

Mathematical Modeling of Non-ideal Zeta Converter Using Tags in MATLAB/Simulink

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Abstract—There are various computer-aided software available to simulate dc-dc power converters using discrete semiconductor devices. Specifically, MATLAB is widely popular for simulation of power electronics converters. For these simulations, MATLAB provides a dedicated toolbox namely ‘SimPowerSystem’. However, this paper presents an alternative approach to simulate power electronic circuits without having this toolbox. This approach is explained with example of a fourth-order non-ideal dc-dc Zeta converter. This method avoids the use of discrete semiconductor devices, resulting in fast and efficient simulation model. The effect of non-idealities of Zeta converter is also included in the simulated model. The simulation results verify that the different voltage and current responses are exactly same as obtained with ‘SimPowerSystem’ toolbox.

Index Terms—Dc-dc converter, modeling, MATLAB/Simulink, non-idealities, simulation, zeta converter

I. INTRODUCTION

The mathematical modeling of any physical system is necessary whether it is mechanical system or electrical system. The mathematical model provides the replica of the physical system which can further be used for analysis and control design. The simulation is the process for verifying the main characteristics and behavior of the obtained mathematical model with the selected physical system. In electrical engineering, the power electronic converters are widely used in variety of applications [1]–[6]. Therefore, the design and analysis of various power electronic converters is important for an electrical engineer. Computer simulation software provide a powerful tool for understanding the dynamical and steady-state behavior of any system before its actual physical/hardware implementation [7], [8]. In last few decades, many computer aided simulation software (such as MATLAB, PSpice, PSIM, SABER *etc.*) have been developed for simulation of power electronic systems [9]–[13]. However, all these software have their own advantages and limitations in terms of accuracy, speed, functionality, user-friendliness, computer memory requirement, cost *etc.* MATLAB is widely used tool for simulation of power electronics systems because it makes the design of complex feedback control very easy [14]. MATLAB can also be used in “hardware-in-loop”

simulations in conjunction with various real-time control systems [15].

MATLAB provides a dedicated toolbox namely “SimPowerSystems” to simulate power electronic system models [16]. This toolbox contains all commonly used semiconductor devices, passive and active components, voltage sources, current sources *etc.* However, to use SimPowerSystem toolbox, one is required to have the knowledge and expertise in power electronics circuits. For the complex power electronics circuits, it suffers from calculations of complex algebraic loops and convergence problems [14]. These problems become more dominant, as the system order and non-idealities increase [17]. In many cases, specifically for control engineers, user may not procure this toolbox. The control engineers just need to implement their control methods on the given system topology. In such cases, there is need of a simple simulation approach. In this paper, the study represents a simplified model which can be easily realized using the general blocks of Simulink. For this purpose, the user is just required to derive the various current and voltage equations of the power converter circuit. There is no requirement to implement the resistors, inductors, switches and diodes for simulation purpose. Therefore, it is free from algebraic loops and convergence problem. The motivation and aim of this paper is to provide a simple simulation model of power electronics converter which can be used easily for analysis and control design.

To explain this approach, a non-ideal dc-dc Zeta converter is chosen. The equivalent series resistances (ESRs) of inductors and capacitors are considered in order to show the general and complex model behavior. Only basic knowledge of Kirchhoff’s voltage and current laws (KVL and KCL) is used to write the output voltage, inductor current and capacitor voltage equations.

The remaining paper is constructed as follows: In section-II, the state-space representation of the non-ideal dc-dc Zeta converter is obtained in order to derive mathematical model and to explain the circuit behavior. In section-III, these mathematical equations are realized using various tags available in Simulink library to obtain the different currents and voltages. The simulation results are analyzed and discussed in section-IV. The conclusion of the paper is given in section-V followed by the significant references.

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II. STATE-SPACE DESCRIPTION OF ZETA CONVERTER

The power circuit of a Zeta converter is shown in Fig. 1. It consists of a switch S , diode D_d , two capacitors C_1 , C_2 (with equivalent series resistances r_{C1} and r_{C2} , respectively) and two inductors L_1 , L_2 (having series internal resistances r_{L1} and r_{L2} , respectively). The converter is operating in continuous current conduction mode (CCM) with duty cycle D and switching period T (or switching frequency f). The principle of operation of Zeta converter has been discussed in literature extensively [18]–[20] and it is therefore not discussed here. In brief, Zeta converter provides the output voltage with same polarity as of input voltage whereas the magnitude can be lower, equal or greater than the input voltage. Zeta converter has continuous output current and discontinuous input current. The dc-dc Zeta converter can operate in two modes: continuous conduction mode (CCM) and discontinuous conduction mode (DCM). However, in subsequent discussion, the converter is assumed to be operating in CCM. The equivalent circuits of Zeta converter during switch on ($0 < t \leq DT$) and switch off ($DT < t \leq T$) are shown in Fig. 2(a) and Fig. 2(b) respectively. The state-space equations (voltage and current equations) for each mode can be written easily by simply applying KVL and KCL. The combined state-space description of Zeta converter can be expressed as-

$$v_{L1}(t) = L_1 \frac{di_{L1}(t)}{dt} = -r_{L1}i_{L1}(t) + uv_g(t) - \bar{u}(r_{C1}i_{L1}(t) + v_{C1}(t)) \quad (1)$$

$$v_{L2}(t) = -r_{L2}i_{L2}(t) - v_o(t) + u(-r_{C1}i_{L2}(t) + v_{C1}(t) + v_g(t)) \quad (2)$$

$$i_{C1}(t) = C_1 \frac{dv_{C1}(t)}{dt} = -ui_{L2}(t) + \bar{u}i_{L1}(t) \quad (3)$$

$$i_{C2}(t) = i_{L2}(t) - \frac{v_o(t)}{R} \quad (4)$$

$$v_o(t) = v_{C2}(t) + r_{C2}i_{C2}(t) \quad (5)$$

Here u is a binary switching signal, which is either 1 (when switch is on) or 0 (when switch is off) and $\bar{u} = 1 - u$. i_{L1} , i_{L2} are currents through inductors L_1 and L_2 respectively. v_{C1} is voltage across capacitor C_1 and v_o is output voltage of Zeta converter. By substituting $u=1$, the state-space equations for equivalent circuit in Fig. 2(a) can be obtained whereas by putting $u=0$, the state-space equations for equivalent circuit in Fig. 2(b) can be obtained. In the next section, this mathematical description of Zeta converter will be used to simulate its various voltage and current waveforms.

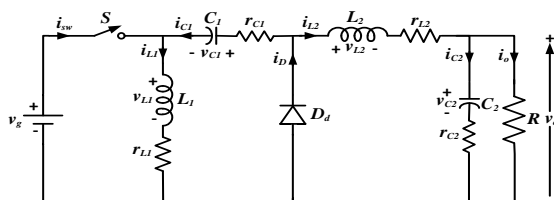


Figure 1. Power circuit of non-ideal dc-dc zeta converter

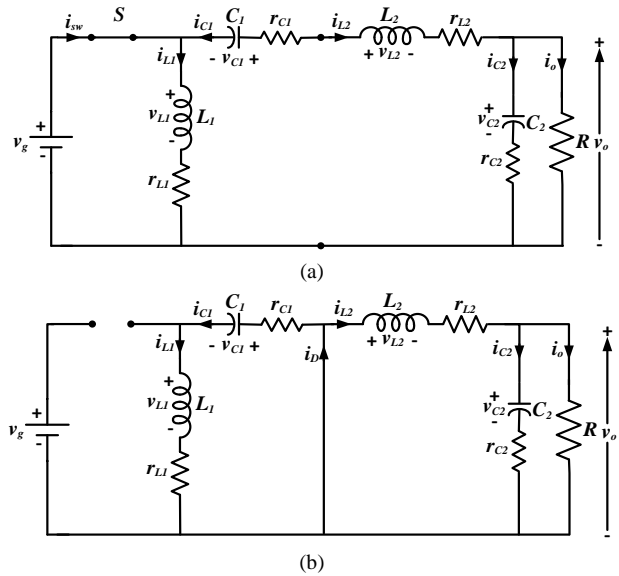


Figure 2. Equivalent circuits of Zeta converter (a) when switch is on (b) when switch is off

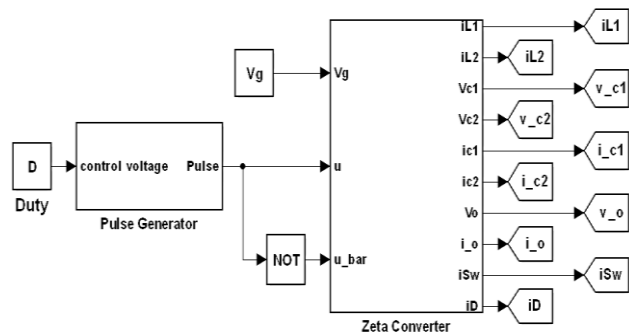
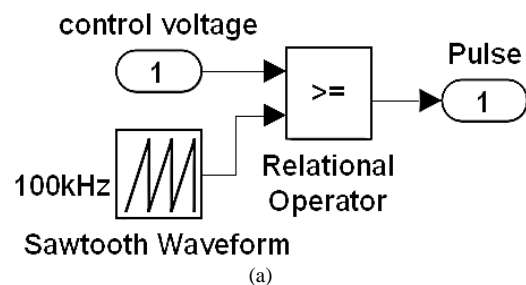


Figure 3. MATLAB/Simulink model of Zeta converter

III. MATLAB/SIMULINK MODEL OF NON-IDEAL DC-DC ZETA CONVERTER

Having obtained the state-space model of Zeta converter in eqs. (1) - (5), the next step is to realize these equations using basic blocks of MATLAB/Simulink. There are many blocks in the Simulink library. The main Simulink block model of non-ideal dc-dc Zeta converter is shown in Fig. 3. This model shows that for a given duty cycle D and input voltage V_g , the various voltages and currents are obtained. This simulation model is realized using various sub-models. The sub-model blocks for pulse generator and power circuit of Zeta converter are shown in Fig. 4.



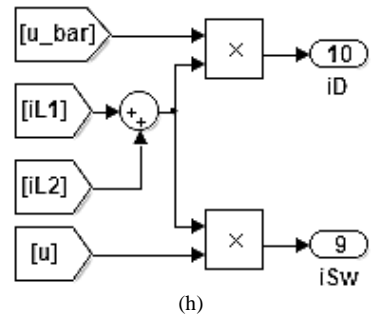
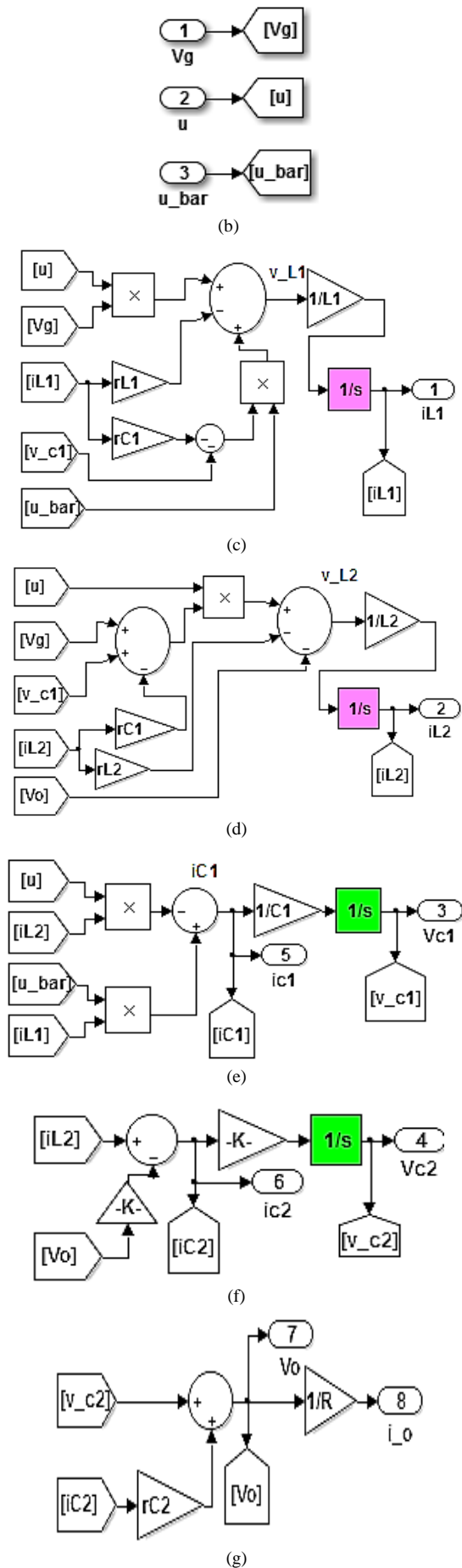


Figure 4. Detailed sub-model blocks in MATLAB/Simulink

The pulse generator block in Fig. 4(a) produces PWM pulses by comparing control voltage signal v_C (received from controller in closed-loop system) with sawtooth waveform of desired fixed switching frequency f [19]. When the magnitude of control signal is greater than sawtooth waveform, then the power switch ‘S’ is on and when the magnitude of control signal is lower, the power switch ‘S’ is off. The comparator is realized using ‘relational operator’ block of Simulink library. The ‘relational operator’ is implemented with greater than equal to (\geq) operation. It functions as comparator for control voltage and sawtooth waveform. In this paper, control voltage is constant and set to a value equal to duty cycle as simulation is carried out in open loop. The switching frequency is selected as 100 kHz and duty cycle 0.25. The output of pulse generator block is 1 for switch on and 0 for switch off. The pulse generator block remains same for all type of dc-dc converters. Its input parameters are converter switching frequency and control voltage, which may vary as per system design.

The other important block is Zeta converter itself. Zeta converter has four state variables namely inductor currents i_{L1} , i_{L2} and capacitors voltages v_{C1} , v_{C2} . In Fig. 4 (c), the realization of inductor current i_{L1} is described implementing eq. (1). The internal resistances r_{C1} , r_{C2} , r_{L1} and r_{L2} are realized using gain blocks. The various signals i_{L1} , u , V_g , v_{C1} , u_bar are sensed using ‘From’ block of signal routing sub-library. The signal ‘ u_bar ’ is inverse of signal ‘ u ’ which is obtained using ‘NOT’ logic block. The inductor voltage v_{L1} is formed by multiplying and then adding these signals as per eq. (1). Then v_{L1} is first multiplied by gain ‘ $1/L_1$ (inverse of L_1)’ and then integrated to generate inductor current i_{L1} . This inductor current is connected to ‘Goto’ block of signal routing sub-library for further use in subsystem to realize other circuit variables. In the same fashion, the other state variables are also realized using integrator, suitable gain, and corresponding signal variables as shown in Fig. 4(d)-4(g). The inductor current i_{L2} can be calculated by first manipulating inductor voltage v_{L2} as given in eq. (2). For calculating capacitor voltage v_{C1} , i_{C1} is first multiplied by gain ‘ $1/C_1$ (inverse of C_1)’ and then integrated. This capacitor voltage is connected to ‘Goto’ block of signal routing sub-library for further use in subsystem to realize other circuit variables. The capacitor current i_{C1} is generated by implementing eq. (3). Similarly, capacitor current i_{C2} is calculated by implementing eq. (4) and then from it, capacitor voltage v_{C2} is obtained.

As shown in Fig. 4(g), the output current i_o is obtained by multiply output voltage v_o with gain '1/R (inverse of resistance)'. For this, first output voltage is calculated using sum block by adding v_{C2} and product of i_{C2} and r_{C2} .

The switch current and diode current can also be simulated implementing following relations. These relations can easily be obtained from the equivalent circuit in Fig. 2.

$$i_{sw}(t) = u(i_{L1}(t) + i_{L2}(t)) \quad (6)$$

$$i_D(t) = \bar{u}(i_{L1}(t) + i_{L2}(t)) \quad (7)$$

As shown in Fig. 4(h), firstly inductor current i_{L1} and inductor current i_{L2} are added together and then the resultant sum is multiplied by signal 'u' to get switch current. The resultant sum is multiplied by signal 'u_bar' to get diode current.

IV. SIMULATION RESULTS

The circuit parameters used in simulation are as follows [18]: input voltage $V_g=15$ V, output voltage $V_o=5$ V, inductors $L_1=100$ μ H, $L_2=55$ μ H, capacitors $C_1=100$ μ F, $C_2=200$ μ F, load resistance $R=1$ Ω , capacitor ESRs $r_{c1}=0.19$ Ω , $r_{c2}=95$ m Ω , inductor ESRs $r_{L1}=1$ m Ω , $r_{L2}=0.55$ m Ω , switching frequency $f=100$ kHz, duty cycle $D=0.25$.

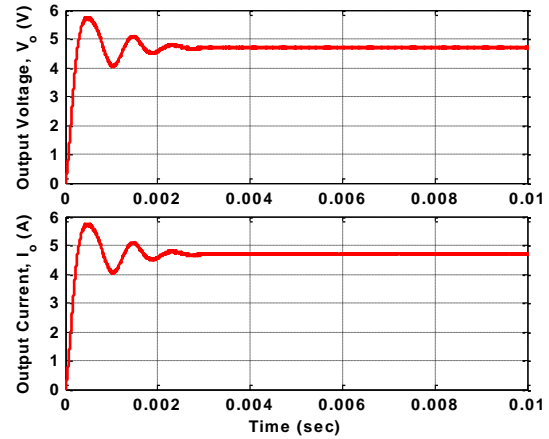
The MATLAB/Simulink simulation results of non-ideal dc-dc Zeta converter are shown in Fig. 5. The various voltage and current waveforms have been plotted using 'scope' block of Simulink. It is observed that all simulation waveforms match exactly with the simulation results of 'SimPowerSystems' toolbox and PSIM. However, owing to space limitation and to avoid repetition of similar results, the simulation results of the later have not been presented in the paper.

The output voltage v_o and output current i_o are shown in Fig. 5(a). The output voltage of ideal Zeta converter is expressed as $V_o=V_g \cdot D/(1-D)$. Therefore, the expected output voltage of ideal Zeta converter should be 5 V for an input voltage (V_g) 15 V and duty cycle (D) 0.25. But due to voltage drop across equivalent series resistances (ESRs) of inductors and capacitors, it comes out to be less than 5 V which can easily be observed in simulation results [21]. The output voltage of non-ideal dc-dc Zeta converter is 4.7 V. The output current profile is same as output voltage for a resistive load of 1 Ω .

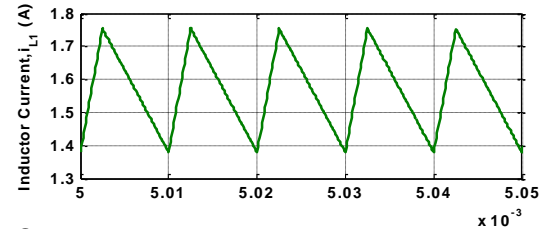
The inductor current waveforms in steady-state are shown in Fig. 5(b). The steady-state waveforms are shown from time $t=5$ msec to $t=5.05$ msec (for five switching cycles). The minimum, maximum and average values of inductor current i_{L1} are 1.38 A, 1.76A and 1.57 A, respectively. It has current ripple of 0.38 A. The minimum, maximum and average values of inductor current i_{L2} are 4.38 A, 5.02A and 4.7 A, respectively. It has current ripple of 0.64 A. The steady-state capacitor voltages v_{C1} , v_{C2} are shown in Fig. 5(c). The minimum, maximum and average values of capacitor voltage v_{C1} are 4.64 V, 4.76A and 4.7 V, respectively. The capacitor

ripple voltage is 0.12 V. The minimum and maximum values of output capacitor voltage v_{C2} are 4.698 V, 4.701A respectively. The output capacitor ripple voltage is 0.003 V.

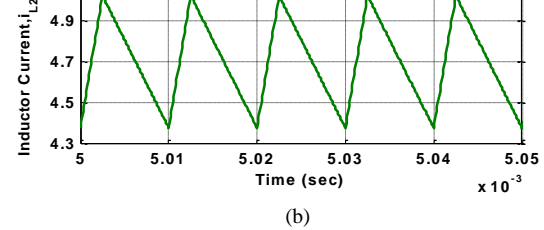
The steady-state simulated waveforms of switch current i_{sw} , diode current i_D are shown in Fig. 5(d). It can be observed that when the switch is on, the switch current is equal to sum of both inductor currents and the diode current is zero. When diode is on, the diode current is equal to sum of both inductor currents and switch current is zero. The peak switch current and diode current is same and its magnitude is 6.778 A.



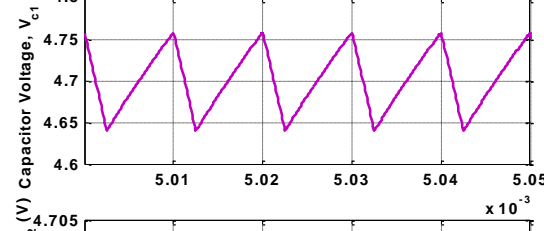
(a)



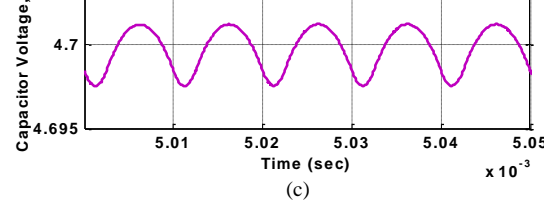
(b)



(c)



(e)



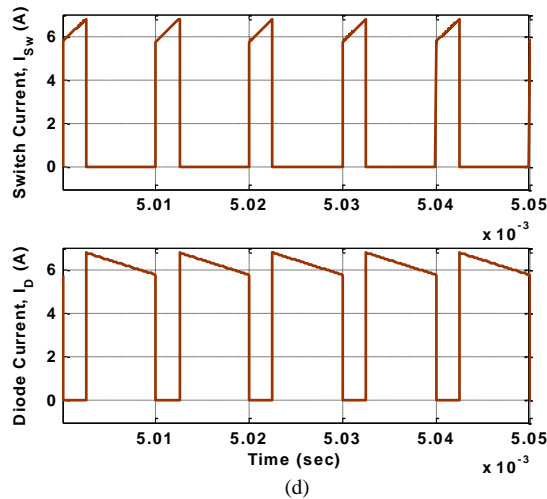


Figure 5. Simulation results (a) Output voltage v_o and output current i_o (b) inductor currents i_{L1} and i_{L2} (c) capacitor voltages v_{C1} and v_{C2} (d) switch current i_{sw} and diode current i_D

V. CONCLUSION

A step-by-step procedure for simulating a non-ideal dc-dc Zeta converter with MATLAB Simulink tags has been presented. This modeling procedure helps to non-power electronics researcher to simulate any power electronic converter with simple steps. The simulation results validate the importance and simplicity of the presented MATLAB/Simulink model for fourth-order dc-dc converters having non-idealities like internal resistances. The approach avoids the use of discrete semiconductor power devices like MOSFETs & Diodes and passive elements such as inductors & capacitors. The approach is presented for dc-dc Zeta converter, however, it is equally valid for other power electronics systems as well. This simulation model can also be used for feedback control design.

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