number. Substituting (6) into (1) - (3), the dimensionless equations become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$\left[\left(1-\phi\right)+\phi\frac{\rho_s}{\rho_f}\right]\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\left[\left(1-\phi\right)+\phi\frac{\rho_s}{\rho_f}\right]u_e\frac{\partial u_e}{\partial x}+\frac{1}{\left(1+\phi\right)^{2.5}}\frac{\partial^2 u}{\partial y^2}+K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right]+\left[\left(1-\phi\right)+\phi\frac{\left(\rho\beta\right)_s}{\left(\rho\beta\right)_f}\right]\lambda\theta\sin\left(x\right), \tag{8}$$

$$\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right]=\frac{k_{nf}}{k_f}\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}, \quad (9)$$

The transformed boundary conditions are

$$u=0,\quad v=0,\quad \frac{\partial\theta}{\partial y}=-\gamma(1-\theta)\quad \text{at } y=0,\ x\ge 0,$$

$$u=u_e(x),\ \frac{\partial u}{\partial y}=0,\ \theta=0\quad \text{as } y\to\infty,\ x\ge 0, \quad (10)$$

where $K=k_o Gr^{5/2}/a^2$ represents the viscoelastic parameter, $\gamma=ah_f\,\mathrm{Re}^{-1/2}/k$ denotes the conjugate parameter, Pr is the Prandtl number, ϕ nanoparticles volume fraction and λ is the mixed convection parameter given by, $\lambda=Gr/\mathrm{Re}^2$ with $Gr=g\beta(T-T_\infty)a^3/v^2$ is the Grashof number.

Further, we introduce the stream function defined as

$$\psi=xF(x,y),\qquad \theta=\theta(x,y), \quad (11)$$

where ψ is the stream function that defines as

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x}. \quad (12)$$

Substituting (12) and (13) into (8) and (9) lead to obtain

$$\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[\left(\frac{\partial F}{\partial y}\right)^2+x\frac{\partial F}{\partial y}\left(\frac{\partial^2 F}{\partial x\partial y}\right)-x\frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial y^2}-F\frac{\partial^2 F}{\partial y^2}\right]=\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\frac{\sin x\cos x}{x}+\frac{1}{(1+\phi)^{2.5}}\frac{\partial^3 F}{\partial y^3}$$
$$+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta\frac{\sin x}{x}+K\left[2\frac{\partial F}{\partial y}\frac{\partial^3 F}{\partial y^3}-F\frac{\partial^4 F}{\partial y^4}-\left(\frac{\partial^2 F}{\partial y^2}\right)^2+x\left(\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^3 F}{\partial y^3}-\frac{\partial F}{\partial x}\frac{\partial^4 F}{\partial y^4}+\frac{\partial F}{\partial y}\frac{\partial^4 F}{\partial x\partial y^3}-\frac{\partial^2 F}{\partial y^2}\frac{\partial^3 F}{\partial x\partial y^2}\right)\right], \quad (13)$$

$$\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\left[x\frac{\partial F}{\partial y}\frac{\partial\theta}{\partial x}-x\frac{\partial F}{\partial x}\frac{\partial\theta}{\partial y}-F\frac{\partial\theta}{\partial y}\right]=\frac{k_{nf}}{k_f}\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}. \quad (14)$$

The boundary conditions (10) are transformed into

$$F=0,\quad \frac{\partial F}{\partial y}=0,\quad \frac{\partial\theta}{\partial y}=-\gamma(1+\theta),\ \text{at } y=0,\quad x\ge 0,\ \frac{\partial F}{\partial y}=\frac{\sin x}{x},\quad \frac{\partial^2 F}{\partial y^2}=0,\ \theta=0,\qquad \text{as } y\to\infty,\quad x\ge 0. \quad (15)$$

At the lower stagnation point of the horizontal circular cylinder $x\approx 0$, (14)-(15) are reduced to the following ordinary differential equations

$$\frac{1}{(1+\phi)^{2.5}}f'''-\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[f'^2-ff''\right]+K\left(2ff'''-ff^{iv}-f'^2\right)+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta=0, \quad (16)$$

$$\frac{k_{nf}}{k_f}\frac{1}{\Pr}\theta'+\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]f\theta'=0, \quad (17)$$

with the boundary condition as

$$f(0)=f'(0)=0,\quad \theta'(0)=-\gamma(1-\theta(0)),\qquad f'(\infty)=1,$$
$$f''(\infty)=0,\qquad \theta(\infty)=0, \quad (18)$$

The physical quantities of principal interest are shearing stress, and the rate of heat transfer in terms of the skin friction coefficient C_f and the local wall temperature $\theta_w(x)$ respectively, which can be written as

$$C_f=\frac{1}{(1-\phi)^{2.5}}x\frac{\partial^2 f}{\partial y^2}(x,0),\quad \theta_w(x)=-\frac{k_{nf}}{k_f}\frac{\partial\theta}{\partial y}(x,0). \quad (19)$$

III. Results And Discussion

The numerical computation has been carried out using the method described in the previous section for various values of K, λ, ϕ and γ to analyze the results. The figures are plotted, tables are drawn and physical explanations are given to illustrate the computed results. The comparison of results has been made with those of Merkin [10] and Rashad *et al.* [7] for the verification purposes. It is worth mentioning that the constant wall temperature results were recovered when a large value of γ is applied in the boundary conditions. Table II presents the results of this comparison. It can be seen from this table that good agreement between the results exists.

TABLE II. Comparison of the Results for Wall Temperature with $K=0$, $\phi=0$, $\Pr=1$, $\gamma\to\infty$

λ	Merkin [10]	Rashad *et al.* [7]	Present
-1.5	0.4576	0.4576	0.4592
0.0	0.5705	0.5706	0.5709
2.0	0.6497	0.6518	0.6519
5.0	0.7315	0.7319	0.7319

In Table III, we can see the numerical values of skin friction and wall temperature for the various values of viscoelastic parameter *K*. It shows that, as the viscoelastic parameter *K* increases, both values of skin friction and wall temperature are decreased. This can be attributed to the thickening of momentum and thermal boundary layers as *K* increases.

TABLE III. VALUES OF $C_f(0)$ AND $\theta_w(0)$ FOR DIFFERENT VALUES OF *K* WHEN $\lambda = 1$, $\phi = 0.1$, $\Pr = 6.2$, $\gamma = 1$

K	$C_f(0)$	$\theta_w(0)$
0	1.5748	0.5179
2	0.8465	0.4718
8	0.4985	0.4345
50	0.2187	0.3873
100	0.1574	0.3731

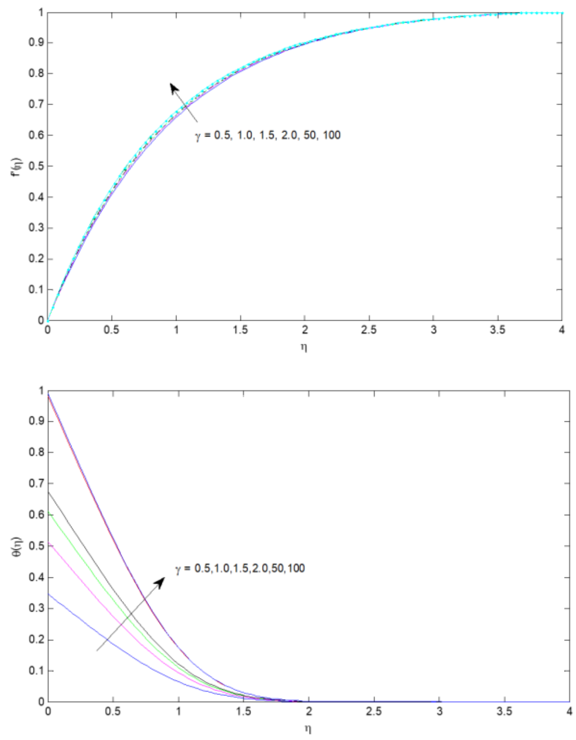

Figure 1. Velocity and temperature profiles for various values of γ when K=1, Pr=6.2, $\phi = 0.1$ and λ=1.0.

Fig. 2 displays the effect of γ on the velocity and temperature profiles. It is observed that increasing γ leads to the increase of the temperature and velocity profiles. As γ increases, the convective heat transfers from the hot fluid on the surface of the cylinder to the cold side increases leading to increases in both velocity and temperature profiles.

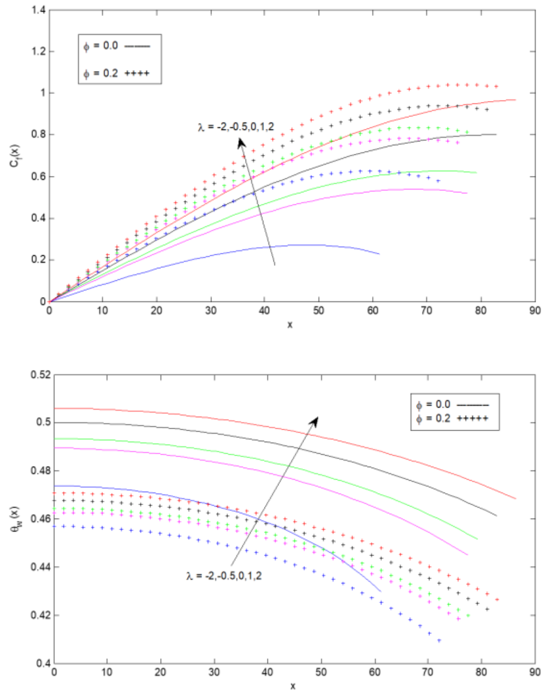

Figure 2. Velocity and temperature profiles for various values of γ when *K*=1, Pr=6.2, $\phi = 0.1$ and λ=1.0.

Fig. 3 display the effect of λ and ϕ on the skin friction coefficient and wall temperature with $\phi = 0.0$ and $\phi = 0.2$ respectively, and for various values of λ positive and negative. From that figure, for fixed ϕ, as the value of λ increases, the magnitude of the skin friction coefficient increases, while the magnitude of wall temperature decreases.

IV. CONCLUSION

In this paper, the effect of the mixed convection parameter λ, viscoelastic parameter *K*, nanoparticle volume fraction ϕ and conjugate parameter γ on the flow and heat transfer characteristic have been discussed. The dimensionless equations being solved numerically using the Keller-box method with FORTRAN software. We could draw the following conclusions: both skin friction and wall temperature decrease for the increase in *K*. On the other hand, increasing γ leads to the increase of the temperature and velocity profiles. Lastly, for fixed ϕ, as the value of λ increases, the magnitude of both the skin friction coefficient and wall temperature also increases.

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