



Research Paper

STRESS ANALYSIS OF SPLINE SHAFT USING FINITE ELEMENT METHOD AND ITS EXPERIMENTAL VERIFICATION BY PHOTO ELASTICITY

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This research work deals with the stress in the Spline shaft under various loading condition of given torque. Finite element method along with experimental technique of photo elasticity is used. The stress analysis of TATA G750 (LPK2518TC) spline shaft model is carried out analytically and by FEM. Its results are validated using photo elasticity. From this analysis it has been observed that results obtained are in close agreement with each other and maximum shear stress concentration occurs at root of the spline teeth.

Keywords: Spline Shaft, Involute Spline, Finite Element Analysis, Photo elasticity

INTRODUCTION

Stress analysis is complete and comprehensive study of stress distribution in a component. As the spline shaft is used in many applications to transmit the torque while permitting the axial movement, it is necessary to know main cause of failure of the spline. The tooth engagement, tolerance and pressure distribution was studied. Very little information is found available on the stress concentration area on spline shaft. In the present work stress analysis of actual model of TATA G750 (LPK2518TC) of spline shaft as shown in figure1 is carried out for different operating torque using analytical, experimental and finite element method. The

analytical and FEA results are compared and validated with experimental results.

Figure 1: Spline Shaft



ANALYTICAL STRESS ANALYSIS OF SPLINE SHAFT

The design parameters for the Spline shaft were taken from actual model measurement as shown in Table 1.

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Table 1: Dimensions of Spline Shaft

Outer Diameter(Do) (D0)	Inner Diameter(Di)	Length of Spline(Le)	Tooth thickness(t)	Number of Spline	Mean Diameter(Dm)
37.4 mm	26.5 mm	125.69 mm	6.28 mm	10	32 mm

Stress calculation has been carried out for Spline Shaft using the technical specifications mentioned above and with following assumptions.

- i. The shaft rotates at a constant speed about its longitudinal axis.
- ii. The shaft has a uniform, circular cross section.
- iii. The joints are perfect and there are no geometrical irregularities.
- iv. Splined application factor $K_a=2.8$ for intermittent shock load.
- v. Load distribution factor $K_m= 1$ for fixed spline.
- vi. Fatigue life factor $K_f =0.3$ for 100000 cycle.

Shear stress at the pitch diameter of teeth. (N/mm²).

$$S_s = \frac{4T K_a K_m}{D_p N L_e t K_f}$$

Where,

T = Torque. N-mm

K_a = Spline application factor.

K_m = Load distribution factor.

K_f = Fatigue life factor.

D_p = Pitch diameter in mm.

N = Number of teeth.

t = Tooth thickness in mm

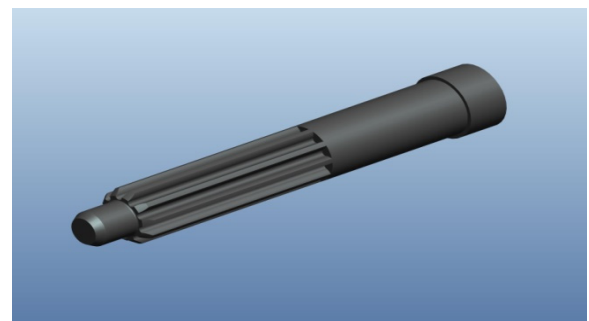
L_e = Length of spline in mm

FINITE ELEMENT ANALYSIS OF SPLINE SHFAT

Modeling

The Spline shaft of model Tata (LPK2518) was created in Pro-E wildfire 5.0 software as shown in figure 2 by taking the dimension from table1 and exported to ANSYS12 using “.iges” format. Thin Shell 187 was considered as the element type and material properties were given for Structured Steel. Then meshing was done for areas using Quad meshing. After this the shaft is fixed at one end and torque is applied at other end. Then the model is solved and FEA results are plotted.

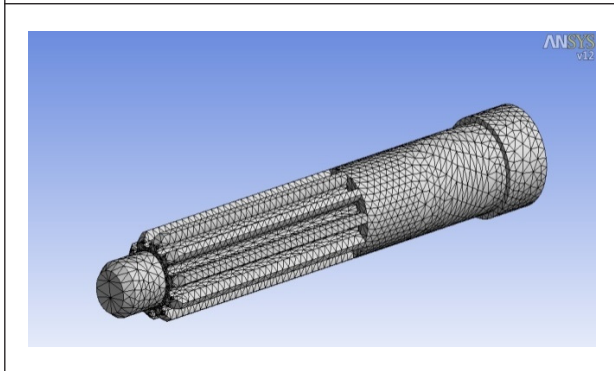
Figure 2: CAD Model of Spline Shaft



Meshing and Boundary Condition

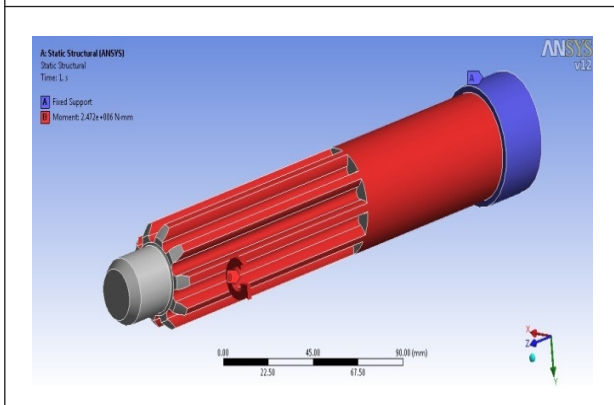
Finite element analysis is a numerical method in which a particular body is subdivided into discrete partitions (called elements) that are bound by nodes. Each element is connected to adjacent elements by the nodes. The best element type that can be employed to mesh the model is solid tetrahydral187 in Figure 3.

Figure 3: Meshing of Spline Shaft



Boundary conditions and environmental factors are applied to the subdivided model. The equations governing the individual elements are then combined and solved to obtain the solution for the overall problem. Boundary conditions are restricted at shank end as shown in Figure 4 (Fixed support) of spline shaft.

Figure 4: Boundary Condition



Applying Torque

A Torque of 2472 N/mm² is applied on spline shaft as indicated by circular arrow "B". The following procedure is adopted in ANSYS.

Preference → Loads → Define load → Apply → Structural Force/Moment → On nodes → Define load → Apply → Moment= 2472 N/mm².

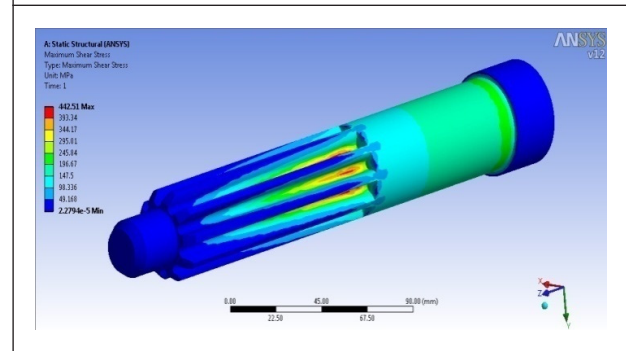
Solution

A Finite Element model is submitted to ANSYS solver as per following steps:

Stress>Max Shear>Solve>Close>Close.

The solved model of spline shaft is shown in Figure 5.

Figure 5: Maximum Shear Stress



EXPERIMENTAL STRESS ANALYSIS OF SPLINE SHAFT USING PHOTO ELASTICITY

Determination of Material Fringe Value Using Circular Calibration Disc

The circular disc of 65 mm diameter made of photo elastic material is loaded under diametral compression in polariscope. The material fringe value is determine as shown in observation Table 2 and isocromatic fringes developed in disc under load as shown in Figure 6.

Figure 6: Calibrating Disc Under Diametral Compression

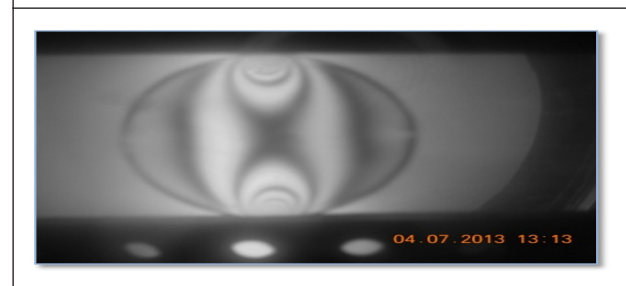


Table 2: Determination of Material Fringe Value

Sr. No.	Load in pan W (kgf)	Load on disc $P=(W \times 3.3) \times 9.81$ (Newton)	Fringe Order (N)			Material Fringe value $F\sigma = \frac{8P}{\pi DN}$ (N/mm)	Average Material Fringe Value $F\sigma$ (N/mm)
			Higher order (N1)	Lower order (N2)	Fringe Order $N = \frac{N1+N2}{2}$		
1	5	161.865	$0.5+0.14 = 0.64$	$1.5-0.85 = 0.65$	0.645	9.83	10.19
2	6	194.238	$0.5+0.25 = 0.75$	$1.5-0.76 = 0.74$	0.745	10.21	
3	7	226.611	$0.5+0.37 = 0.87$	$1.5-0.64 = 0.86$	0.865	10.26	
4	8	258.984	$0.5+0.51 = 1.01$	$1.5-0.50 = 1.0$	1.005	10.09	
5	9	291.357	$0.5+0.63 = 1.13$	$1.5-0.41 = 1.09$	1.11	10.28	
6	10	323.73	$0.5+0.71 = 1.21$	$1.5-0.29 = 1.21$	1.21	10.48	

Determination of Stress of Spline in Photo Elastic Model

For the photo elastic analysis, the loading fixture as shown in Figure 7 is designed and the analysis is carried out in bright field setup circular polariscope arrangement. The shear stress at the root of spline tooth is determined by calculating the fringe order under different torque for photo elastic model as shown in observation Table 4. The analytical shear stress for photo elastic model is shown in Table 3.

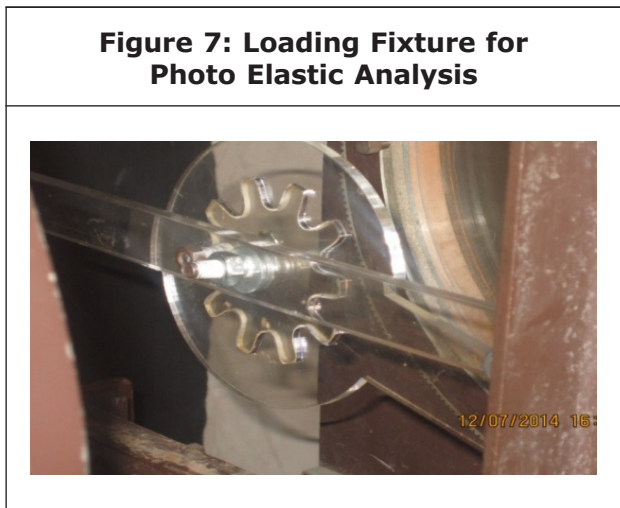


Table 3: Analytical Result: Shear Stress for Photo Elastic Model

Sr. No.	Weight in Pan (kgf)	Weight in Newton(N)	Lever length (mm)	Torque (N-mm)	Torque (N-m)	Theoretical Stress (N/mm ²)
1	2.5	25	260	6500	6.50	7.19
2	3.5	35	260	9100	9.10	10.07
3	4.5	45	260	11700	11.70	12.94
4	5	50	260	13000	13.00	14.38
5	6	60	260	15600	15.60	17.26

Shear Stress Observation on Photo Elastic Model During Experiment

In experimental analysis on photo elastic model shown in Figure 8, it is observed that maximum shear stress found on the root of spline teeth which is clearly shown during experimentation on photo elastic model.

Following data is used for stress analysis by photo elasticity:

- Material fringe value $F_{\sigma} = 10.19 \text{ N/mm}$
- Model thickness $h = 5 \text{ mm}$
- Experimental stress = N/mm^2
- Fringe order = N

Figure 8: Experimental Observation on Photo Elastic Model

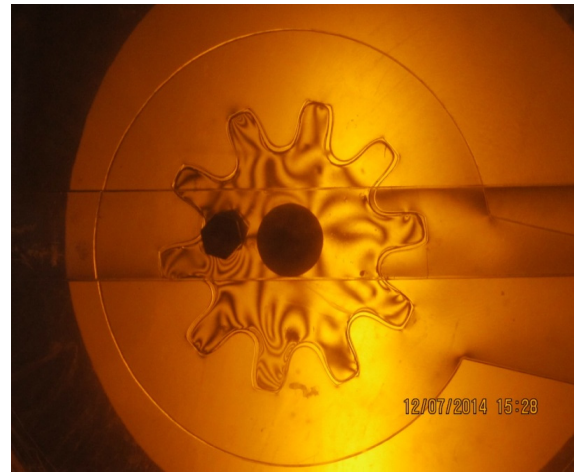


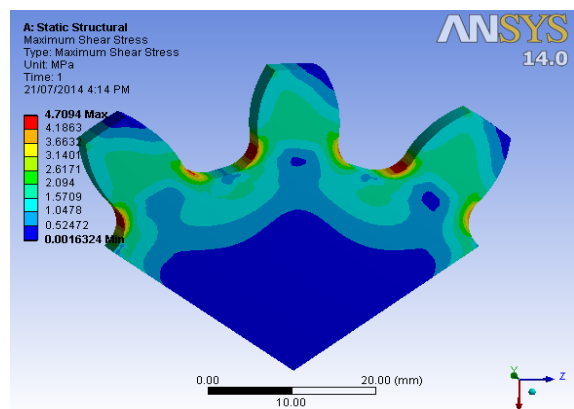
Table 4: Experimental Results: - Observation Table for Determination of Stress

Sr. No.	Load in Pan (N)	Lever Length (m)	Torque (N-m)	Fringe order(N)	Experimental Stress $\sigma_{ex} = NF\sigma/2h$ (N/mm ²)
1	25	0.26	6.50	7.41	7.55
2	35	0.26	9.10	10.57	10.77
3	45	0.26	11.70	13.96	14.23
4	50	0.26	13.00	15.24	15.53
5	60	0.26	15.60	17.95	18.29

FINITE ELEMENT MODEL HAVING SAME SPECIFICATION AS THAT OF PHOTO ELASTIC MODEL (PREPARED FOR COMPARISONS OF RESULTS)

Maximum stress values obtained for various loading conditions for photo elastic model using Finite element analysis. With the help of images obtained from FEA and photo elasticity experiment stress pattern is similar and maximum stress concentration occurs at root of the spline as shown in Figure 9.

Figure 9: FE Model (Photo Elastic) of Spline Shaft

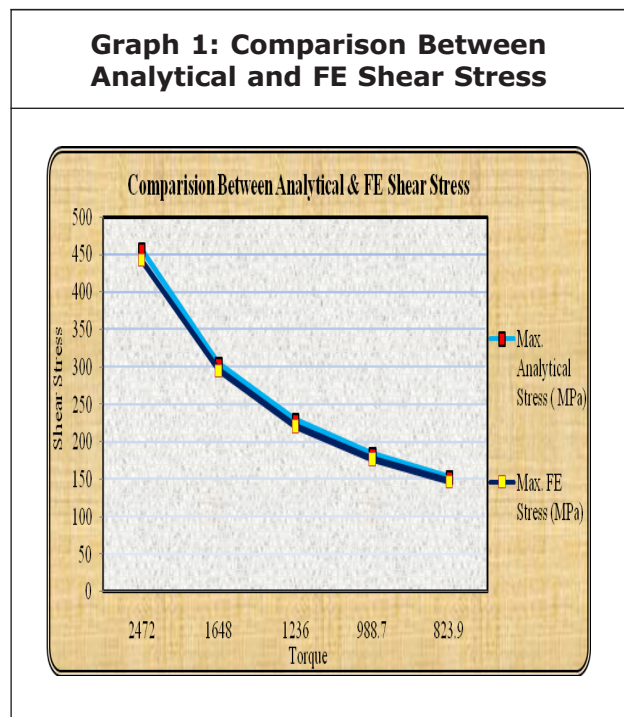


RESULTS AND DISCUSSION

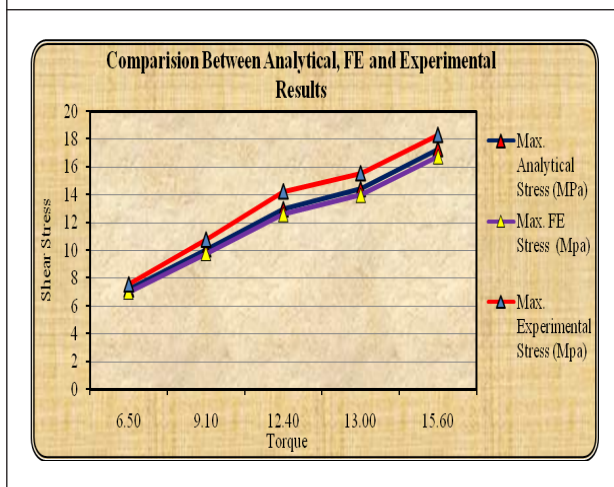
The results of stress analysis obtained experimentally by using photo elastic analysis and numerically by using Finite Element Analysis, (ANSYS package), Version 12.0, are investigated. In the numerical F.E.A models have been analyzed with different operating condition of torque. Photo elastic study was performed to check the numerical analysis by determined values of shear stress.

The Shear stress obtained from Analytical calculation is compared with the results obtained from FEA for actual model of spline shaft as shown in Table 5. It is observed that the results are in close agreement with each other with a small percentage of error as depicted in Graph 1. The shear stress is found maximum at root of the spline.

Sr. No.	Torque (N-m)	Max. Analytical Stress (MPa)	Max. FE Stress (MPa)	% Error
1	2472	456.90	442.51	3.14
2	1690	312.12	305.12	2.24
3	1648	304.31	295.01	3.05
4	1236	228.41	221.25	3.13
5	988.7	182.70	176.86	3.19
6	823.90	152.35	147.32	3.30



Sr. No.	Load (N)	Torque T (N-m)	Max. Shear Stress (Analytical) (MPa)	Max. Shear Stress (FEA) (MPa)	Max. Shear Stress (Experimental) (Mpa)	% Error between A and B	% Error between A and C
			A	B	C		
1	25	6.500	7.19	6.98	7.55	2.92	4.76
2	35	9.100	10.07	9.77	10.77	2.97	6.49
3	45	12.40	12.94	12.57	14.23	2.85	9.06
4	50	13.00	14.38	13.97	15.53	2.85	7.40
5	60	15.60	17.26	16.76	18.29	2.89	5.63

Graph 2: Comparison Between Analytical, FE and Experimental Shear Stress

Analysis of photo elastic model the shear stress obtained from Analytical calculation is compared with the results obtained from FEA and experimental as shown in Table 6. It is observed that the results are in close agreement with each other with a small percentage of error as depicted in Graph 2. The shear stress is found maximum at root of the spline.

CONCLUSION

The stress analysis of spline shaft of TATA G750 (LPK2518TC) is carried out theoretically for various loading conditions and these results are verified by FE analysis and experimental analysis. From above discussion it is observed that maximum shear stress is found at the root section of spline. From the FE analysis of spline shaft it can be concluded that the shear stress is maximum at the root section of spline near the rigid end as compare to free end. The maximum shear stress found to be increasing from free end to rigid end. This is due to stiffening effect of rigid end.

FUTURE SCOPE

- Future studied of this type includes the use of gap element to model the contact stresses developed between mating spline teeth.
- The Stress analysis by varying fillet radius at root of spline tooth would give additional insight into ways that the stress distribution within the splined shafts can be influenced.

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