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Research Paper

MODELLING OF SURFACE ROUGHNESS INDEX FOR A ROLLER BURNISHING PROCESS USING GRAPH THEORY AND MATRIX METHOD

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In the present paper, Graph Theoretic Approach is used for the analysis and comparison of roller burnishing process in terms of surface roughness. The surface roughness is affected by different parameters. In the present work, the important parameters are identified and an index of surface roughness is proposed with the help of Graph Theoretic Approach which evaluates the inhibiting power of these parameters. This index can be used in comparison of different roller burnishing processes.

Keywords: Roller burnishing process, Parameters, Intensity, Graph theoretic approach

INTRODUCTION

Roller Burnishing (RB) is one of the no-chip finishing processes for surface engineering. Roller Burnishing is a cold working, surface treatment process in which plastic deformation of surface irregularities occurs by exerting pressure through a very hard and smooth roller on a surface to generate a uniform and work-hardened surface. The burnishing process is an attractive finishing technique which can increase the work piece surface strength as well as decreasing its surface roughness (Figure 1).

A number of approaches are available in the literature for the analysis of roller burnishing process. The various methods used so far are as following:

- Taguchi method (Philip Selvaraj and Chanramohan, 2010; and Ugur Esme, 2010).
- Surface Roughness Methodology (Dabeer and Purohit, 2010).
- Neural network (Stankovic *et al*., 2012).
- Slip-line field model (Black *et al*., 1997).

We conclude that Graph Theory has not been used for the analysis of Roller Burnishing

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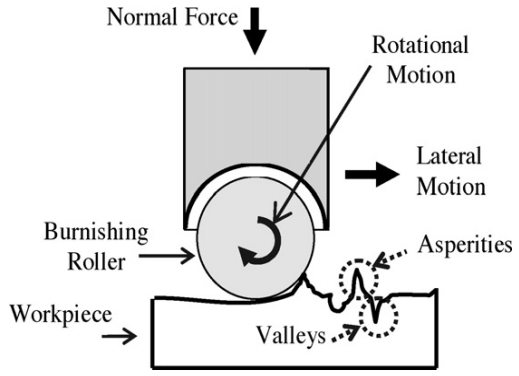
Figure 1: Schematic Illustration of Burnishing Mechanism



Source: El-Tayeb et al. (2007)

process. So, we are going to apply Graph theory and Matrix Approach for the analysis of Roller burnishing process.

Graph theory is a systematic and logical approach. Graph model representation has proved to be useful for modelling and analysing various kinds of systems and problems in numerous fields of science and technology .The matrix approach is useful in analysing the graph models expeditiously to derive the system function and index to meet the objectives. Moreover, representation of the graph by a matrix offers ease in computer handling. In view of these, graph theory and matrix methods are proposed in this paper for quantification of parameters.

To the best of our knowledge, quantification of these parameters was not reported in the literature.In this paper, an attempt is made to analyse and quantify these parameters using digraph and matrix approach. The main objectives of this paper are as follows:

- To identify and segregate parameters in to categories through literature study and mathematical modeling.
- To develop a mathematical model of these performance parameters using graph theoretic approach (GTA).
- To propose a single numerical index representing the value of the multinomial of the model.

IDENTIFICATION OF PERTINENT ATTRIBUTES AND THE FACTORS

- Feed Direction: Feed direction means the direction in which the tool approaches the workpiece. There are two types of feed direction: 1) along, 2) counter.

  1. Along: It implies that the tool is moving in the direction of workpiece.
  2. Counter: It implies that the tool and workpiece are moving in the opposite direction.

  By the study of literature, it was found that surface roughness will improve more in case of counter feed direction.

- Workpiece materials: By the study of literature, we found that the Roller Burnishing process has been applied to different materials. RB has a positive effect on the surface roughness of O1 alloy steel. The improvement percentage on the surface quality was 12.5% (Dabeer and Purohit, 2010).

- Spindle Speed: The spindle speed is the rotational frequency of the spindle of the machine, measured in revolutions per minute (RPM). The preferred speed is determined by working backward from the desired surface speed (m/min) and incorporating the diameter (of workpiece or tool). Excessive spindle speed will cause

premature tool wear, breakages, and can cause tool chatter, all of which can lead to potentially dangerous conditions. Using the correct spindle speed for the material and tools will greatly enhance tool life and the quality of the surface finish. In general, an increase in burnishing speedup to about 100 m/min leads to a decrease in mean roughness. With a further increase in burnishing speed mean roughness gradually increases. The deterioration of the surface roughness in the burnishing process at high burnishing speeds is believed to be caused by the chatter that results in instability of the burnishing tool across the workpiece surface. In addition, the low surface roughness can be interpreted by the low deforming action of the roller sat high speeds and also because the lubricant loses its effect due to the insufficient time for it to penetrate between the roller and the workpiece surfaces. It is better then to select low speeds because the deforming action of the burnishing tool is greater and metal flow is regular at low speed (El-Khabeery and El-Axir, 2001).

- Feed Rate: Feed rate is the velocity at which the tool is fed, that is, advanced against the workpiece. It is expressed in units of distance per revolution for burnishing process. Feed Rate is dependent on the:
  - Type of tool.
  - Surface finish desired.
  - Power available at the spindle.
  - Rigidity of the machine and tooling setup.
  - Strength of the work piece.

  The surface roughness is increased with increasing the feed rate up to certain point (0.06 mm/rev) then it begins to decrease, it is also observed that higher reduction in surface roughness happened at very low burnishing feed rate (Stankovic *et al*., 2012).
- No of passes: It was found that the number of burnishing tool passes is an important parameter for the surface roughness and hardness of burnished components (Hassan, 1997; and Hassan *et al.*, 1998; and Luo *et al.*, 2005). The surface roughness can be increased with increasing the burnishing force or by number of passes to certain limit. The surfaces starts to deteriorate, as the surface of the metal is over work hardened due to the plastic deformation caused by the burnishing force or the number of tool passes exceeding the limits.
- Coolant: A coolant is a fluid which flows through a device to prevent its overheating, transferring the heat produced by the device to other devices that use or dissipate it. An ideal coolant has high thermal capacity, low viscosity, is low-cost, non-toxic, and chemically inert, neither causing nor promoting corrosion of the cooling system.

GRAPH THEORETIC APPROACH (GTA)

GTA is a systematic and logical approach that is applied in various disciplines (Gandhi and Agarwal, 1992; Gandhi and Agarwal, 1996; and Rao and Padmanabhan, 2006). The conventional representations like block diagrams, cause and effect diagrams and flowcharts do not depict interactions among factors and are not suitable for further analysis and cannot be processed or expressed in

mathematical form. The following features highlight the uniqueness of this approach over the other similar approaches:

- It a single numerical index for all the parameters.
- It is a systematic methodology for conversion of qualitative factors to quantitative values, and mathematic modelling gives an edge to the proposed technique over conventional methods.
- It permits presents modelling of interdependence of parameters under consideration.
- It allows visual analysis and computer processing.
- It leads to self-analysis and comparison of different organisations.

Main Elements of GTA

- The digraph representation.
- The matrix representation.
- The permanent function representation.

The Digraph Representation of Roller Burnishing Parameters

A directed graph (or a digraph) is nothing but a graph with directed edges. A digraph is used to represent the elements (Roughness parameters) and their interdependencies in terms of nodes and edges (Figures 2 and 3). In an undirected graph, no direction is assigned to the edges in the graph, whereas directed graphs or digraphs have directional edges. A Roller Burnishing parameter diagraph is prepared to represent the parameters of Roller Burnishing in terms of nodes and edges. It represents parameters (B_i's) through its nodes and dependence of parameters (b_{ij}'s) through its edges. P_i indicates the inheritance of parameters and pij indicates the degree of dependence of j^{th} parameter on i^{th} parameter. In the present work six parameters such as feed direction (B_1), workpiece material (B_2), speed (B_3), feed (B_4), number of passes (B_5), coolant (B_6) are identified.

Figure 2: Schematic Arrangement of Roller Burnishing Surface Roughness Parameters

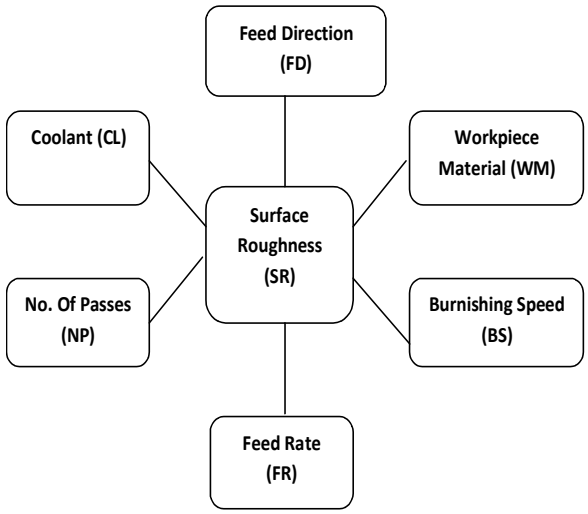
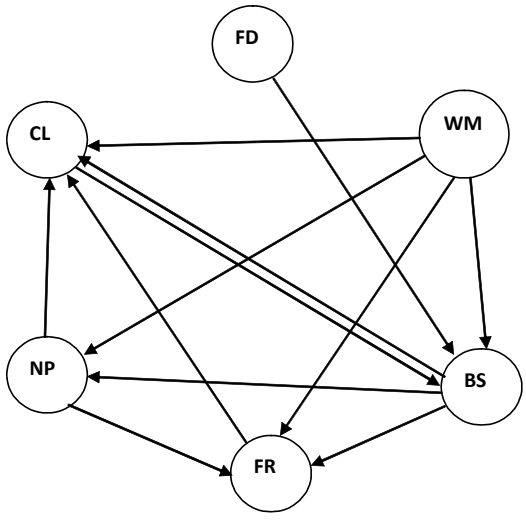


Figure 3: Roller Burnishing Surface Roughness Parameter Digraph



The System Structure Matrix Representation

A digraph is a visual representation so it helps in analyse to a limited extent only. To establish the expression for Roller Burnishing Surface Roughness (RBSR) parameters, the digraph is represented in matrix form which is also a convenient form of computer processing. Consider a digraph of n parameters leading to nth order symmetric (0, 1) matrix $A = [b_{ij}]$. The rows and columns in the matrix represent interactions among parameters, i.e., b_{ij} represents the interaction of i-th parameter with the jth parameter. The matrix (6×6) showing is represented as:

$$A = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL & \text{Parameters} \end{matrix} \\ \begin{bmatrix} B_1 & b_{12} & b_{13} & b_{14} & b_{15} & b_{15} \\ b_{21} & B_2 & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & B_3 & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & B_4 & b_{45} & b_{46} \\ b_{51} & b_{52} & b_{53} & b_{54} & B_5 & b_{56} \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & B_6 \end{bmatrix} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix} \end{array} \quad ...(1)$$

Generally $b_{ij} \neq b_{ji}$ as RBSR parameters are directional and $b_{ii} = 0$, as a parameter is not interacting with itself. This RBSR parameter matrix is square and nonsymmetrical and is analogous to adjacency matrix in graph theory. On putting the values of interactions like:

b_{ij} = 1; if parameter *i* is connected to parameter *j*

= 0 otherwise

The RBSR parameter matrix representing the graph shown in Figure 3 is written as:

$$A = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix} \end{array}$$

The interdependency of RBSR parameters is shown by off-diagonal elements with 0 or 1. The diagonal elements are 0 since the effect of roughness parameters is not taken in to account. To consider this, another matrix known as characteristic matrix is defined.

Characteristic Matrix of RBSR Parameter (CM_{RB})

The characteristic matrix is used to characterize parameters of RBSR. Consider I as an identity matrix and b as the variable representing RBSR parameters. The RBSR parameter (CM_{RB}) characteristic matrix is written as

$$C = [BI - A]$$

$$C = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL \end{matrix} \\ \begin{bmatrix} B_1 & 0 & b_{13} & 0 & 0 & 0 \\ b_{21} & B_2 & b_{23} & b_{24} & b_{25} & b_{26} \\ 0 & 0 & B_3 & b_{34} & b_{35} & b_{36} \\ 0 & 0 & 0 & B_4 & b_{45} & b_{46} \\ 0 & 0 & 0 & b_{54} & B_5 & b_{56} \\ 0 & 0 & b_{63} & 0 & 0 & B_6 \end{bmatrix} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix} \end{array} \quad ...(3)$$

In the matrix, the value of all diagonal elements is the same, i.e., all surface roughness parameters were assigned the same values depending on various parameters affecting them. Moreover

interdependencies were assigned values of 0 and 1 depending on whether it is there or not. To consider this, another matrix, the variable characteristic matrix of RB surface parameters is considered.

Variable Characteristic Matrix of RB Parameter (VCM_{RB})

The variable characteristic matrix of RB surface roughness parameter (VCM_{RB}) takes in to consideration the effect of different parameters and their interactions. The digraph in figure 3 is considered for defining $VCM_{RB.}$ Consider a matrix D with off diagonal elements b_{ij} representing interactions between RBSR parameters, i.e., instead of 1 (as shown in matrix A).Consider another matrix E is taken with diagonal elements B_i, i = 1,2,3,4,5,6 where the B_i represents the effect of various parameters, i.e. instead of B only (as shown in matrix C). Considering matrices D and E, VCM_{RB} is expressed as:

$$H = [E - D]$$

$$D = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL \end{matrix} \\ \begin{bmatrix} B_1 & 0 & -b_{13} & 0 & 0 & 0 \\ -b_{21} & B_2 & -b_{23} & -b_{24} & -b_{25} & -b_{26} \\ 0 & 0 & B_3 & -b_{34} & b_{35} & -b_{36} \\ 0 & 0 & 0 & B_4 & b_{45} & -b_{46} \\ 0 & 0 & 0 & -b_{54} & B_5 & -b_{56} \\ 0 & 0 & -b_{63} & 0 & 0 & B_6 \end{bmatrix} \end{array} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix}$$

This matrix provides a powerful tool through its determinant, called the variable characteristic RBSR parameter multinomial.

The determinant of the matrix H contains positive and negative sign with some of its coefficients. Hence, complete information in the RB environment will not be obtained as some will be lost due to addition and subtraction of numerical values of diagonal and off-diagonal elements (i.e., B_i's and b_{ij}'s). Thus the determinant of the variable characteristic matrix, i.e., the matrix H, does not provide complete information concerning the RBSR system. For this, another matrix known as Variable Permanent Matrix of RBSR parameter (VPM_{RB}) is introduced.

Variable Permanent Matrix of RBSR Parameter (VPM_{RB})

Since the total quantitative value is not obtained in $VCM_{RB,}$ VPM_{RB} is defined for the organization in general (assuming interactions among all parameters) as:

$$P = [E + D]$$

$$P = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL \end{matrix} \\ \begin{bmatrix} B_1 & b_{12} & b_{13} & b_{14} & b_{15} & b_{15} \\ b_{21} & B_2 & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & B_3 & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & B_4 & b_{45} & b_{46} \\ b_{51} & b_{52} & b_{53} & b_{54} & B_5 & b_{56} \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & B_6 \end{bmatrix} \end{array} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix}$$

...(5)

where, E and D have the meaning as in matrix H.

Thus the VPM_{RB} corresponding to the six surface roughness parameters RB digraph (Figure 3) is given by:

$$H = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL \end{matrix} \\ \begin{bmatrix} B_1 & 0 & b_{13} & 0 & 0 & 0 \\ b_{21} & B_2 & b_{23} & b_{24} & b_{25} & b_{26} \\ 0 & 0 & B_3 & b_{34} & b_{35} & b_{36} \\ 0 & 0 & 0 & B_4 & b_{45} & b_{46} \\ 0 & 0 & 0 & b_{54} & B_5 & b_{56} \\ 0 & 0 & b_{63} & 0 & 0 & B_6 \end{bmatrix} \end{array} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix}$$

...(6)

The diagonal elements B_1, B_2, B_3, B_4, B_5 and B_6 represent the effect of the six critical surface roughness parameters on Roller Burnishing and off-diagonal elements represent interdependencies of each element in the matrix. The contribution can be expressed quantitatively.

Permanent Representation

Both digraph and matrix representations are not unique as they are changed by changing the labelling of nodes. To develop a unique representation, independent of labelling, a permanent function of the matrix VPM_{RB} is proposed. The permanent function is obtained in a similar manner as its determinant. This computational process results in a multinomial whose every term has a physical significance. This multinomial representation includes all the information regarding surface roughness parameters and is called Variable Permanent Function of RBSR parameters (VPF_{RB}), also known as Pm (per Pm) and in general form, it is written as:

$$VPF_{RB} = perB = +\prod_{1}^{6} B_i + \sum_{1}\sum_{2}\sum_{3}\sum_{4}\sum_{5}\sum_{6}(b_{ij}b_{ji})B_kB_lB_mB_n$$

$$+\sum_{1}\sum_{2}\sum_{3}\sum_{4}\sum_{5}\sum_{6}(b_{ij}b_{jk}b_{ki} + b_{ik}b_{kj}b_{ji})B_lB_mB_n$$

$$+\left[\sum_{1}\sum_{2}\sum_{3}\sum_{4}\sum_{5}\sum_{6}(b_{ij}b_{ji})(b_{kl}b_{lk})B_mB_n\right.$$

$$\left.+\sum_{1}\sum_{2}\sum_{3}\sum_{4}\sum_{5}\sum_{6}(b_{ij}b_{jk}b_{kl}b_{li} + b_{il}b_{lk}b_{kj}b_{ji})B_mB_n\right]$$

$$+\left[\sum_{1}\sum_{2}\sum_{3}\sum_{4}\sum_{5}\sum_{6}(b_{ij}b_{jk}b_{ki} + b_{ik}b_{kj}b_{ji})(b_{lm}b_{ml})B_n\right.$$

$$\left.+\sum_{1}\sum_{2}\sum_{3}\sum_{4}\sum_{5}\sum_{6}(b_{ij}b_{jk}b_{kl}b_{li} + b_{il}b_{lk}b_{kj}b_{ji})(b_{mn}b_{nm})\right.$$

$$\left.+\sum_{1}\sum_{2}\sum_{3}\sum_{4}\sum_{5}\sum_{6}(b_{ij}b_{jk}b_{ki} + b_{ik}b_{kj}b_{ji})(b_{lm}b_{mn}b_{nl} + b_{ln}b_{nm}b_{ml})\right] \quad ...(7)$$

The VPF_{RB} is a mathematical expression in symbolic form and it ensures an estimate of the RBSR parameters existing in an organization. It is a complete expression for RBSR parameters as it considers the presence of all parameters and their interdependencies.

Equation (6) contains 6! Terms and these terms are arranged in n + 1 grouping, where n is the number of elements (parameters). The value of n is equal to six in the present case. The physical significance of various groupings is explained as follows:

- The first grouping contains only one term. It represents a set of N unconnected RBSR parameters, i.e., B_1, B_2, ..., B_6.
- The second grouping is absent as there is no self loop in the digraph (Figure 3).
- The third grouping contains a set of two RBSR parameter interdependence, i.e., b_{ij} b_{ji} and measure of remaining (n – 2) RBSR parameters.
- Each term of fourth grouping represents a set of three RBSR parameter interdependence, i.e., b_{ij} b_{jk} b_{ki} or its pair b_{ik} b_{kj} b_{ji} and measure of remaining (n – 3) RBSR parameters.
- The terms of fifth grouping are arranged in two subgroups. The first subgrouping is a set of two RBSR parameter interdependence i.e., b_{ij} b_{jk} b_{kl} b_{li} or its pair

$b_{il}\, b_{lk}\, b_{kj}\, b_{ji}$ and measure of remaining (n – 4) RBSR parameters.

- The terms of sixth grouping are also arranged in two subgroups. The first subgrouping is a set of two RBSR parameter interdependence, i.e., $b_{ij}\, b_{jk}\, b_{kl}\, b_{lm}\, b_{mn}\, b_{ni}$ or its pair $b_{im}\, b_{ml}\, b_{lk}\, b_{kj}\, b_{ji}$.

Quantification of B_i's and b_{ij}'s

Quantification of RBSR parameters (i.e., B_i) is carried out on the basis of Equation (5). To get the complete value of multinomial (Equation 6) the diagonal as well as off-diagonal elements in VPM_{RB} are to be assigned some numerical values. As already discussed, the diagonal elements represent different parameters and the off-diagonal elements represent interdependencies among surface roughness parameters. As the influence of all parameters may not be equal and the dependence among parameters measured directly, hence these values are assigned only after proper interpretation through a team of experts. It is suggested to use Tables 1 and 2 for assigning these values.

Table 1: Quantification of RBSR Parameters

S. No.	Qualitative Measure of Parameter	Assigned Value of Parameter
1.	Exceptionally low	1
2.	Very low	2
3.	Low	3
4.	Below average	4
5.	Average	5
6.	Above average	6
7.	High	7
8.	Very high	8
9.	Exceptionally high	9

Table 2: Quantification of RBSR Parameter Interdependencies

S. No.	Qualitative Measure of Parameter	p_{ij}
1.	Very strong	5
2.	Strong	4
3.	Medium	3
4.	Weak	2
5.	Very weak	1

METHODOLOGY

The Graph theoretic approach evaluates the permanent qualitative index of Roller burnishing in terms of single numerical index, which takes into account the individual effect of various surface roughness parameters and their interdependencies while analysing and evaluating the RBSR. The various steps of the proposed approach, which would be helpful in evaluation of the RBSR, are enlisted in sequential manner as below;

- Identify the various parameters of the RB affecting the Surface Roughness of the workpiece.
- Develop a digraph among the RBSR parameters based on interactions among them.
- Develop a performance parameter matrix on the basis of digraph developed in step2.

  This will be of size M × M, with diagonal elements representing surface roughness parameters and the off-diagonal elements representing interactions among them.
- Develop a variable permanent matrix (VPM_{RB}).
- Find the value of permanent function for the surface roughness parameters of RB.

- Compare different roller burnishing process in terms of surface roughness parameter permanent function obtained in step 5.
- Document the results for future analysis/ reference.

Based on the methodology discussed above, the organization can evaluate the extent of parameters present in roller burnishing.

WORKING EXAMPLE

For the demonstration of proposed methodology, an organization is taken as an example. It is proposed to find the value of RB permanent function of surface roughness parameters. For this purpose, some numerical values of all surface roughness parameters and their interdependencies are required. The methodology discussed in the previous section is used to evaluate the permanent function of parameters of RBSR in this example.

The various parameters affecting the Surface roughness are identified and presented and explained in the previous section. The dependencies of performance parameters are visualized in digraph in Figure 3.

The variable permanent matrix for digraph is written as

$$H = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL & Parameters \end{matrix} \\ \begin{bmatrix} B_1 & 0 & b_{13} & 0 & 0 & 0 \\ b_{21} & B_2 & b_{23} & b_{24} & b_{25} & b_{26} \\ 0 & 0 & B_3 & b_{34} & b_{35} & b_{36} \\ 0 & 0 & 0 & B_4 & b_{45} & b_{46} \\ 0 & 0 & 0 & b_{54} & B_5 & b_{56} \\ 0 & 0 & b_{63} & 0 & 0 & B_6 \end{bmatrix} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix} \end{array} \quad ...(8)$$

By taking the numerical values of six Surface roughness parameters from Table 1 as: $B_1 = 5$, $B_2 = 5$, $B_3 = 8$, $B_4 = 6$, $B_5 = 4$, $B_6 = 7$.

For interdependencies, the values taken from Table 2 are: $b_{13} = 4$, $b_{21} = 2$, $b_{23} = 3$, $b_{24} = 4$, $b_{25} = 1$, $b_{26} = 2$, $b_{34} = 5$, $b_{35} = 3$, $b_{36} = 4$, $b_{46.} = 3$, $b_{54} = 2$, $b_{56} = 2$, $b_{63} = 1$.

Substituting these values in above matrix:

$$VPM_{RB} = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL & Parameters \end{matrix} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix} \end{array} \quad ...(9)$$

The value of permanent function as calculated by c++ Software is

Variable Permanent Function, VPF = 38850

$$\text{Max } VPM_{RB} = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL \end{matrix} \\ \begin{bmatrix} 9 & 0 & 4 & 0 & 0 & 0 \\ 2 & 9 & 3 & 4 & 1 & 2 \\ 0 & 0 & 9 & 5 & 3 & 4 \\ 0 & 0 & 0 & 9 & 0 & 3 \\ 0 & 0 & 0 & 2 & 9 & 2 \\ 0 & 0 & 1 & 0 & 0 & 9 \end{bmatrix} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix} \end{array} \quad ...(10)$$

The maximum value of variable permanent function is

Max $VPF_{RB} = 574452$

$$\text{Min } VPM_{RB} = \begin{array}{c} \begin{matrix} FD & WM & BS & FR & NP & CL \end{matrix} \\ \begin{bmatrix} 1 & 0 & 4 & 0 & 0 & 0 \\ 2 & 1 & 3 & 4 & 1 & 2 \\ 0 & 0 & 1 & 5 & 3 & 4 \\ 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{matrix} \end{array} \quad ...(11)$$

The minimum value of variable permanent function is, Min VPF_{RB} = 44

With the above method, the effect of each individual surface roughness parameter can be seen by putting the maximum (9) and minimum (1) values of each individual parameter.

For Feed Direction, the maximum and minimum values of permanent function are calculated as;

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 9 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \end{array} \quad ...(12)$$

The value of maximum permanent function for FD is, Max VPF_1 = 69930

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 1 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \end{array} \quad ...(13)$$

The value of minimum permanent function for FD is, Min VPF_1 = 7770

For WM, the maximum and minimum values of permanent function are calculated as;

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 9 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \end{array} \quad ...(14)$$

The value of maximum permanent function for WM is, Max VPF_2 = 69930

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 9 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \end{array} \quad ...(16)$$

The maximum value of variable permanent function for BS is, Max VPF_3 = 43050

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 1 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \end{array} \quad ...(17)$$

The minimum value of variable permanent function for BS is, Min VPF_3 = 9450

For FR, the maximum and minimum values of permanent function are calculated as;

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 9 & 0 & 3 \\ 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \end{array} \quad ...(18)$$

The maximum value of variable permanent function for FR is, Max VPF_4 = 57300

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \end{array} \quad ...(19)$$

The minimum value of variable permanent function for FR is, Min $VPF_4 = 8100$

For NP, the maximum and minimum values of permanent function are calculated as;

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 2 & 9 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \end{array} \quad ...(20)$$

The maximum value of variable permanent function for NP is, Max $VPF_5 = 85725$

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 7 \end{bmatrix} \end{array} \quad ...(21)$$

The minimum value of variable permanent function for NP is, Min $VPF_5 = 10725$

For CL, the maximum and minimum values of permanent function are calculated as;

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 9 \end{bmatrix} \end{array} \quad ...(22)$$

The maximum value of variable permanent function for CL is, Max $VPF_6 = 38850$

Table 3: Variable Permanent Function (Max and Min) of Surface Roughness Parameter

S. No.	Parameter	Max. VPF	Min. VPF
1.	FD	69930	7770
2.	WM	69930	7770
3.	BS	43050	9450
4.	FR	57300	8100
5.	NP	85725	10725
6.	CL	38850	10050

$$VPM_{RB} = \begin{array}{c} \\ FD \\ WM \\ BS \\ FR \\ NP \\ CL \end{array} \begin{array}{c} \begin{array}{cccccc} FD & WM & BS & FR & NP & CL \end{array} \\ \begin{bmatrix} 5 & 0 & 4 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 \\ 0 & 0 & 8 & 5 & 3 & 4 \\ 0 & 0 & 0 & 6 & 0 & 3 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad ...(23)$$

The minimum value of variable permanent function for CL is, Min VPF_6 = 10050

RESULTS AND DISCUSSION

- Graph Theory and Matrix Method is a multi-attribute decision making method (MADM) which is successfully applied on the roller burnishing process.
- By the application of Graph Theory and Matrix Method, we have identified the critical parameter affecting the surface roughness of roller burnishing process. In the present work, number of passes is the most critical parameter having value of maximum variable permanent function = 85725.
- Graph Theory and Matrix Method is used in the selection of an improved process among various roller burnishing processes by the comparison of variable permanent functions of the different processes.
- The Graph Theory and Matrix Method is used in the arrangement of the parameters according to their relative importance in the process. The sequence of parameters in our work is number of passes, feed direction, workpiece material, feed rate, coolant, burnishing speed.
- If some improvement is done in the parameters affecting the surface roughness of the process, then we can compare the improved process with the previous process.

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