Comparative Analysis of MPC Based on Integer and Non-Integer Order Models: Case Studies

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Abstract— The present work makes use of the fractional order modeling in realizing Model Predictive Control (MPC) for some processes i.e. DC motor and Water Bath system, whose models are derived from their open loop experimental data. The process behavior is investigated from experimental setup, then for the controller performance evaluation, simulation based analysis is made. Firstly, open loop experimental data is collected by applying a step change in the manipulating variable. Based on these data, integer and non-integer order model parameters of the processes are estimated using Genetic Algorithm (GA) respectively, by minimizing Integral of Squared Error (ISE) between open loop step data and model response. Once the models are available, a model based control technique MPC is designed and simulated using MATLAB MPC toolbox. Before using fractional order model for MPC design, it is firstly converted to equivalent higher integer order model using Oustaloup's recursive approximation. Since the performance of such type of control technique depends on the accuracy of model of the plant. Therefore, MPC based on non-integer model give better performance as compared to integer order model, as former is able to capture the model dynamics more accurately than later. Different simulations performed in MATLAB also approves the same.

Index Terms—Fractional order modeling, model predictive control, water bath system, DC motor, model identification, genetic algorithm.

Nomenclature

MPC:	Model Predictive Control
GA:	Genetic Algorithm
ISE:	Integral of Squared Error
IAE:	Integral of Absolute Error
FO:	Fractional Order
PWM:	Pulse Width Modulation
RPM:	Rotation per Minute
IR:	Infrared
SSR:	Solid State Relay
OP-AMP:	Operational Amplifier
FOPD:	First Order plus Delay

I. INTRODUCTION

Model Based Control (MBC) is one of the established control strategy that has many applications in process

industries. Model of the plant or process plays a vital role in determining the performance of such controllers, as they are embedded in control law. Accurate models will enhance the control performance, while the inaccurate one will deteriorate it. Since non-integer operators are comparatively more efficient in capturing the dynamics of real world systems, rather than integer one [1, 2], therefore non-integer models are bound to give better performances in such model based control technique. In the present work, a very popular model based control technique, i.e. Model Predictive Control (MPC), has been adopted. The model required for the design of MPC is estimated based on the information provided by open loop response of the experimental setup. The system is not only modelled as integer order but also as non-integer, thereby enhancing the controller performance. Two different cases that are considered, for performance analysis of the proposed controllers are as follows.

- Case I: Speed control of a DC motor
- Case II: Temperature control of a water bath system.

The term fractional order or non-integer has been used interchangeably in various literatures and so is the present case. Fractional order modelling refers to the class of modelling of systems where fractional derivative or integral operators are used. The branch of mathematics dealing with such type of operators is known as *"Fractional Calculus"* rather than simply *"Calculus"*. The definition of such type of fractional fundamental operator is given by (1), where α is the operator and 'a' & 't' are the limits of the operator.

$${}_{a}\mathcal{D}_{t}^{\alpha} = \begin{cases} d^{\alpha} / dt^{\alpha} &, \alpha > 0 \\ 1 &, \alpha = 0 \\ \int_{a}^{t} (d\tau)^{-\alpha} &, \alpha < 0 \end{cases}$$
(1)

Despite the fact that fractional calculus is as old as conventional calculus, it was not being preferred by scientific community, as the solutions of problems consisting such fractional operator were not available. In recent decades, the hard work of the researchers has provided numerous techniques to deal with such type of operators, thus making its application feasible in varied

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field of engineering. Monje, Vinagre, Xue and Feliu have presented their analysis on fractional order system and control in great detail [3]. A number of research articles are also available, where the applications of fractional calculus is being applied in control engineering to give further improved results [4-7]. Also for simulation work related to fractional order system, FOMCON toolbox [8] has been used in MATLAB.



Figure 1. Work outline

The rest of the paper is organised as follows. The step by step outline of the work carried out is given in Section II. Experimental setup for both the cases are described in section III. Section IV gives the transfer function modelling of the processes based on their open loop data. The control technique implemented here, i.e. MPC is discussed briefly in section V. Simulations results are presented and discussed in section VI. After analysing the results conclusion is drawn in Section VII.

II. OUTLINE OF THE WORK

Stepwise working procedure for the design of MPC for both the cases i.e. DC motor speed control and Water Bath temperature control can be well understood by Fig.1. For each steps, the tool required to achieve that particular step is also listed. Following are the different stepwise description.

- Step 1: Firstly, open loop data are collected when excited to different step input.
- Step 2: The data obtained in Step 1 are then used to estimate the integer and fractional order

model parameters using Genetic Algorithm (GA).

- Step 3: MPC are designed based on these integer and fractional order models obtained in Step 2 respectively, using MATLAB MPC toolbox.
- Step 4: The controllers obtained in Step 3 is tested against step change in the set-point. Also the regulatory responses are noted by adding disturbance at the output. All these testing are done through MATLAB simulations.
- Step 5: Based on above simulations, different performance indices are calculated to make comparative study between the controllers obtained based on integer and fractional order models respectively.

III. EXPERIMENTAL SETUP

A. CASE I: Speed Control of a DC Motor

DC motor is having a prominent role in almost all industrial and robotic applications. Its simplicity,

reliability, economical feasibility and the ease with which such types of DC motor speed can be controlled, makes it omnipresent. With the recent advancements power electronics, various switching control techniques emerged. Pulse Width Modulation (PWM) technique is one of them [9, 10], which has been used here for the speed control of DC motor by manipulating its duty cycle.

The experimental setup for the DC motor speed control system is shown in Fig. 2. It includes one 12 volts / 1amp DC motor having maximum RPM of 2000. To the shaft of this motor is attached one circular disc, whose one half of the circular area is colored in silver and another half in black. An infrared (IR) proximity sensor having one IR transmitter and one IR receiver is engaged to sense the RPM of motor. The IR transmitter continuously transmits the IR ray on the rotating disc which is reflected back to the IR receiver to give positive output when falls on silver area and zero output when falls on black area, as the black area absorbs the IR ray rather than reflecting it. Thus depending on the rotational speed of the motor a pulsating signal of specific frequency is generated at the output of IR transmitter.



Figure 2. DC motor speed control system

At the interval of every 0.1 second, the output frequency is measured and updated which is communicated to control algorithm employed in LabVIEW environment through a LabJack make U3-UV [11], data acquisition card. Based on current RPM information, the controller generates control signal in the form of PWM signal through the same data acquisition card. This signal is then fed to L298N DC motor driver circuit, through which the speed of motor is regulated to desired set point. This is how a real time closed loop speed control system prototyping is realized.

B. CASE II: Tempertaure Control of a Water Bath System

Temperature monitoring plays a decisive role in controlling the product quality of various process industries. Water Bath is one of the very useful system associated with chemical and food processing sector [12, 13]. In chemical industries, the water bath system proves to be crucial to gain control over chemical reactions, whereas in food industries it is used for pasteurization of

different milk products and also for preparing other products. Here a simple experimental setup has been established depicting the water bath system

Fig. 3, represents the experimental setup of water bath temperature system used for collecting open loop data. A 240V, 50Hz electric kettle of 300 wattages serves as a water bath system. One very common temperature sensor, LM35[14] is installed at the lower portion of kettle using a resin based epoxy compound, thereby making it waterproof. The kettle is filled with 1000 ml of water whose temperature profile is recorded.



Figure 3. Water bath system

The real time experimental setup is interfaced with LabVIEW environment using Labjack U3-HV, Data Acquisition Card (DAC) [11]. This DAC is also having the provision of generating PWM signal of desired duty-cycle (in %) through inbuilt PWM signal generator function just by using it in that specific mode. Thus to obtain the step response in open loop configuration, the step change is made in the duty cycle of PWM manually, which then goes to Solid State Relay (SSR) [15] through the non-inverting OP-AMP arrangement to regulate the power flow to the electric kettle. The output of the DAC is 3.3V and SSR used here operates at 5V, therefore a non-inverting OP-AMP arrangement is employed to amplify 3.3V signal to 5V. This is how for different input of PWM signal, the open loop data is collected.

IV. INTEGER AND NON-INTEGER ORDER MODELLING OF THE SYSTEMS

A. Case I: DC Motor

For the control system under study i.e. speed control of a DC motor, first of all different open loop step responses are noted to analyze the system behavior. Since PWM is the input i.e. manipulating variable and RPM of the motor is output variable i.e. controlled variable, therefore the duty cycle (in percentage) of PWM signal is varied to obtain the steady state RPM corresponding to each input signal. This input-output data is then plotted as shown in Fig. 4. From the nature of the plot, it can be concluded that within the specific input range of duty cycle i.e. 25% to 75%, DC motor speed response almost exhibits linear characteristic. So, the controller designed for any operating condition within this range should work well throughout the RPM range of 550 to 1600.

Now the next step is to estimate the DC motor model parameter based on the step response. A step change in duty cycle of PWM signal is made from 50% to 75%, and RPM response data is obtained. Since this response very much matches the characteristics of step response of first order plus delay system. Therefore, it is deduced that the current system can be modelled as first order plus delay system. Making use of the flexible feature [16, 17] of fractional calculus in modelling, the present system is not only modelled as integer but also as non-integer order system given by (2)and as (3)respectively.



Figure 4. Input output steady state characteristics for DC motor

$$G_{\text{integer}} = \frac{K}{\tau s + 1} e^{-Ls} \tag{2}$$

$$G_{\text{non-integer}} = \frac{K}{\tau s^{\alpha} + 1} e^{-Ls}$$
(3)

B. Case II: Water Bath System

In very same manner as above, the duty cycle of PWM signal is varied from 20% to 40%, at interval of every 2.5% and corresponding steady state temperature profile is noted. Then these input-output data are plotted as shown in Fig.5, from which it is inferred that within the specified range of input, water bath temperature profile almost exhibits linear characteristic.



Figure 5. Open loop input-output characteristic of Water Bath system

After that, a step change is made in the duty cycle around the operating condition to observe the open loop step response and is found to be very much akin to the step response of First Order Plus Delay (FOPD) system just like in the case of above DC motor response. Hence water bath system is modelled by the transfer function given by (2), where the parameters Gain (K), Time constant (τ) and Delay (L) are estimated. The system is also modelled as fractional FOPD system, where one

additional parameter i.e. ' α ', order of the model along with (K), (τ) & (L) is also estimated as given by (3).

For parameters estimation of models for both the systems designated as CASE I & CASE II, given by (2) & (3), a very popular nature inspired optimization technique, i.e. Genetic Algorithm (GA) [18, 19], has been employed. Prior to the start for search of optimal parameters using GA, one has to initialize the optimal parameter values. Also an objective function, whose fitness depends on these parameters are defined which is being minimized or maximized as per need of the problem. For the present case, Integral of Squared Error (ISE), given by (4) is taken as objective function subject to minimization. By comparing the available open loop data with model step response data at every simulation run, the error is obtained, based on which ISE value is calculated. It is then fed back to GA, executed in MATLAB as shown in Fig.6. This loop keeps on running until optimality is reached.

$$ISE = \int_{0}^{\infty} \{y_{sp}(t) - y(t)\}^{2} dt = \int_{0}^{\infty} \{e(t)\}^{2} dt$$
(4)



Figure 6. GA optimized Model Parameters

Table I gives the considered GA characteristics.

TABLE I. GA SETTINGS

Population Size	50
Fitness Scaling Function	Rank
Crossover Function	0.8
Crossover Fraction	Scattered
Migration Fraction	0.2
Ending Criterion	Function tolerance of 1e-4

The estimated integer & non-integer order transfer function model parameters for DC motor are given in Table II, with their respective ISE values. Table III gives the same for Water Bath system. The open loop step response of the obtained models along with the real system, shown in Fig.7, helps in validating the estimated models for DC motor. Fig.8 gives the same for Water Bath system

TABLE II. TRANSFER FUNCTION MODELS FOR DC MOTOR

Nature of Model	Transfer Function	ISE
Integer Order	$\frac{22.58}{0.235s+1}e^{-0.347s}$	440
Non- Integer Order	$\frac{23.358}{0.25s^{0.91}+1}e^{0.364s}$	154

Nature of Model	Transfer Function	ISE
Integer Order	$\frac{1.114}{1170s+1}e^{-122s}$	690
Non-Integer Order	$\frac{1.122}{1313s^{1.021}+1}e^{112s}$	230

TABLE III. TRANSFER FUNCTION MODELS FOR WATER BATH SYSTEM



Open Loop Step Responses for DC motor Figure 7.



Open Loop Step Responses for Water Bath System Figure 8.

V MODEL PREDICTIVE CONTROL

Model Predictive Control (MPC) is one of the advanced model based control technique, where the accuracy of process model significantly affects the overall closed loop control performance. Two very renown companies Shell Oil and ADERSA around 1980, proposed the elementary idea of MPC [20, 21]. This technique gained a wide acceptance worldwide from different process industries, mainly oil refineries and petrochemical plants. The main adorable feature of MPC is that, it can easily deal with the control aspect of multiinput multi-output (MIMO) system and also fulfilling the desired constraints on input as well as on output at the same time. Although the present control problem limits to single input single output system.

In MPC, the future control moves are calculated based on the model response. Depending on the error obtained by comparing the predictions and actual measurements, an appropriate change in manipulated variable is made. Fig.9 shows the MPC block diagram, where the residual or error is fed back to Prediction block [20]. At every sampling instant, the predicted output is used for updating the set-point and control moves calculations. This control calculation block mainly involve optimizer, which

optimizes an already defined objective function to future generate the control moves. Different terminologies associated with MPC are as follows:

- Sampling time (T_s) : It is the time interval at which the control action takes place after performing all the control calculations. In present case the control interval is taken same as sampling time.
- Prediction Horizon (P): It is number of interval for which MPC predicts the future control moves based on MPC calculations.
- Control Horizon (M): It is number of manipulated variable moves to make at the specified control interval.
- Objective function: It is function of error whose value needs to be minimized to obtain optimum control moves. Generally quadratic objective function is taken and so is the present case. It is defined as the sum of squares of predicted errors.



Figure 9. MPC Structure

Table IV gives the MATLAB MPC toolbox setting used for deriving the controller for speed control of the DC motor. Table V gives the MPC settings for temperature control of the Water Bath System.

TABLE IV. MPC SETTINGS FOR DC MOTOR

Sampling time (T_s)	0.1 second
Prediction Horizon (P)	20
Control Horizon (M)	1
Objective function	Quadratic

TABLE V. MPC SETTINGS FOR WATER BATH SYSTEM

Sampling time (T_s)	1 second
Prediction Horizon (P)	2000
Control Horizon (M)	5
Objective function	Quadratic

For designing MPC based on fractional order model [22], the model is firstly converted the equivalent higher integer order model using Oustaloup's method [23].

VI. RESULTS AND DISCUSSION

For testing the control performance of the designed MPC controllers, various simulations were performed using MATLAB SIMULINK. As observed in Section IV,

non-integer model is more accurate than integer order model, therefore for simulation purpose the fractional order model is assumed to be representing the actual system. Then the MPC based on integer order model is implemented in closed loop followed by the MPC based on fractional order model.

A. CASE I: Speed Control of a DC Motor

A set-point change is made to have the desired speed of DC motor at 600 rpm. Fig.10 gives the step responses for both type of MPC's, from which different time domain performance indices were obtained and tabulated in Table VI. Fig. 11 show the servo responses for the MPC obtained from both integer and fractional model, for the step change of 1600 rpm to 1400 rpm, at t=2 seconds once it stabilizes at 1600 rpm, and 1400 rpm to 1800 rpm at t=4 seconds once it stabilizes at 1400 rpm, along with the control signal profile also. Fig. 12 gives the regulatory response for the same, when a positive and negative disturbance of 10% with respect to set-point are applied at t=2 seconds and t=4 seconds respectively. Analysis different closed loop response as shown in Fig. 9, 10 & 11 and the data of Table VI highlight the fact the MPC based on non-integer model outperforms MPC based on integer model.

TABLE VI. PERFORMANCE INDICES FOR DC MOTOR

Performance Indices	MPC based on integer order model	MPC based on fractional order model
Peak time (t_p)	1.42 s	1.35 s
Rise time (t_r)	1.11 s	1.31 s
Max. overshoot (M_p)	3.2%	Nil
Settling time (t_s), $\pm 2\%$	1.8 s	1.3 s
ISE	1.411e+6	1.244e+6
IAE	1108	941

B. CASE II: Motor Tempertaure Control of a Water Bath System

A step change is made to have the desired water bath temperature of 50 °C. Fig.13 gives the step responses for both type of MPC's obtained using integer & non-integer model, from which different time domain performance indices were obtained and tabulated in Table VII. The results of Table VII show the superiority of MPC based on fractional order model over integer order, only rise time, t_r being the exception, which is not so relevant in the present case. Fig. 14 show the servo responses for the MPC obtained from both integer and fractional model, for the step change of 50 $^{\circ}$ C to 60 $^{\circ}$ C, at t=2000 seconds once it stabilizes at 50 °C and 60 °C to 40C at t=4000 seconds once it stabilizes at 60 °C, along with the control signal profile also. Fig. 15 gives the regulatory response for the same, when a positive and negative disturbance of 20% with respect to set-point are applied at t=2000 seconds and t=4000 seconds respectively.

TABLE VII. PERFORMANCE INDICES FOR WATER BATH SYSTEM

Performance Indices	MPC based on integer order model	MPC based on fractional order model
Peak time (t_p)	1130 s	1020 s
Rise time (<i>t_r</i>)	750 s	900 s
Max. overshoot (M_p)	3%	0.8%
Settling time (t_s) , $\pm 2\%$	1200 s	900 s
ISE	7.835 e+5	7.820 e+5
IAE	2.21e+4	2.15e+4



Figure 10. Closed loop Step response for DC motor system



Figure 11. Closed loop Servo Response for DC motor system







Figure 13. Closed loop Step response for Water Bath system





Figure 15. Closed loop Regulatory Response for Water Bath system

VII. CONCLUSION

Based on real time open loop data collected using Data Acquisition Card (DAC), models of the processes are estimated. Not only integer order model but also fractional order model is estimated as the later one proves to be efficient in capturing the dynamics of both the plants, thus reducing model error. These models are then used for predictions in MPC. Through results obtained from various simulations performed in MATLAB, it is deduced that MPC designed based on fractional order model gives better control performance as compared to integer order model in terms of performance indices. Also the Integral Error criteria such as ISE & ITAE ratifies the same. Hence the proposed technique can also be extended to other model based control techniques to obtain better control performance.

REFERENCES

 S. Das and A. Gupta, "Fractional order modeling of a PHWR under step-back condition and control of its global power with a robust Pl^AD^μ controller," *IEEE Transactions on Nuclear Science*, vol. 58, no. 5, pp. 2431-2441, Oct. 2011

- [2] W. Yu, Y. Luo, and Y. Pi, "Fractional order modeling and control for permanent magnet synchronous motor velocity servo system," *Mechatronics*, vol. 27, no. 7, pp. 813-820, 2013.
- [3] C. A. Monje, B. M. Vinagre, D. Xue, and V. Feliu, Fractionalorder Systems and Controls: Fundamentals and Applications, Springer-Verlag, London, 2010.
- [4] R. K. H. Galvao, S. Hadjiloucas, K. H. Kienitz, H. M. Paiva, and R. J. M. Afonso, "Fractional order modeling of large threedimensional rc networks," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 3, pp. 624–637, March 2013.
- [5] A. S. Elwakil, "Fractional-order circuits and systems: An emerging interdisciplinary research area," *IEEE Circuits and Systems Magazine*, vol. 10, no. 4, pp. 40–50, Fourthquarter 2010.
- [6] M. O. Efe, "Fractional order systems in industrial automation: a survey," *IEEE Transactions on Industrial Informatics*, vol. 7, no. 4, pp. 582–591, Nov 2011.
- [7] A. W. Nasir and A. K. Singh, "IMC based Fractional Order Controller for Non-Minimum phase system," in *Proc. 2015 Annual IEEE India Conference (INDICON)*, New Delhi, 2015, pp. 1-6.
- [8] A. Tepljakov, E. Petlenkov, and J. Belikov, "FOMCON: a MATLAB toolbox for fractional-order system identification and control," *International Journal of Microelectronics and Computer Science*, vol. 2, no. 2, pp. 51–62, 2011.
- [9] B. Najjari, S. M. Barakati, A. Mohammadi, M. J. Futohi, and M. Bostanian, "Position control of an electro-pneumatic system based

on PWM technique and FLC," ISA Transactions, vol. 53, no. 2 pp. 647-657 March 2014.

- [10] L. Petru and G. Mazen, "PWM Control of a DC Motor Used to Drive a Conveyor Belt," Procedia Engineering, vol. 100, 2015, pp. 299-304.
- [11] "U3 datasheet " [Online]. Available: http://labjack.com/sites/default/files/LabJack-U3-Datasheet-xport-20160108.pdf
- [12] O. P. Verma, R. Singla, and R. Kumar, "Intelligent temperature controller for water-bath system," International Journal of Computer, Electrical, Automation, Control and Information Engineering, vol. 6, no. 9, pp.1178 - 1184, 2012.
- [13] "Cooking in food industry." [Online]. Available: http://wiki.zeroemissions.at/index.php?title=Cooking in food industry "Lm35 precision centigrade temperature sensors."
- [14] [Online]. Available: http://www.ti.com/lit/ds/symlink/lm35.pdf
- g3mb." state relay [15] "Solid [Online]. Available: http://www.electronica60norte.com/mwfls/pdf/g3mb-202.pdf
- [16] A. Vishwesh and P. S. V. Natraj, "Fractional order modeling of neutron transport in a nuclear reactor," *Applied Mathematical Modelling*, vol. 37, no. 23, pp. 9747-9767, 2013.
- [17] T. Djamah, R. Mansouri, S. Dfennounce, and M. Bettayeb, "Optimal low order model identification of fractional dynamic systems," Applied Mathematics and Computation, vol. 206, no. 2, pp. 543-554, 2008.
- [18] J. Holland, Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence. University of Michigan Press, 1975.
- [19] D. Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning, ser. Artificial Intelligence. Addison-Wesley Publishing Company, 1989.
- [20] D. Seborg, D. Mellichamp, T. Edgar, and F. Doyle, Process Dynamics and Control. John Wiley & Sons, 2010.
- [21] J. B. Rawlings, "Tutorial overview of model predictive control," IEEE Control Systems, vol. 20, no. 3, pp. 38-52, 2000.
- [22] M. M. Joshi, V. A. Vyawahare, and M. D. Patil, "Model predictive control for fractional-order system a modeling and approximation based analysis," in Proc. 2014, 4th International Conference On Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH), pp. 361–372.
- [23] Z. C. C. Y. Xue, D., "A modified approximation method of fractional order system," in *Proc.* 2006 International Conference on Mechatronics and Automation, June 2006, pp. 1043-1048.



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