Modeling and Designing Hierarchical Sliding Mode Controller for a 4-DOF Solar Autonomous Underwater Vehicles

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***Abstract*—Autonomous Underwater Vehicles (AUV) are automatic equipment that can move in 6 degrees of freedom according to the motion in the water. Modeling accurately AUV is very difficult because of the influence of factors such as hydrodynamic forces, time error, and environmental noise, etc. It is important that the controller designing needs to meet the requirements of stability and suitability to specific diving equipment models. The hydrodynamic equations are established with the assumed conditions. Controlling self-propelled diving equipment is a major challenge for researchers because of the complex, and nonlinear correlation between diving and operating environments. Therefore, high-quality control systems for the AUV should exhibit the ability to update the variability of the device's hydrodynamic coefficients to achieve the desired control quality. In this study, the authors focus on building a Hierarchical Sliding Mode Controller (HSMC) for Solar Autonomous Underwater Vehicles (S-AUV), the kinematics and dynamics of the underactuated attitude control adjusting system are analyzed. More precisely, the controller is designed based on the hydrodynamic model of the S-AUV. By employing the propulsion speed, the position of the steering blades as design variables, the dive of the S-AUV is stably controlled in location, velocity, and depth. For a given set of operating parameters, the simulation result shows that the developed controller exhibits errors within the allowed range of values.**

***Keywords*—solar autonomous underwater vehicles, underactuated system, hierarchical sliding mode controller, control AUV, S-AUV**

I. INTRODUCTION

Autonomous Underwater Vehicles are being applied in many industries such as oceanography, fishery, environmental monitoring, security, defense, operation in dangerous waters, photometric surveys, pipeline surveys, mapping profound, rescue, and salve, etc [1–3]. With the ability to move automatically in water without a driver, AUV is suitable for exploring deep water and/or a long-term mission. Determining the variables that affect the AUV during an operation in water is commonly difficult, it is more complicated to correlate theoretical and experimental configurations because of various complex phenomena, for instance, turbulent flow around the AUV. Several experimental studies on AUV at the sea have been done by mounting laboratory equipment, for instance in [4, 5], however, the obtained information is not much. Consequently, the process of the modeling, simulation, analysis before making any experiments is necessary to take into account the effects of the environment, structure, shape of designing on the operability of the AUV.

One of the major challenges is the controller needs to self-adjust to match the change during operation. To this aim, researchers and scientists often use linear controller methods rather than nonlinear ones because of their ease of application. However, in a linear controller mode, it is often to omit or to simplify the parameters which make the model exhibiting certain errors. The AUV's hydraulic features are quite complicated because of their high nonlinearity, for instance, the hydraulic forces aretime-dependent, unstable and noise due to the flow and the wave. Recently, many techniques have been proposed for developing the controller models. The linear controllers introduced in [6–9] worked well together with different assumed and omitted parameters. Simple proportional integral derivative controller is mainly applied to the linear system, however there are also some extensions of such a controller for the nonlinear systems [10]. Other types of controller can be found in the literature such as the sliding mode controllers (SMC) [11, 12], adaptive controllers [13, 14], fuzzy logic controllers [15], model predictive controller [16], and controllers based on neural networks [17, 18]. On the other hand, object oriented modelling method has been introduced in [19]. The underactuated control laws can be used for the underwater vehicles [20].

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Advantages and disadvantages of the control methods can be listed as follows. One of the most discussed weakness points of linear controllers is linear quadratic regulator, which provides a poor and unstable performance because it does not consider the non-linearity. Besides, the neural network controller also has some weaknesses such as unsatisfactory convergence to a correct model, long time for learning process, high configuration requirement, so there are not many AUV systems have been built by this controller. On the other hand, the HSMC control can be considered as a good solution for nonlinear systems. However, there are several inconveniences such as it produces high frequency oscillation that affects the actuator, wastes energy and generating fluctuation on the steering wheel leads to instability of the vehicle. Fuzzy logic controller is easy to apply in industrial process because of its simple structure. However, FLC with fixed ratio factors and fuzzy rules may not provide required performance in the case that an AUV has unstable and nonlinear parameters [21]. Recently, several more modern controllers have been introduced. Several advantages can be listed such as well-response controls, stability adaptation, etc. However, the mentioned above controllers require an AUV model composing of accurate and complete parameters.

In this paper, the authors focus on HSMC for a small S-AUV which is an underactuated system. The outstanding advantage of the HSMC control is the stability and steady of the whole system in the case of noise or the object's parameters that change over time. HSMC control have been studied. Thus, the goal of this study is to design an efficient and stable HSMC control for a S-AUV diving equipment working in water environment that has been built by the authors. The theory of SMC was first designed based on structural test variables in the 1960s. However, the first paper was published by Utkin in 1977 [22]. Since then, the HSMC approach has attracted many researchers to implement HSMC control in many practical applications such as mechatronics systems, automatic system, robots, ... Recently, despite instability system, noise environment, parameter variations, many applications have been successful thanks to HSMC control and sliding control theory developed in recent decades [23]. It is due to the superiority of HSMC control that is stable and robust. The combination of HSMC control with other control techniques also gives better results in both theoretical studies and experimental studies [24–26]. SMC control design method consists of steps: Design a stable sliding surface for the control system with the desired performance and design rule control to put the system in orbit, it is defined the time and maintaining that movement [11, 12]. Designing the sliding surface needs to address all restrictions and digital requirements. Therefore, it is necessary to make a plan that replies to all the requests. The limit of SMC is the chattering phenomenon and performing intermittently to switch functions which generate controlling oscillation signal. Another system which is also made, the oscillation system is used of digital control with sampling rate [27, 28]. Vibration (high frequency oscillation) may make system failure because it can affect the stability and degree of the mechanical system, wear of the moving mechanism and the power components. Some methods are used to reduce vibration are hierarchical sliding mode controller.

II. DYNAMICS MODEL OF S-AUV

A. Coordinates of S-AUV

The dynamic model of the S-AUV is built on the basis of mechanical theory, the principles of kinetics and statics. Hydrodynamic models of the S-AUV are used to design control systems for the S-AUV that meet specific objectives such as motion trajectory control, dive depth control, direction control, ... In general, the movement of S-AUV can be represented by equations of motion with six degrees of freedom [29, 30]. Parameters such as the direction of motion, force and torque, speed and position for the S-AUV are shown in Table I and Fig. 1.

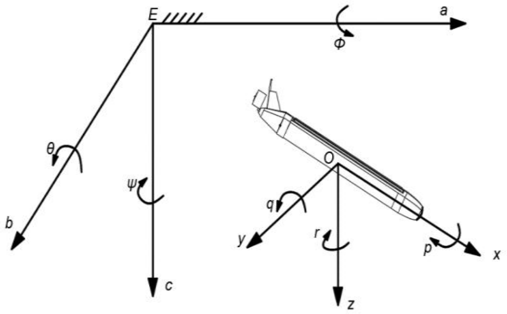


Figure 1. The dynamic coordinate system xyz and fixed coordinate system abc.

TABLE I. PARAMETER SYMBOLS ARE REPRESENTED IN DYNAMIC AND FIXED COORDINATE SYSTEMS.

Motion	Forces and moments	Velocity	Position and Euler angles
Surge	X	u	x
Sway	Y	v	y
Heave	Z	w	z
Roll	K	p	ϕ
Pitch	M	q	θ
Yaw	N	r	ψ

Velocity vector v, reference vector η can be represented as follows:

$$\begin{cases} \eta = [\eta_1^T, \eta_2^T]^T \in R^6 \\ \nu = [\nu_1^T, \nu_2^T]^T \in R^6 \end{cases} \quad (1)$$

where: $\begin{cases} \eta_1 = [x, y, z]^T \in R^3 \\ \eta_2 = [\phi, \theta, \psi]^T \in R^3 \end{cases}$ and $\begin{cases} \nu_1 = [u, v, w]^T \in R^3 \\ \nu_2 = [p, q, r]^T \in R^3 \end{cases}$

The first-order derivative of the position vector is related to the velocity vector through the below transformation:

$$\begin{cases} \dot{\eta}_1 = J_1(\eta_2)\nu_1 \\ \dot{\eta}_2 = J_2(\eta_2)\nu_2 \end{cases} \quad (2)$$

Combining Eqs. (1) and (2) creates an equation which describes the position and direction of S-AUV:

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} J_1(\eta_2) & 0_{3x3} \\ 0_{3x3} & J_2(\eta_2) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Leftrightarrow \dot{\eta} = J(\eta)v \qquad (3)$$

With:

$$J_1(\eta_2) = \begin{bmatrix} c(\theta).c(\psi) & s(\phi).s(\theta).c(\psi) - c(\phi).s(\psi) & c(\phi).s(\theta).c(\psi) + s(\phi).s(\psi) \\ c(\theta).s(\psi) & s(\phi).s(\theta).s(\psi) + c(\phi).c(\psi) & c(\phi).s(\theta).s(\psi) - s(\phi).c(\psi) \\ -s(\theta) & s(\phi).c(\theta) & c(\phi).c(\theta) \end{bmatrix}$$

$$J_2(\eta_2) = \begin{bmatrix} 1 & s(\phi)\,t(\theta) & c(\phi)\tan(\theta) \\ 0 & c(\phi) & -s(\phi) \\ 0 & s(\phi)\sec(\theta) & c(\phi)\sec(\theta) \end{bmatrix}$$

$\sin = s; \quad \cos = c$

B. Model S-AUV with 4 Degrees of Freedom

The motion equation of S-AUV consists of 6 degrees of freedom which is expressed through the forces and moments as follows [30]:

$$\begin{aligned} & m\left[\dot{u} - vr + wq - x_g(q^2 + r^2) + y_g(pq - \dot{r}) + z_g(pr + \dot{q})\right] = X \\ & m\left[\dot{v} - wp + ur - y_g(r^2 + p^2) + z_g(pr - \dot{p}) + x_g(qp + \dot{r})\right] = Y \\ & m\left[\dot{w} - uq + vp - z_g(q^2 + p^2) + x_g(rp - \dot{q}) + y_g(rq + \dot{p})\right] = Z \qquad (4) \\ & I_{xx}\dot{p} + (I_{zz} - I_{yy})qr + m\left[y_g(\dot{w} - uq + vp) - z_g(\dot{v} - wp + ur)\right] = K \\ & I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + m\left[z_g(\dot{u} - vr + wq) - x_g(\dot{w} - uq + vp)\right] = M \\ & I_{zz}\dot{r} + (I_{yy} - I_{xx})qp + m\left[x_g(\dot{v} - wq + ur) - y_g(\dot{u} - vr + wq)\right] = N \end{aligned}$$

Depending on the specific applications that we choose the appropriate number of degrees of freedom, the less the number of degrees of freedom is, the less complicated the control will be. The device operates in water, the exact controlling of the positions and coordinates of all 6 degrees is very complicated. To simplify the small types of AUV we can remove 2 unnecessary degrees of freedom: angle θ (pitch) and angle ϕ (roll). Hence, the equations of movement of the S-AUV 4 degrees of freedom are expressed through the quantities. Coordinate position (x, y), direction of AUV ψ (yaw) and position on axis z (diving depth).

In this study, the author focuses on constructing the hierarchical sliding mode controller with the S-AUV's parameter model which is appropriately calculated and selected as shown in Fig. 2.

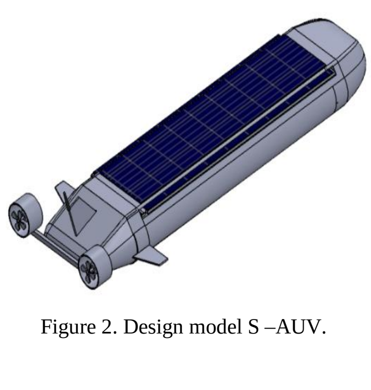


Figure 2. Design model S –AUV.

Four degrees of freedom movement model of S-AUV includes: $\eta = [x, y, z, \psi]^T$ are the position vector of the device in axes Ox, Oy, Oz and the directional angle of the S-AUV rotates around the axis Oz; $v = [u, v, w, r]^T$ is a vector of long velocity in the directions Ox, Oy, Oz and the rotational velocity around axis Oz. It is assumed that the fixed coordinate origin of S-AUV attached to the center gravity of S-AUV. Eq. (4) after removing 2 unnecessary degrees of freedom is written:

$$\begin{aligned} & m\left[\dot{u} - vr + wq - x_g(q^2 + r^2) + y_g(pq - \dot{r}) + z_g(pr + \dot{q})\right] = X \\ & m\left[\dot{v} - wp + ur - y_g(r^2 + p^2) + z_g(pr - \dot{p}) + x_g(qp + \dot{r})\right] = Y \\ & m\left[\dot{w} - uq + vp - z_g(q^2 + p^2) + x_g(rp - \dot{q}) + y_g(rq + \dot{p})\right] = Z \qquad (5) \\ & I_{zz}\dot{r} + (I_{yy} - I_{xx})qp + m\left[x_g(\dot{v} - wq + ur) - y_g(\dot{u} - vr + wq)\right] = N \end{aligned}$$

The nonlinear dynamic equations for the 4 °of freedom S-AUV is as follows:

$$\begin{cases} \dot{\eta} = J(\eta)v \\ M\dot{v} + C(v)v + D(v)v = \tau \end{cases} \qquad (6)$$

In which the matrix rotates around the axis Oz is presented as:

$$J(\eta) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 & 0 \\ \sin(\psi) & \cos(\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

System inertia matrix:

$$M = \begin{bmatrix} m + X_{\dot{u}} & 0 & X_{\dot{w}} & -my_g \\ 0 & m + Y_{\dot{v}} & 0 & Y_{\dot{r}} + mx_g \\ Z_{\dot{u}} & 0 & m + Z_{\dot{w}} & 0 \\ -my_g & mx_g + N_{\dot{v}} & 0 & I_z + N_{\dot{r}} \end{bmatrix}$$

Coriolis and system radial force matrix:

$$C = \begin{bmatrix} 0 & -mr & 0 & -mx_g r - a_2 \\ mr & 0 & 0 & -my_g r + a_1 \\ 0 & 0 & 0 & 0 \\ mx_g r + a_2 & my_g r - a_1 & 0 & 0 \end{bmatrix}$$

Hydrodynamic attenuation matrix:

$$D(v) = \begin{bmatrix} X_u + X_{u|u|}|u| & 0 & 0 & 0 \\ 0 & Y_v + Y_{v|v|}|v| & 0 & 0 \\ z_0|u| & 0 & Z_w + Z_{w|w|}|w| & 0 \\ 0 & 0 & 0 & K_p + K_{p|p|}|p| \end{bmatrix}$$

With $M, J(\eta), C(v), D(v)$ matrixes satisfy the following properties:

(1) $M = M^T > 0$

(2) $C(v) = C^T(v)$

(3) $D(v) > 0$

(4) $J(\eta)$ is the matrix that rotates around Oz and is the orthogonal matrix $J^{-1}(\eta) = J^T(\eta)$

III. Design of Control for S-AUV

A. Analysis Dynamic Model for Underactuated Systems S-AUV

In this case the S-AUV is an underactuated system, consisting of 2 input signals and 4 output signals. Therefore, we separate the mathematical model into two parts, including the underactuated and full actuated system. Position vector η will be separated into 2 parts $\eta = [\eta_1 \ \eta_2]^T$ and $\eta_1 = [x \ y]^T$ for a state of full actuated and $\eta_2 = [z \ \psi]^T$ for a state of underactuated. Similarly, velocity vector v is divided into two parts with $v = [v_1 \ v_2]^T$. The diving gear dynamic equation was rewritten as follows:

$$\begin{cases} \dot{\eta}_1 = J_{11}v_1 + J_{12}v_2 \\ \dot{\eta}_2 = J_{21}v_1 + J_{22}v_2 \\ M_{11}\dot{v}_1 + (C_{11} + D_{11})v_1 + M_{12}\dot{v}_2 + (C_{12} + D_{12})v_2 = \tau \\ M_{21}\dot{v}_1 + (C_{21} + D_{21})v_1 + M_{22}\dot{v}_2 + (C_{22} + D_{22})v_2 = 0 \end{cases} \quad (7)$$

with:

$$M_{11} = \begin{bmatrix} m + X_{\dot{u}} & 0 \\ 0 & m + Y_{\dot{v}} \end{bmatrix}; \ M_{12} = \begin{bmatrix} X_{\dot{w}} & -my_g \\ 0 & Y_{\dot{r}} + mx_g \end{bmatrix};$$

$$M_{21} = \begin{bmatrix} Z_{\dot{u}} & 0 \\ -my_g & mx_g + N_{\dot{v}} \end{bmatrix}; \ M_{22} = \begin{bmatrix} m + Z_{\dot{w}} & 0 \\ 0 & I_z + N_{\dot{r}} \end{bmatrix};$$

$$C_{11}(v) = \begin{bmatrix} 0 & -mr \\ mr & 0 \end{bmatrix}; \quad C_{12}(v) = \begin{bmatrix} 0 & -mx_g r - a_2 \\ 0 & -my_g r + a_1 \end{bmatrix};$$

$$C_{21}(v) = \begin{bmatrix} 0 & 0 \\ mx_g r + a_2 & my_g r - a_1 \end{bmatrix}; \ C_{22}(v) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};$$

$$J_{11}(v) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix}; \quad J_{12}(v) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};$$

$$J_{21}(v) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad J_{22}(v) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$D_{11}(v) = \begin{bmatrix} X_u + X_{u|u|}|u| & 0 \\ 0 & Y_v + Y_{v|v|}|v| \end{bmatrix};$$

$$D_{12}(v) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad D_{21}(v) = \begin{bmatrix} Z_0|u| & 0 \\ 0 & 0 \end{bmatrix};$$

$$D_{22}(v) = \begin{bmatrix} Z_{\mathrm{w}} + Z_{\mathrm{w|w|}}|\mathrm{w}| & 0 \\ 0 & K_p + K_{p|p|}|p| \end{bmatrix};$$

Since M_{22} is a positive determined matrix, so from the fourth equation in Eq. (7), we have:

$$\dot{v}_2 = -M^{-1}{}_{22}\left[M_{21}\dot{v}_1 + (C_{21} + D_{21})v_1 + (C_{22} + D_{22})v_2\right] \quad (8)$$

Replace Eq. (8) into the third equation in Eq. (7):

$$\begin{aligned} &M_{11}\dot{v}_1 + (C_{11} + D_{11})v_1 - M_{12}M^{-1}{}_{22}[M_{21}\dot{v}_1 + (C_{21} + D_{21})v_1 + \\ &(C_{22} + D_{22})v_2] + (C_{12} + D_{12})v_2 = \tau \end{aligned} \quad (9)$$

Simplify Eq. (9) we get:

$$\bar{M}\dot{v}_1 + \bar{C}_1 v_1 + \bar{C}_2 v_2 = \tau \quad (10)$$

with:

$$\bar{M} = M_{11} - M_{12}M^{-1}{}_{22}M_{21}$$

$$\bar{C}_1 = (C_{11} + D_{11}) - M_{12}M^{-1}{}_{22}(C_{21} + D_{21})$$

$$\bar{C}_2 = (C_{12} + D_{12}) - M_{12}M^{-1}{}_{22}(C_{22} + D_{22})$$

With the assumption that we can choose the parameters for assuring that the $\bar{M}$ is positive definite matrix, from the Eq. (10), we have:

$$\dot{v}_1 = \bar{M}^{-1}(-\bar{C}_1 v_1 - \bar{C}_2 v_2) + \bar{M}^{-1}\tau \quad (11)$$

Replace Eq. (11) into the Eq. (8):

$$\dot{v}_2 = -M^{-1}{}_{22}\begin{bmatrix} M_{21}\bar{M}^{-1}(\tau - \bar{C}_1 v_1 - \bar{C}_2 v_2) + \\ (C_{21} + D_{21})v_1 + (C_{22} + D_{22})v_2 \end{bmatrix}$$

$$= -M^{-1}{}_{22}\begin{bmatrix} M_{21}\bar{M}^{-1}(-\bar{C}_1 v_1 - \bar{C}_2 v_2) + \\ (C_{21} + D_{21})v_1 + (C_{22} + D_{22})v_2 \end{bmatrix} - M^{-1}{}_{22}M_{21}\bar{M}^{-1}\tau \quad (12)$$

Replace Eqs. (11) and (12) into the Eq. (7), We have the dynamic equations of S-AUV as follows:

$$\begin{cases} \dot{\eta}_1 = J_{11}v_1 \\ \dot{v}_1 = \bar{M}^{-1}(-\bar{C}_1 v_1 - \bar{C}_2 v_2) + \bar{M}^{-1}\tau \\ \dot{\eta}_2 = J_{22}v_2 \\ \dot{v}_2 = f(v_2) \end{cases} \quad (13)$$

With: $J_{12}(v) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$; $J_{21}(v) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$;

$$f(v_2) = -M^{-1}{}_{22}\begin{bmatrix} M_{21}\bar{M}^{-1}(-\bar{C}_1 v_1 - \bar{C}_2 v_2) \\ +(C_{21} + D_{21})v_1 + (C_{22} + D_{22})v_2 \end{bmatrix} - M^{-1}{}_{22}M_{21}\bar{M}^{-1}\tau$$

B. Design HSMC for S-AUV

The problem of controlling motion through position vector and velocity vector that include $\eta = [x, y, z, \psi]^T$ are the position vector of the device in axes Ox, Oy, Oz and the directional angle of the S-AUV rotates around the axis Oz; $v = [u, v, w, r]^T$ is a vector of linear velocity in the directions Ox, Oy, Oz and the angular velocity around axis Oz. To solve the above problem, the paper proposes to use HSMC controller because this is the most suitable method to control underactuated systems. Therefore, the algorithm structure is used to ensure the stability of the system while still sticking to the given value as shown in Fig. 3.

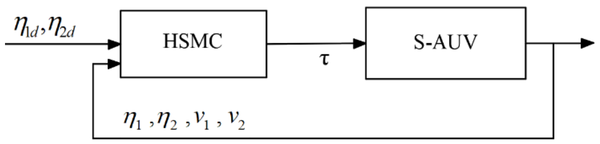


Figure 3. HSMC controller structure block diagram.

From the system of Eq. (13), we rewrite in the generalized form as follows:

$$\begin{cases} \dot{\eta}_1 = J_{11}v_1 \\ \dot{v}_1 = f_1(X) + g_1(X)\tau \\ \dot{\eta}_2 = J_{22}v_2 \\ \dot{v}_2 = f_2(X) + g_2(X)\tau \end{cases} \tag{14}$$

With:

$$X = [\eta_1 \quad v_1 \quad \eta_2 \quad v_2]^T$$

$$f_1(X) = \bar{M}^{-1}(-\bar{C}_1 v_1 - \bar{C}_2 v_2)$$

$$g_1(X) = \bar{M}^{-1}$$

$$f_2(X) = -M^{-1}_{22}\begin{bmatrix} M_{21}\bar{M}^{-1}(-\bar{C}_1 v_1 - \bar{C}_2 v_2) \\ +(C_{21}+D_{21})v_1 + (C_{22}+D_{22})v_2 \end{bmatrix}$$

$$g_2(X) = -M^{-1}_{22}M_{21}\bar{M}^{-1}$$

The definition of the error vector between the output signal and the set signal is as follows:

$$e(t) = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} \eta_1 - \eta_{1d} \\ v_1 \\ \eta_2 - \eta_{2d} \\ v_2 \end{bmatrix} \tag{15}$$

With:

$$\dot{e}_1 = \dot{\eta}_1 - \dot{\eta}_{1d} = J_{11}v_1 - \dot{\eta}_{1d} = J_{11}v_1$$

$$(\eta_{1d} = const \Rightarrow \dot{\eta}_{1d} = 0)$$

$$\dot{e}_3 = \dot{\eta}_2 - \dot{\eta}_{2d} = J_{22}v_2 - \dot{\eta}_{2d} = J_{22}v_2$$

$$(\eta_{2d} = const \Rightarrow \dot{\eta}_{2d} = 0)$$

The definition of the sliding surface is as follows

$$\begin{cases} s_1 = k_1 e_1 + e_2 (k_1 > 0) \\ s_2 = k_2 e_3 + e_4 (k_2 > 0) \\ S = \lambda s_1 + \beta s_2 (\lambda, \beta > 0) \end{cases} \tag{16}$$

According to the control method HSMC for underactuated system, the controller signal is divided into two components:

$$\tau_n = \tau_{eqn} + \tau_{swn} \tag{17}$$

With + τ_{eqn} is the signal that is used to control the subsystem in the controller structure Hierarchical Sliding Mode Controller.

+ τ_{swn} is the signal that is used to control the switching of the system sliding surface.

Consider the first subsystem model:

$$\begin{cases} \dot{\eta}_1 = J_{11}v_1 \\ \dot{v}_1 = f_1(X) + g_1(X)\tau \end{cases} \tag{18}$$

Applying Eq. (18), we have the control signal for the first and second subsystem as follows:

$$\begin{cases} \tau_1 = \tau_{eq1} + \tau_{sw1} \\ \tau_2 = \tau_{eq2} + \tau_{sw2} \end{cases} \tag{19}$$

The sliding surface derivative s_1 with respect to time we get:

$$\begin{aligned} \dot{s}_1 &= k_1\dot{e}_1 + \dot{e}_2 \\ &= k_1\dot{e}_1 + f_1(X) + g_1(X)\tau_1 \\ &= k_1\dot{e}_1 + f_1(X) + g_1(X)(\tau_{eq1} + \tau_{sw1}) \\ &= k_1\dot{e}_1 + f_1(X) + g_1(X)\tau_{eq1} - as_1 - bsign(s_1) \\ &+ as_1 + bsign(s_1) + g_1(X)\tau_{sw1} \end{aligned} \tag{20}$$

We choose the control signal for the first subsystem as follows:

$$\begin{cases} \tau_{eq1} = -(g_1^{-1}(\mathrm{X}))(k_1 J_{11} v_1 + f_1(\mathrm{X})) \\ \tau_{sw1} = -(g_1^{-1}(\mathrm{X}))(as_1 + bsign(s_1)) \end{cases} \tag{21}$$

Substituting the system of Eq. (21) into Eq. (20) we have:

$$\dot{s}_1 = -as_1 - bsign(s_1) \tag{22}$$

The control signal for the system consists of two subsystems as follows:

$$\tau = \tau_1 + \tau_2 \tag{23}$$

Select the sliding surface for the first and second subsystems as follows:

$$S = \lambda s_1 + \beta s_2 \tag{24}$$

To ensure stability for the S-AUV, consider Lyapunov function for the closed system as follows:

$$V = \frac{1}{2}S^T.S \tag{25}$$

Derivative V over time:

$$\frac{\partial V}{\partial t} = S^T.\dot{S} \tag{26}$$

From Eqs. (14)–(16) and Eq. (26) we have below equation:

$$\frac{\partial V}{\partial t} = S^T.\dot{S}$$

$$= S^T.\left[\lambda\dot{s}_1 + \beta\dot{s}_2\right]$$

$$= S^T.\begin{bmatrix}\lambda(k_1 J_{11} v_1 + f_1(X) + g_1(X)\tau - k_1\dot{\eta}_{1d}) + \\ \beta(k_2 J_{22} v_2 + f_2(X) + g_2(X)\tau - k_2\dot{\eta}_{2d})\end{bmatrix} \quad (27)$$

Since η_{1d}, η_{2d} are constant values so $\dot{\eta}_{1d} = \dot{\eta}_{2d} = 0$. Inferred:

$$\frac{\partial V}{\partial t} = S^T.\dot{S}$$

$$= S^T.\begin{bmatrix}\lambda(k_1 J_{11} v_1 + f_1(X) + g_1(X)\tau) + \\ \beta(k_2 J_{22} v_2 + f_2(X) + g_2(X)\tau)\end{bmatrix}$$

$$= S^T.\begin{bmatrix}\lambda[k_1 J_{11} v_1 + f_1(X) + \beta[k_2 J_{22} v_2 + f_2(X) + g_1(X)(\tau_{eq1} \\ +\tau_{sw1} + \tau_{eq2} + \tau_{sw2})] + g_2(X)(\tau_{eq1} + \tau_{sw1} + \tau_{eq2} + \tau_{sw2})]\end{bmatrix} \quad (28)$$

$$= S^T.\begin{bmatrix}\lambda[k_1 J_{11} v_1 + f_1(X) + g_1(X)\tau_{eq1}] + \beta[k_2 J_{22} v_2 \\ + f_2(X) + g_2(X)\tau_{eq2}] \\ +[\lambda g_1(X) + \beta g_2(X)]\tau_{sw1} + [\lambda g_1(X) + \beta g_2(X)]\tau_{sw2} \\ +\lambda g_1(X)\tau_{eq2} + \beta g_2(X)\tau_{eq1} \\ +k.S + \delta\,\mathrm{sgn}(S) - (k.S + \delta\,\mathrm{sgn}(S))\end{bmatrix}$$

To ensure the stability of the system through the principle of stability of Lyapunov so that $\frac{\partial V}{\partial t}$ is defined negative, we choose the following control signals:

$$\begin{cases} \tau_{eq1} = -\left(g_1^{-1}(X)\right)\left(k_1 J_{11} v_1 + f_1(X)\right) \\ \tau_{eq2} = -\left(g_2^{-1}(X)\right)\left(k_2 J_{22} v_2 + f_2(X)\right) \\ \tau_{sw2} = -\left(\lambda g_1(X) + \beta g_2(X)\right)^{-1}\left(\lambda g_1(X)\tau_{eq2} + \beta g_2(X)\tau_{eq1}\right) \\ \quad -\left(\lambda g_1(X) + \beta g_2(X)\right)^{-1}\left(k.S + \delta\,\mathrm{sgn}(S)\right) - \tau_{sw1} \end{cases} \quad (29)$$

IV. SIMULATION RESULTS

To verify the quality of the HSMC, simulation was performed for the new S-AUV as shown in Fig. 4 with parameters as shown in Table II below:

Figure. 4. Model S-AUV was built.

TABLE II. MODELING PARAMETER S-AUV.

Parameters	Values
m	18.5
x_g, y_g	0.15
z_g	0
X_u	6.53
$Z_{u\|u\|}$	−0.58
Y_v	0.08
Y_r	−1.03
$Y_{\dot{v}}$	−0.85
$Y_{v\|v\|}$	−0.62
Z_w	4.57
$Z_{\dot{u}}$	0.32
N_r	−12.32
$X_{\dot{w}}$	-1.13×10^{-4}
$N_{\dot{v}}$	0.32
$N_{\dot{r}}$	−2.15
I_z	1.57
$N_{r\|r\|}$	0.5×10^{-4}
$X_{\dot{u}}$	6.83×10^{-4}
$Z_{\dot{w}}$	0.32×10^{-4}

The parameters of the HSMC law were set to k =100, λ =500, β =2.5, δ =5, k_1 =0.05 and k_2 = 5. The block diagram of function in MATLAB Simulink is shown in Fig. 5.

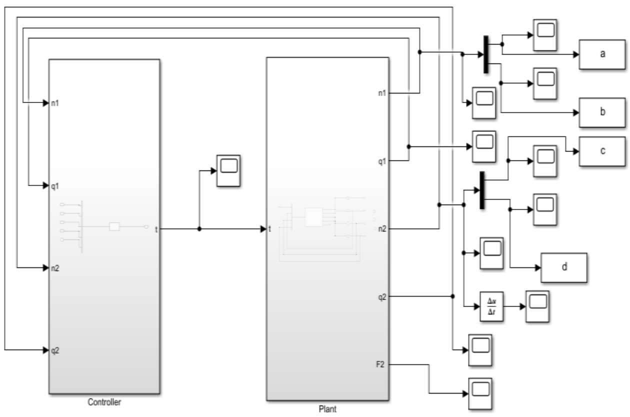
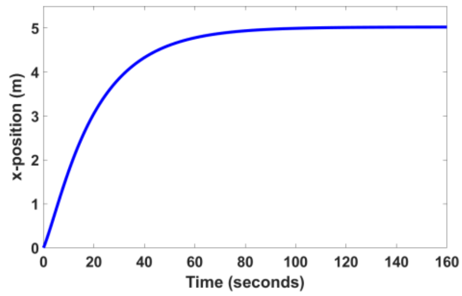
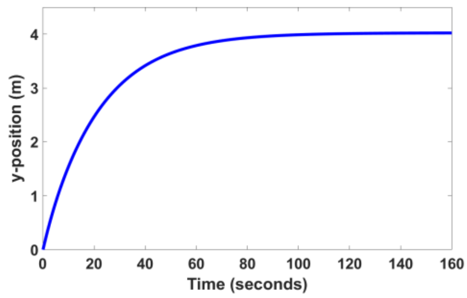


Figure 5. Block diagram of function in MATLAB Simulink

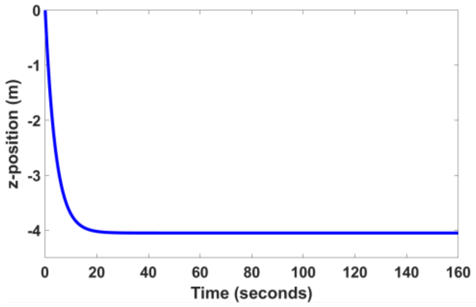
Case 1: $\eta_{1d} = [5 \quad 4]^T$; $\eta_{2d} = [-4 \quad -0.05]^T$



(a)



(b)



(c)

Figure 6. Simulation of the position of the S-AUV in the direction: (a) in the *Ox* direction , (b) in the *Oy* direction, (c) in the *Oz* direction

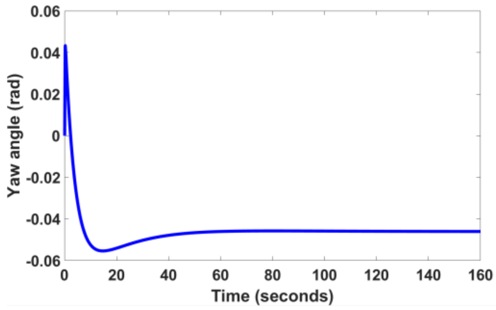
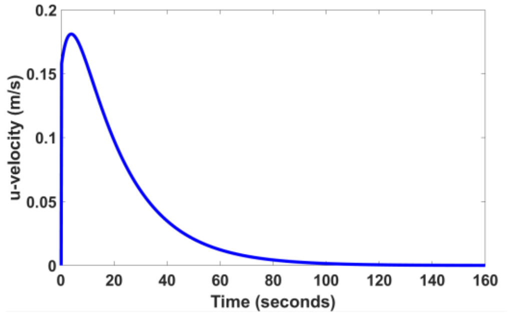
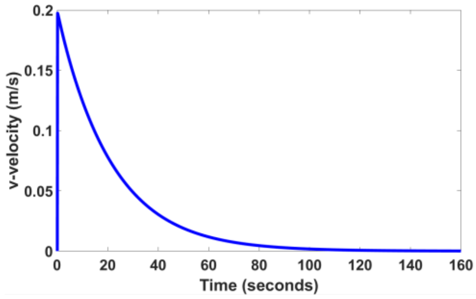


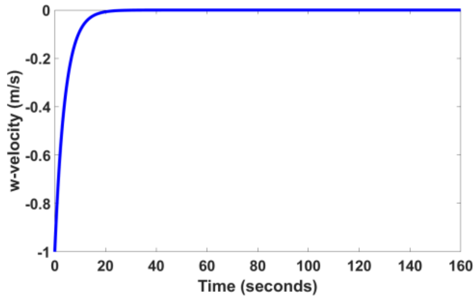
Figure 7. Navigation angle of S-AUV in Case 1



(a)



(b)



(c)

Figure 8. Simulation of the velocity of the S-AUV in the direction: (a) in the *Ox* direction , (b) in the *Oy* direction, (c) in the *Oz* direction

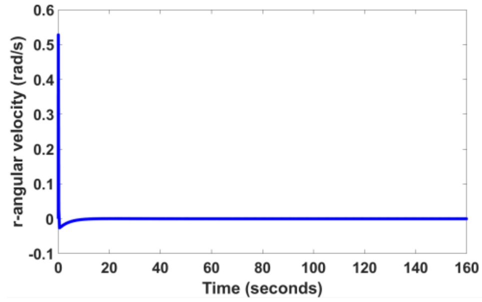
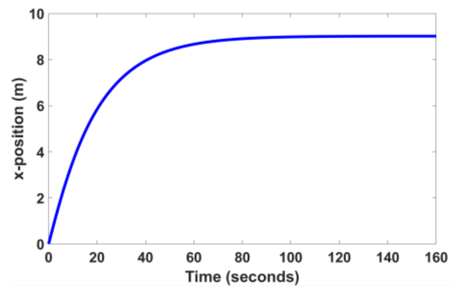
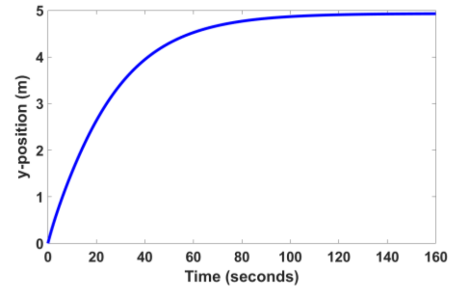


Figure 9. Angular velocity of S-AUV navigation in Case 1

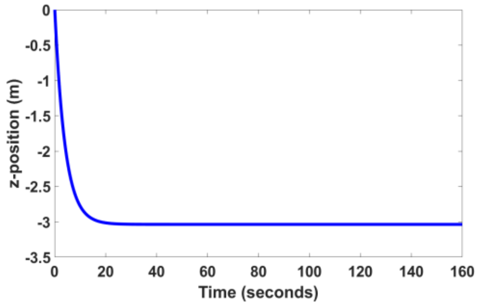
Case 2: $\eta_{1d} = [9 \quad 5]^T$; $\eta_{2d} = [-3 \quad -0.1]^T$



(a)



(b)



(c)

Figure 10. Simulation of the position of the S-AUV in the direction: (a) in the *Ox* direction , (b) in the *Oy* direction, (c) in the *Oz* direction

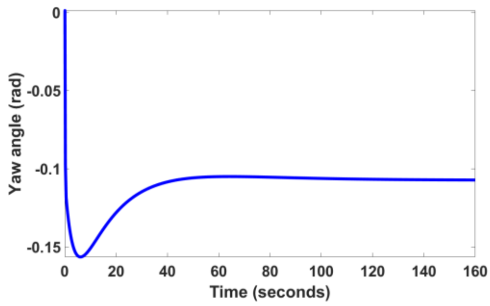
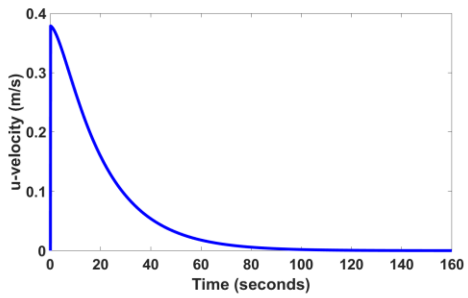
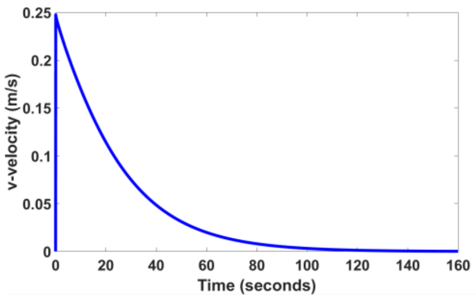


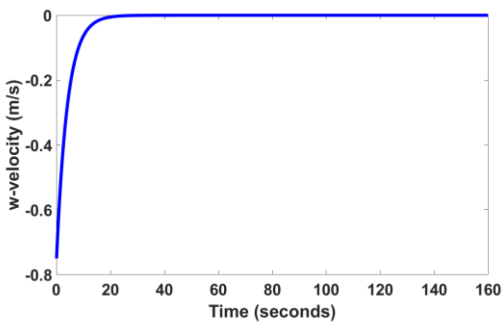
Figure 11. Navigation angle of S-AUV in Case 2



(a)



(b)



(c)

Figure 12. Simulation of the velocity of the S-AUV in the direction: (a) in the *Ox* direction , (b) in the *Oy* direction, (c) in the *Oz* direction

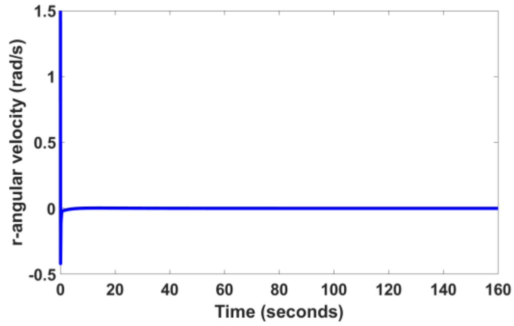


Figure 13. Angular velocity of S-AUV navigation in Case 2

The simulation results from Figs. 6–13 shows that the HSMC is applied to new S-AUV with good control quality in terms of grip positions, direction angle, velocity and angular velocity in three dimensions. Specifically, duration for the system to set the grip position in Ox, Oy is 90s, 100s; duration to set position in Oz is 20s and navigation angle is 40s.

V. Conclusion

The paper applies the Hierarchical Sliding Mode Controller to the new S-AUV model. The simulation results on MATLAB software have proved that: with this algorithm, the S-AUV response to the desired signal with insignificant overshoot, the setting error is zero and the duration to set position sticking in the Ox, Oy, Oz and the navigation angle meets the requirements. The built HSMC gave good quality compared to requirements of controlling for underactuated systems S-AUV. The controller follows the desired signal with a negligible transient of less than 5%. Upcoming, the author group will combine the intelligent controllers to further optimize the control algorithm to achieve high efficiency in controlling the designed S-AUV model. For example, Adaptive Fuzzy Controller for sliding surface parameters and Adaptive Neural Network Controller to approximate parameters which are difficult to determine in practice in order to improve the optimum control quality.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Author Contributions: Tuan Nguyen Van, Phong Dinh Van authors discussed the idea, conceptualization, data curation, methodology, and writing—original draft, writing—review and editing. Tan Nguyen Cong conceptualization, proved stability. Hung Nguyen Chi review and editing the manuscript. All authors have read and agreed to the published version of the manuscript.

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