International Journal of Mechanical Engineering and Robotics Research

ISSN 2278 – 0149 www.ijmerr.com
Vol. 4, No. 1, January 2015

Research Paper

USING FINITE ELEMENT METHOD VIBRATION ANALYSIS OF FRAME STRUCTURE SUBJECTED TO MOVING LOADS

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It was purposed to understand the dynamic response of frame which are subjected to moving point loads. The finite element method and numerical time integration method (New mark method) are employed in the vibration analysis. The effect of the speed of the moving load on the dynamic magnification factor which is defined as the ratio of the maximum dynamic displacement at the corresponding node in the time history to the static displacement when the load is at the mid-point of the structure is investigated. The effect of the spring stiffness attached to the frame at the conjunction points of beam and columns are also evaluated. Computer codes written in Mat lab are developed to calculate the dynamic responses. Dynamic responses of the engineering structures and critical load velocities can be found with high accuracy by using the finite element method.

Keywords: Beam, Finite element methods, Numerical tine integration method, Dynamic magnification factors, Stiffness, Shape function, Critical load velocity

INTRODUCTION

Vibration analysis of structures has been of general interest to the scientific and engineering communities for many years. These structures have multitude of applications in almost every industry. The aircraft industry has shown much interest in this, some of the early solutions were motivated by this industry. This study deals with the finite element analysis of the monotonic behavior of beams, slabs and beam-column joint sub assemblages. It is assumed that the behavior of these members can be described by a plane stress field. Reinforced concrete has become one of the most important building materials and is widely used in many types of engineering structures. The economy, the efficiency, the strength and the stiffness of reinforced concrete make it an attractive material for a wide range of structural applications. For its use as structural material, concrete must satisfy the following conditions:

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The Structure Must be Strong and Safe: The proper application of the fundamental principles of analysis, the laws of equilibrium and the consideration of the mechanical Properties of the component materials should result in a sufficient margin of safety against collapse under accidental overloads.

The Structure Must be Stiff and Appear Unblemished: Care must be taken to control deflections under service loads and to limit the crack width to an acceptable level.

The Structure Must be Economical: Materials must be used efficiently, since the difference in unit cost between concrete and steel is relatively large. Moving loads have considerable effects on the dynamic behavior of the engineering structure. Transport engineering frame structures such as bridges are subjected to loads that vary in both time and space (moving forces), in the form of vehicular traffic, which cause them to vibrate. A moving vehicle on a bridge causes deflections and stresses that are generally greater than those caused by the same vehicular loads applied statically. The dynamic analysis of a structure subjected to a moving load is an old topic of research; hence a lot of literature exists. Olsson (1991) studied the dynamics of a beam subjected to a constant force moving at a constant speed and presented analytical and finite element solutions. Thambiratnam and Zhuge (1996) studied the dynamics of beams on an elastic foundation and subjected to moving loads by using the finite element method. They investigated the effect of the foundation stiffness, travelling speed and the span length of the beam on the dynamic magnification factor, which is defined as the ratio of the maximum displacement in the time history of the mid-point to the static midpoint displacement. Wang (1997) analyzed the multi-span Timoshenko beams subjected to a concentrated moving force by using the mode superposition method and made a comparison between the Euler-Bernoulli beam and Timoshenko beam. Zheng *et al.* (1998) analyzed the vibration of a multi span non uniform beam subjected to a moving load by using modified beam vibration functions as the assumed modes based on Hamilton's principle. The modified beam vibration functions satisfy the zero deflection conditions at all the intermediate point supports as well as the boundary conditions at the two ends of the beam. Numerical results are presented for both uniform and non-uniform beams under moving loads of various velocities. Wang and Lin (1998) studied the vibration of multi-span Timoshenko frames due to moving loads by using the modal analysis. Kadivar and Mohebpour (1998) analyzed the dynamic responses of unsymmetrical composite laminated orthotropic beams under the action of moving loads. Hong and Kim (1999) presented the modal analysis of multi span Timoshenko beams connected or supported by resilient joints with damping. The results are compared with FEM. Ichikawa *et al.* (2000) investigated the dynamic behavior of the multi-span Continuous beam traversed by a moving mass at a constant velocity, in which it is assumed that each span of the continuous beam obeys uniform Euler-Bernoulli beam theory.

Dynamic Analysis by Numerical Integration

Dynamic response of structures under moving loads is an important problem in engineering and studied by many researchers. The numerical solution can be calculated by various methods which are as follows:

1. Duhamel Integral
2. New mark Integration method
3. Central difference Method
4. Houbolt Method
5. Wilson ¯ Method

New Mark Family of Methods

The New mark integration method is based on the assumption that the Acceleration varies linearly between two instants of time. In 1959 new mark presented a family of single-step integration methods for the solution of structural Dynamic problems for both blast and seismic loading. During the past 45 years new mark method has been applied to the dynamic analysis of many practical engineering structures. In addition, it has been modified and improved by many other researchers. In order to illustrate the use of this family of numerical integration methods, we considered the solution of the linear dynamic equilibrium equations written in the following form:

$$[M]\dot{u}_t + [C]\dot{u}_t + [K]u_t = F_t \quad ...(1)$$

where M is the mass matrix, C is the damping matrix and K is the stiffness matrix.

$\ddot{u}$, $u, \wedge u$ are the acceleration, velocity and displacement vectors, respectively F_t is the external loading vector. The direct use of Taylor's series provides a rigorous approach to obtain the following two additional equations:

$$u_t = u_{t-\Delta t} + \Delta t\dot{u}_{t-\Delta t} + \frac{\Delta t^2}{2}\dot{u}_{t-\Delta t} + \frac{t^3}{6}\dddot{u}_{t-t} \quad ...(2)$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + \Delta t\dot{u}_{t-\Delta t} + \frac{\Delta t^2}{2}\dddot{u}_{t-\Delta t} \quad ...(3)$$

New mark truncated these equations and expressed them in the following form:

$$u_t = u_{t-\Delta t} + \Delta t\dot{u}_{t-\Delta t} + \frac{\Delta t^2}{2}\dot{u}_{t-\Delta t} + \quad \Delta t^3\dddot{u} \quad ...(4)$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + \Delta t\dot{u}_{t-\Delta t} + ¥\Delta t^2\dddot{u} \quad ...(5)$$

If the acceleration is assumed to be linear within the time step, the following equation can be written as:

$$\dddot{u} = \frac{\dot{u}_t - \dot{u}_{t-\Delta t}}{\Delta t} \quad ...(6)$$

The substitution of Equation (6) into Equations (4) and (5) produces new mark's equations in standard form

$$u_t = u_{t-\Delta t} + \Delta t\dot{u}_{t-\Delta t} + \left(\frac{1}{2} - \quad\right)\Delta t^2\dot{u}_{t-\Delta t} + \quad \Delta t^2\dot{u}_t \quad ...(7)$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + (1 - ¥)\Delta t\dot{u}_{t-\Delta t} = ¥\Delta t\ddot{u}_t \quad ...(8)$$

Stability of New Mark Method

For zero damping new mark method is conditionally stable if

$$¥ \geq \frac{1}{2}, \quad \leq \frac{1}{2} \text{ and } \Delta t \leq \frac{1}{\mu_{max}\sqrt{\left(\frac{¥}{2} - \quad\right)}} \quad ...(9)$$

where μ_{max} is the maximum frequency in the structural system New mark's method is unconditionally stable if

$$2 \quad \geq ¥ \geq \frac{1}{2} \quad ...(10)$$

However, if ¥ is greater than 1/2, errors are introduced. These errors are associated with "numerical damping" and "period elongation". For large multi degree of freedom structural systems the time step limit, given by Equation (9), can be written in a more usable form as:

$$\frac{\Delta t}{T_{\min}} \le \frac{1}{2® \sqrt{\left(\frac{¥}{2} - \right)}} \quad ...(11)$$

where T_{min} is the minimum time period of the structure. Computer model of larger structures normally contain a large number of periods which are smaller than the integration time step; therefore, it is essential that one select a numerical integration method that is unconditionally stable for all time steps. Table 1 shows the summary of the New mark method for direct integration.

Solution Using Finite Element Method

Finite Element Method (FEM) is a numerical method for solving a differential or integral equation. It has been applied to a number of physical problems, where the governing differential equations are available. The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution.

A promising approach for developing a solution for structural vibration problems is provided by an advanced numerical discretization scheme, such as, Finite Element Method (FEM). The FEM is the dominant discretization technique in structural mechanics.

The basic concept in the physical FEM is the subdivision of the mathematical model into disjoint (non-overlapping) components of simple geometry called finite elements or elements for short. The response of each element is expressed in terms of a finite number of degrees of freedom characterized as the value of an unknown function, or functions, at a set of nodal points. The response of the mathematical model is then considered to be approximated by that of the discrete model obtained by connecting or assembling the collection of all elements.

The Finite Element Method (FEM) is the dominant discretization technique in structural mechanics. The basic concept in the physical interpretation of the FEM is the subdivision of the mathematical model into disjoint (non-overlapping) components of simple geometry called finite elements or elements for short. The response of each element is expressed in terms of a finite number of degrees of freedom characterized as the value of an unknown function, or functions, at a set of nodal points.

Table 1: Dynamic Magnification Factors for the Mid-Points of the Beam and Columns of the Frame and Spring Attached Frame

Frame (Column 1)			
~	**Frame**	**Spring Attached (k1)**	**Spring Attached (k2)**
0.2	1.482	1.120	1.131
0.4	1.371	1.139	1.169
0.6	1.908	1.149	1.183
1.0	1.792	1.202	1.271
1.5	1.032	1.358	1.257
1.9	1.191	1.431	1.420
2.0	1.201	1.576	1.497
4.0	1.632	2.052	1.905
4.5	1.729	2.125	1.936
6.0	1.908	2.253	2.003
7.0	1.778	2.078	1.987
8.0	1.341	1.540	1.635
10.0	0.942	1.005	1.120
12.0	0.789	0.791	0.909
14.0	0.632	0.637	0.640
16.0	0.455	0.451	0.448

The response of the mathematical model is then considered to be approximated by that of the discrete model obtained by connecting or assembling the collection of all elements. The disconnection-assembly concept occurs naturally when examining many artificial and natural systems. For example, it is easy to visualize an engine, bridge, building, airplane, or skeleton as fabricated from simpler components. Unlike finite difference models, finite elements do not overlap in space.

Frame Element

The moving load problem is extended to a frame structure. Axial displacement for longitudinal vibration of the frame element is assumed to be linear so the shape functions for the longitudinal vibration are,

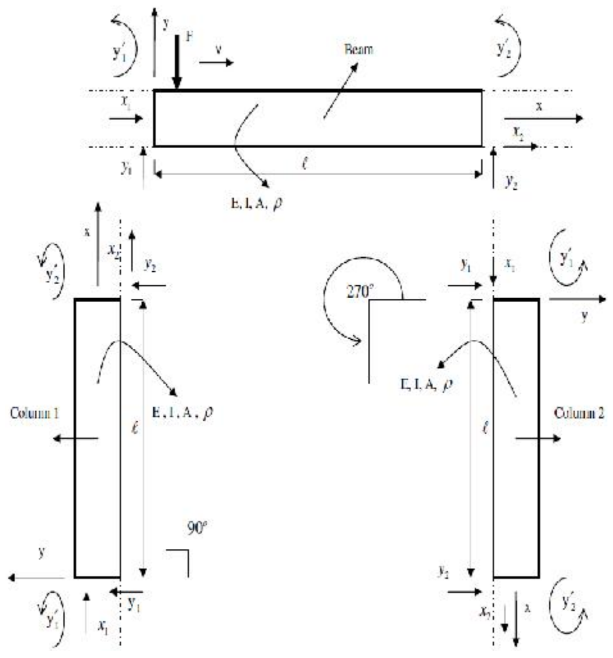
$$H_5(x) = \left(1 - \frac{x}{l}\right) \quad ...(12)$$

$$H_6(x) = \frac{x}{l} \quad ...(13)$$

There are three degrees of freedom per node, translation along x-axis, translation along y-axis and rotation about z-axis is assumed for frame element as shown in Figure 1. The coupling between bending and longitudinal vibrations is neglected. The stiffness and mass matrices for the frame element are constructed by superimposing both the axial and bending matrices

The transformation matrices are used to form the mass and stiffness matrices for the columns of the frame structure. Both columns and beam of the frame structure are modeled with 10 equally sized elements. All the element mass and stiffness matrices ([*K*] and [*M*]) are multiplied by the transformation matrix [*T*] given in as Equation (14).

Figure 1: Modelling the Frame Elements



[*K*] = [*T*]' [*K*][*T*] and [*M*] = [*T*]' [*M*][*T*]

where

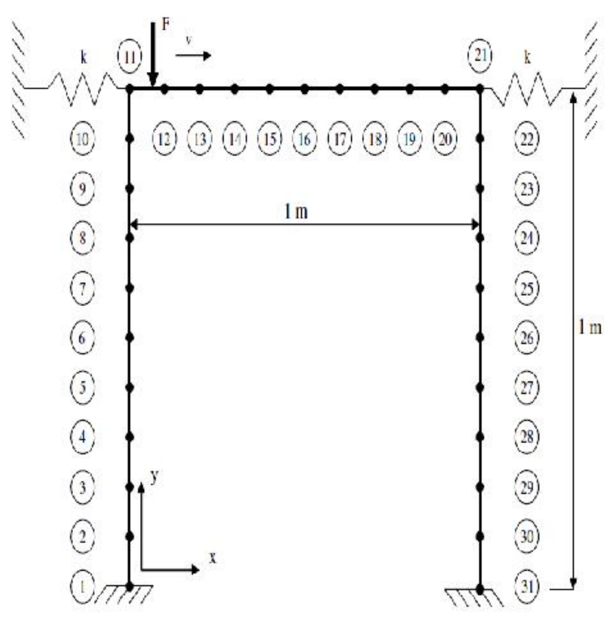
$$[T] = \begin{bmatrix} c_{-} & s_{-} & 0 & 0 & 0 & 0 \\ -s_{-} & c_{-} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{-} & s_{-} & 0 \\ 0 & 0 & 0 & -s_{-} & c_{-} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c_{-} = \cos(^{-}),\ s_{-} = \sin(^{-}) \quad ...(14)$$

¯ for column 1 is 90° and r for column 2 is 270° as shown in Figure 1. A spring is attached to the frame at the column and beam conjunction points in order to analyze the effect of the spring stiffness. Spring has a stiffness *k* in the *x* direction as shown in Figure 2.

The effect of springs on the dynamic response of the frame structure is investigated. The stiffness of the spring *k* is added to column's stiffness matrices as a constant term at the corresponding degree of freedom.

Figure 2: Spring Attached Frame



Equations of Motion of the Frame Structure

The equation of motion for a multiple degree of freedom undamped structural system is represented as follows

$$[M]\{\dot{y}\}+[K]\{y\}=\{F(t)\} \quad ...(15)$$

where and $\dot{y}$ are the respective acceleration and displacement vectors for the whole structure and $F(t)$ is the external force vector.

Under free vibration, the natural frequencies and the mode shapes of a multiple degree of freedom system are the solutions of the Eigen values problem.

$$\left[[K]-\mu^2[M]\right]\{\Phi\}=0 \quad ...(16)$$

where μ is the angular natural frequency and Φ is the mode shape of the structure for the corresponding natural frequency.

The Effect of Viscous Damping

A proportional damping is assumed to show the effect of damping ratio on the dynamic magnification factor. Rayleigh damping, in which the damping matrix is proportional to the combination of the mass and stiffness matrices, is used.

$$[C] = a_0[M] + a_1[K] \quad ...(17)$$

If the damping ratios $\P_m$ and $\P_n$ associated with two specific frequencies μ_m, μ_n Rayleigh damping factors, a_0 and a_1 can be evaluated by the solution of the following equation

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \frac{\mu_m\mu_n}{\mu_m^2-\mu_n^2}\begin{bmatrix} \mu_n & -\mu_m \\ \dfrac{1}{\mu_n} & \dfrac{1}{\mu_m} \end{bmatrix}\begin{Bmatrix} \P_m \\ \P_n \end{Bmatrix} \quad ...(18)$$

The same damping ratio is applied to both control frequencies, μ_1 and μ_2, i.e., $\P = \P_1 = \P_2$ then the proportionality factors can be given in simplified form as

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \frac{2}{\mu_1+\mu_2}\begin{Bmatrix} \mu_1\mu_2 \\ 1 \end{Bmatrix} \quad ...(19)$$

Two different damping ratios, $\P_1 = 0.01$ and $\P_2 = 0.05$ are used to show the effect of the damping ratio on the dynamic magnification factor.

The Application of the Moving Force

The dynamic response of an Euler-Bernoulli beam under moving loads is studied by mode superposition. The inertial effects of the moving load are included in the analysis. The time-dependent equations of motion in modal space are solved by the method of multiple scales.

The transverse point load F has a constant velocity, $v = L/^2$, where t is the travelling time across the beam and L is the total length of the beam.

For the forced vibration analysis an implicit time integration method, called as the

Newmark integration method is used with the integration parameters =1/4 and ϒ =1/2, which lead to constant-average acceleration approximation. The time step is chosen as $\Delta t = T_{20}/20$ during the beam vibration analysis in order to ensure that all the 20 modes contribute to the dynamic response, where T_{20} is the period of the 20th natural mode of the structure.

The time history of the nodal force in the transverse direction is given in Figure 3. The time for the load to arrive i^{th} node, $^2{}_{§} = x_i/v$, where x_i is the location of ith node. The nodal force on the i^{th} node, $F_i = 0$ except $^2{}_{§-1} < t < ^2{}_{§+1}$. The force is applied all the nodes according to Figure 4. Moment effect of the force is ignored, only vertical degree of freedom is affected by this force. The Simple method in which $M_i = 0$ at any time is used for the calculation of the dynamic responses. A non-dimensional velocity parameter a is used as $\sim = T_1/^2$, where T_i is the period of the first natural frequency of the beam.

Results of the Dynamic Analysis of Frame Structure

Figure 4 shows the first three mode shapes of the frame and spring attached frame (k2). Attaching a spring to the frame at the conjunction points of the beam and columns makes the frame more rigid and shifts the mode shapes of the frame structure up. Generally, the first mode of vibration is the one of primary interest. The first mode usually has the largest contribution to the structure's motion. The period of this mode is the longest. (Shortest natural frequency = first eigenvecto.

Figure 10 the effect of a on the dynamic magnification factor of column 2 for normal and spring attached frame at different nodes.

Figure 3: Moving Load Time History for the i^{th} Node

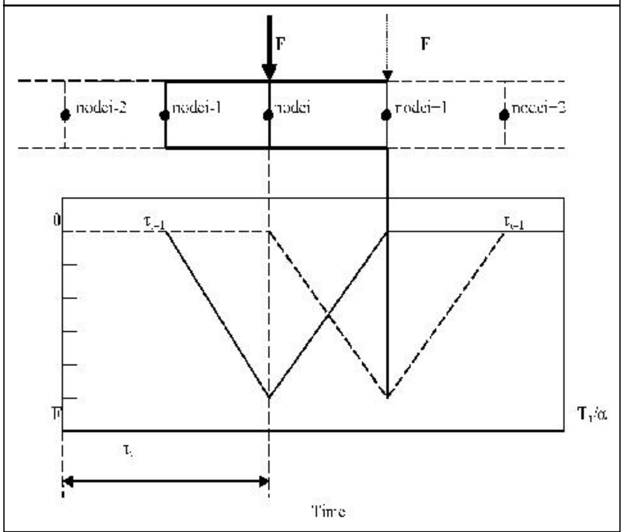


Figure 4: First Three Mode Shape of the Frame and Spring Attached Frame (K_2)

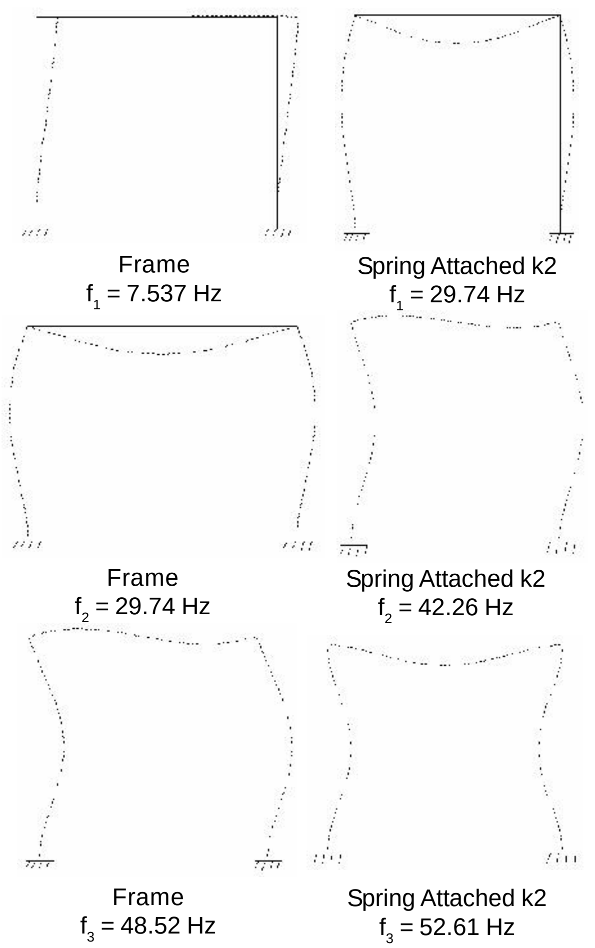


Figure 5: The Effect of the Springs on the Mid-Point Displacement of the Column 1 of the Frame Structure

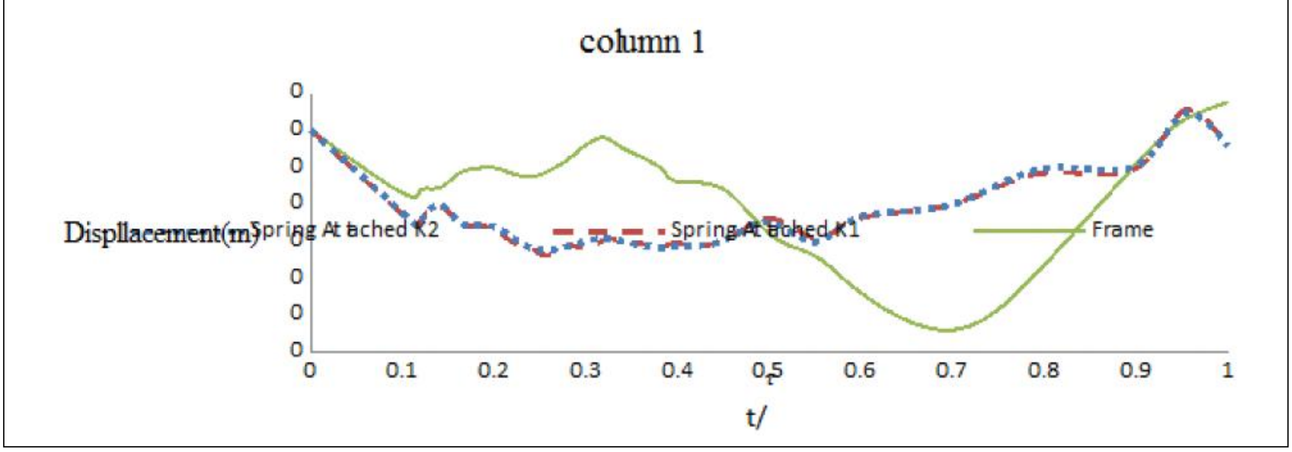


Figure 6: The Effect of the Springs on the Mid-Point Displacement of the Column 2 of the Frame Structure

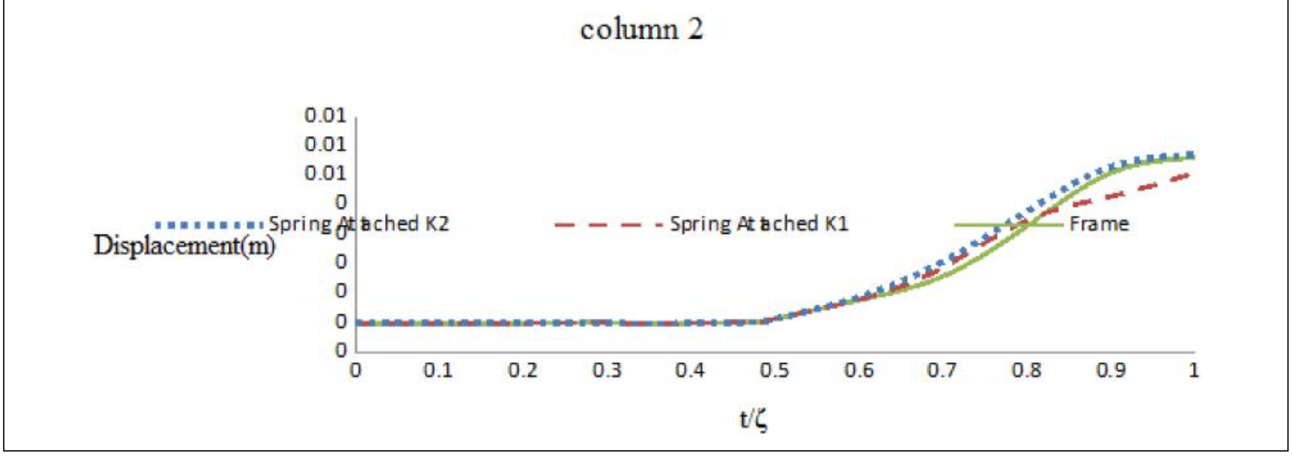


Figure 7: The Effect of the Springs on the Mid-Point Displacement of the Beam of the Frame Structure

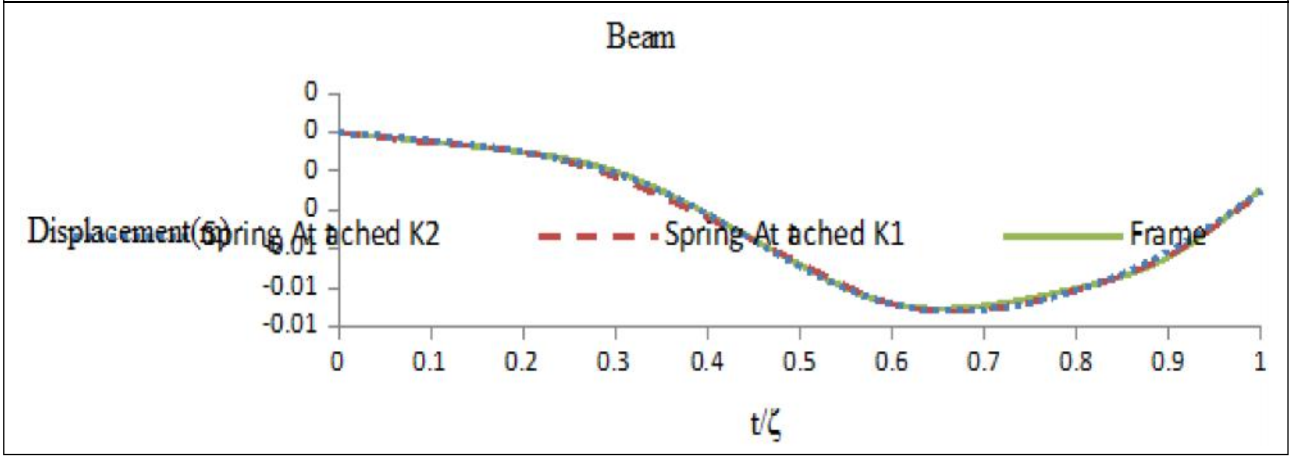


Figure 8: The Effect of a on the Dynamic Magnification Factor of Column 1 for Normal and Spring Attached Frame at Different Node

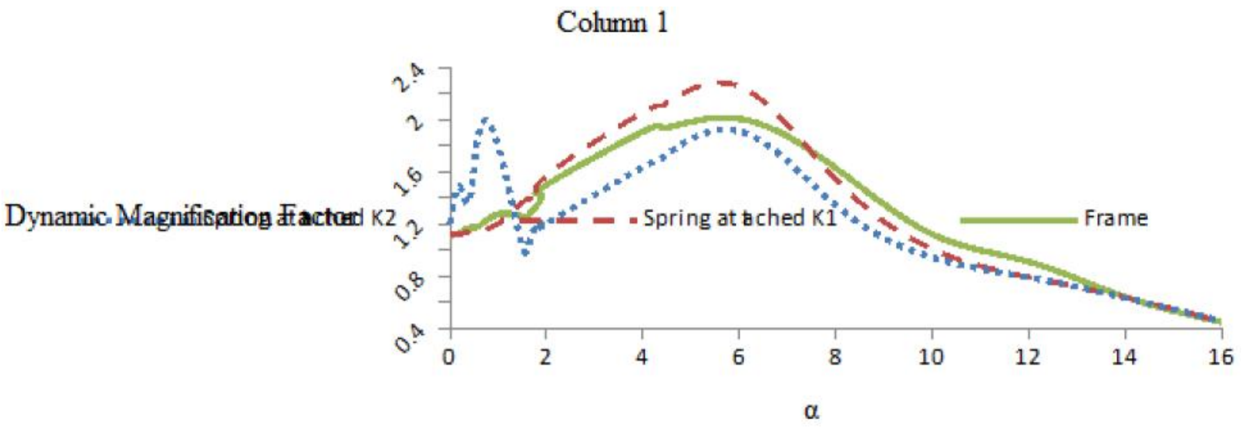


Figure 9: The Effect of a on the Dynamic Magnification Factor of Beam for Normal and Spring Attached Frame at Different Nodes

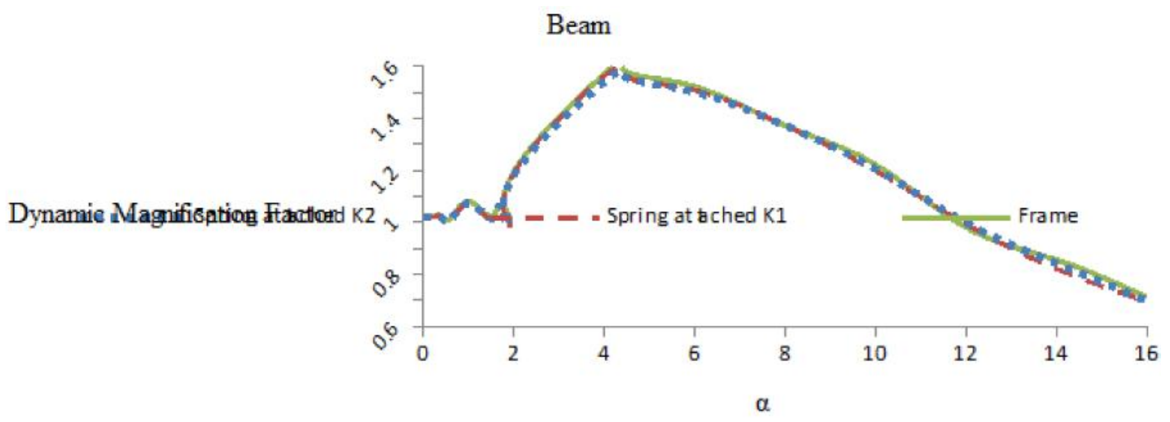
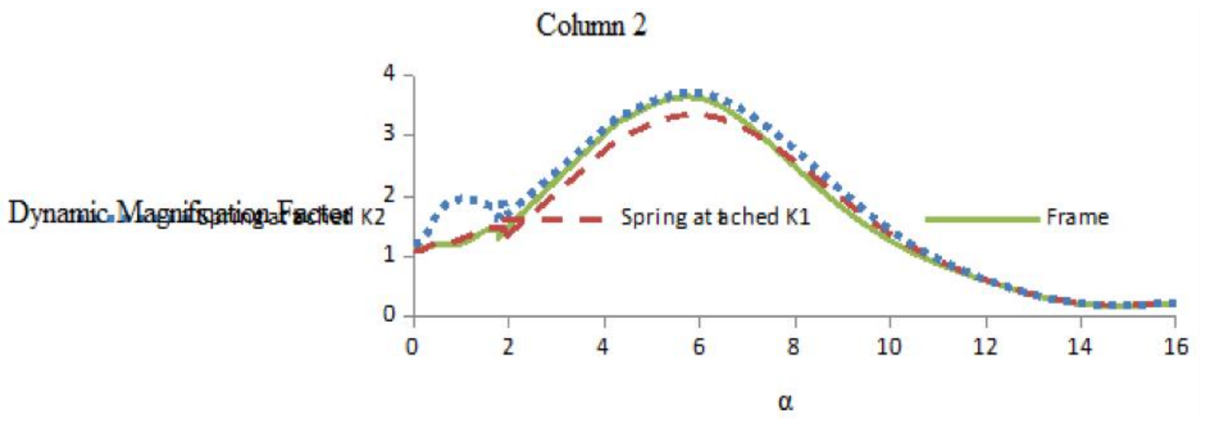


Figure 10: The Effect of a on the Dynamic Magnification Factor of Column 2 for Normal and Spring Attached Frame at Different Nodes



Figures 5, 6, and 7 show the effect of the assumed springs stiffness (K_s) on the mid-point displacements of beam and columns of a Frame structure. Three critical load speeds shown in Figures 8, 9 and 10 are considered for columns and beam. v = 4.522 m/s for column 1 (~ = 0.6), v = 34.67 m/s for beam (~ = 4.6), v = 45.22 m/s for column 2 (~ = 6). Figure 5 shows that springs have more effect on the dynamic response of the mid-point of column 1 for the critical load speed and reduce the maximum dynamic displacements. On the other hand, springs have no contribution for beam displacements as shown in Figure 6 and also no noteworthy contribution for the mid-point displacement of column 2 as shown in Figure 7.

Figures 8-10 shows the effect of a values on the dynamic magnification factor for different nodes of a normal and spring attached frame. Dynamic magnification factors are calculated for frame structure as calculated for beam. D_d is the ratio of the maximum dynamic displacement to the static displacement at the considered node. The static displacements for all the nodes of the beam and columns of the frame are calculated when the force acting on the mid-point of the beam. There is no noteworthy difference between frame and spring attached frame static displacements except conjunction points. Figures 8-10 also show the contribution of springs to D_d. It can be said that for small a values (~ < 1) springs are very effective for all nodes. In this interval, higher D_d values are obtained with increasing spring stiffness. Figures 8-10 show the D_d values only when the moving load is on the beam, so the interpretations are based on this situation. The maximum D_d values occur in the neighborhood of ~ = 6 for nodes 2-5. The attachment of the spring causes higher D_d values in the middle speed region. Lower D_d values are obtained with increasing spring stiffness in this region. The maximum D_d values for nodes 6-11 are observed close to ~ = 0.6. The springs are very effective especially in this low speed region, but higher D_d values are obtained with increasing spring stiffness in this interval. For beam (nodes 12-21) not a noteworthy difference is observed both by attaching a spring or increasing spring stiffness. The maximum D_d occurs at ~ = 4.5 for the mid-point of the beam. Similar to column 1, two critical moving load speeds are observed for column 2, ~ =1 and ~ = 6. The springs have no contribution to the D_d values for both beam and columns at high speed region (~ > 10). Lower D_d values are obtained by attaching spring for column 2 in the middle speed region.

Table 1 Dynamic magnification factors for the mid-points of the beam and columns of the frame and spring attached frame for different ~ values. (*~ values which makes D_d maximum when the moving load is on the beam.

Effect of the Rayleigh Damping

Dynamic analyses are performed to show the effect of the Rayleigh damping on the magnification factors of beam and frame structures. A description of a mechanical structure requires knowledge of the geometry, boundary conditions and material properties. The mass and stiffness matrices of a structure with complicated geometry, boundary conditions and material properties can be obtained experimentally or numerically (for example, using the finite-element method). Unfortunately, present knowledge of damping does not allow us to obtain the damping matrix like the mass and stiffness matrices for complicated systems. For this reason we consider simple systems for which geometry, boundary conditions and material properties are easy to determine.

Figure 11 shows the effect of damping ratio ¶ on the magnification factor of a clamped-clamped beam. The maximum D_d value (1.632), which is observed at ~ = 1.02 for the undamped case is recorded at ~ = 1 (D_d = 1.608) for ¶ = 0.01 and at ~ = 0.97 (D_d = 1.524) for ¶ = 0.05. D_d values are decreased with increasing damping ratio as expected and the time at which the maximum D_d occurs shifts left with increasing damping ratio.

Figures 12-17 show the effect of the Rayleigh damping on the dynamic magnification factor for the mid-point of both columns and beam of frame and spring attached (k2) frame. The maximum D_d values for the mid-points of columns and beam of the frame are; for ¶ = 0.01, at ~ = 6.1 (D_d = 1.849) for column 1, at ~ = 4.4 (D_d = 1.534) for beam and at ~ = 6.1 (D_d = 3.595) for column 2. Similarly the maximum Dd values are observed at ~ = 0.7 (D_d = 1.691) for column 1, at ~ = 4.2 (D_d = 1.468) for beam and at ~ = 6.1 (D_d = 3.172) for column 2 for the damping ratio ¶ = 0.05. Lower D_d values are observed with increasing damping ratio.

The time at which the maximum D_d values occur shifts left for the mid-point of the beam of the frame with increasing damping ratio, but the maximum D_d values are obtained with higher values for the columns of the frame with increasing damping ratio. The dynamic magnification factors for the frame structure

Table 2: Maximum D_d Values for the Mid-Points of the Columns and Beam of the Frame with the Effect of Damping

Frame (Column 2)			
~	**Frame**	**Spring Attached (k1)**	**Spring Attached (k2)**
0.2	1.291	1.113	1.121
0.4	1.572	1.206	1.179
0.6	1.804	1.214	1.203
1.0	1.955	1.284	1.209
1.5	1.873	1.470	1.424
1.9	1.821	1.421	1.419
2.0	1.730	1.372	1.504
4.0	3.123	2.754	3.628
4.5	3.388	3.025	3.312
6.0	3.710	3.358	3.630
7.0	3.467	3.132	3.343
8.0	2.77o	2.571	2.486
10.0	1.433	1.370	1.256
12.0	0.614	0.609	0.598
14.0	0.219	0.215	0.209
16.0	0.230	0.226	0.220

Table 3: Maximum D_d Values for the Mid-Points of the Columns and Beam of the Spring Attached (k2) Frame with the Effect of Damping

Undamped		
Quantity	D_d	~
Column 1	2.016	1.6
Beam	1.550	1.2
Column 2	3.635	1.5
	¶ = 0.01	
Quantity	D_d	~
Column 1	1.969	1.6
Beam	1.532	1.1
Column 2	3.538	1.5
	¶= 0.05	
Quantity	D_d	~
Column 1	1.795	1.6
Beam	1.465	1.1
Column 2	3.189	1.5

Figure 11: The Effect of Rayleigh Damping on the Magnification Factor of Clamped-Clamped Beam

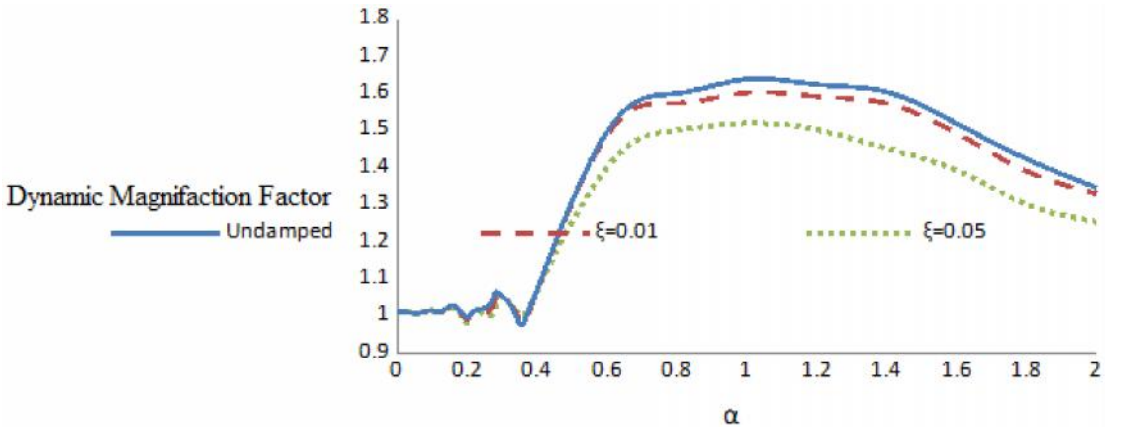


Figure 12: The Effect of Rayleigh Damping on the Dynamic Magnification Factor for the Midpoint of Column 1 of the Frame Structure

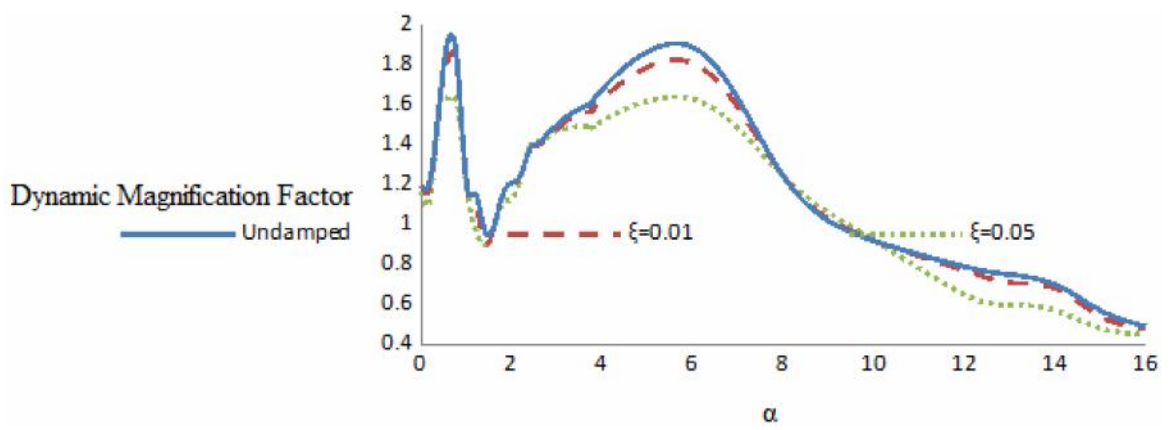


Figure 13: The Effect of Rayleigh Damping on the Dynamic Magnification Factor for the Midpoint of Beam of the Frame Structure

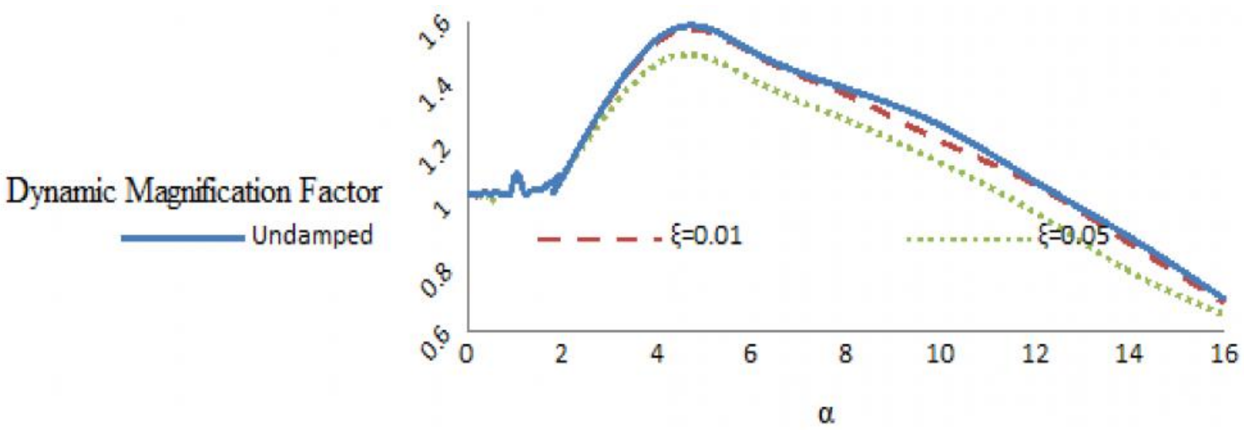


Figure 14: The Effect of Rayleigh Damping on the Dynamic Magnification Factor for the Midpoint of Column 2 of the Frame Structure

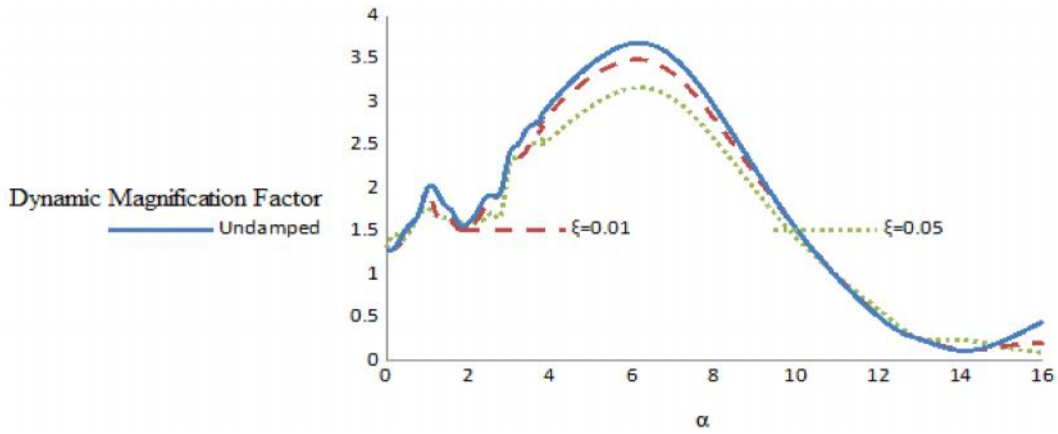


Figure 15: The Effect of Rayleigh Damping on the Dynamic Magnification Factor for the Midpoint of Column 1 of the Spring Attached (k2) Frame Structure

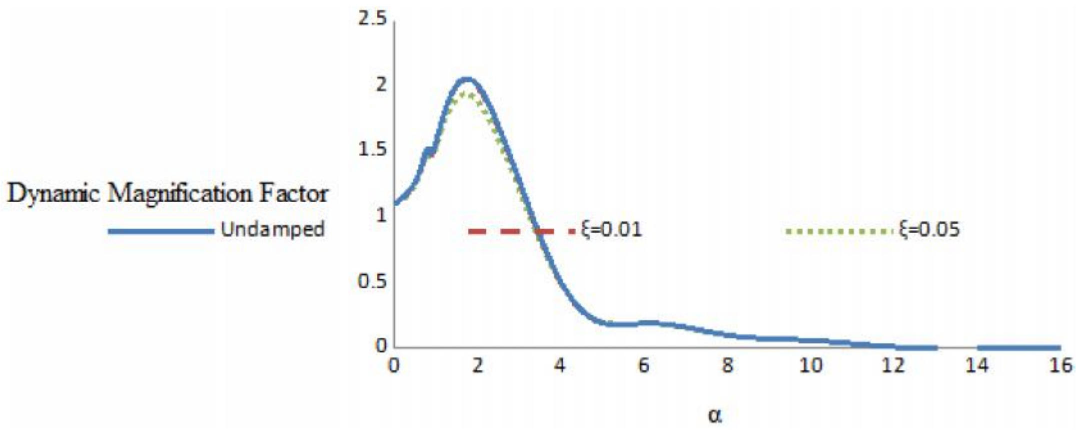


Figure 16: The Effect of Rayleigh Damping on the Dynamic Magnification Factor for the Midpoint of Beam of the Spring Attached (k2) Frame Structure

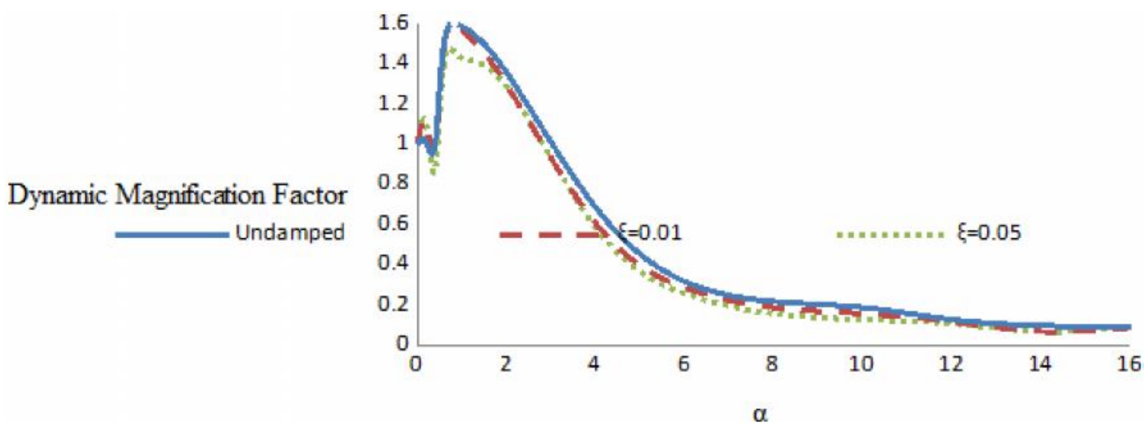
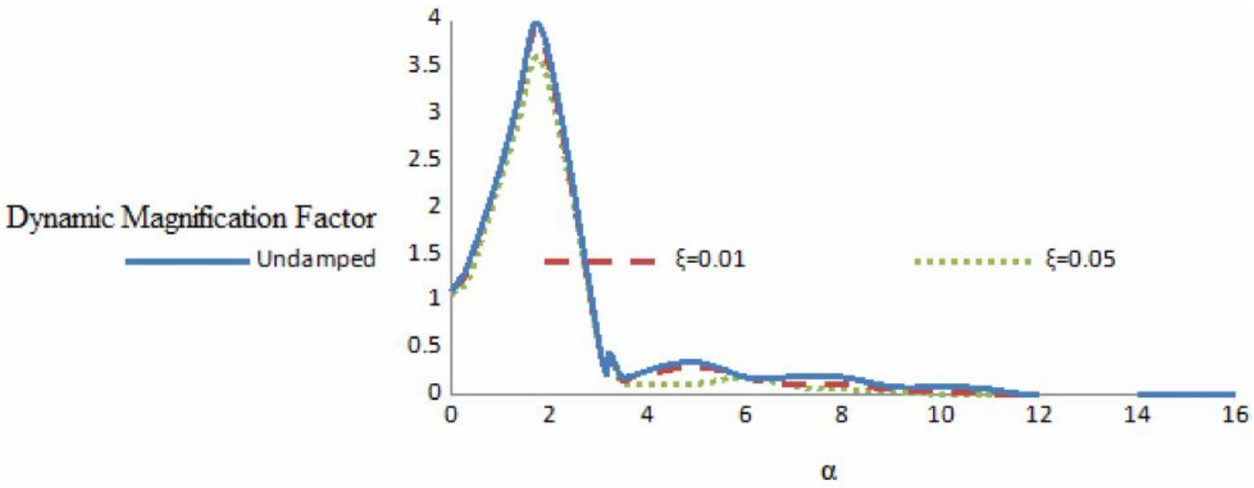


Figure 17: The Effect of Rayleigh Damping on the Dynamic Magnification Factor for the Midpoint of Column 2 of the Spring Attached (K_2) Frame Structure



for different damping ratios are given in Table 2. Table 3 shows the dynamic magnification factor values for the mid-points of beam and columns of spring attached (k2) frame with and without damping effect. Similar to frame structure, smaller dynamic displacements are observed with increasing damping ratio. Negligible difference is observed at the occurring time of the maximum D_d values for the columns of the spring attached frame. The occurring time of maximum dynamic displacement shifts left for the beam of the spring attached frame similar to frame structure.

CONCLUSION

Moving load problem is generally studied for beam structures. In addition to the beam structures, dynamic responses of frames and spring attached frames subjected to the moving point load are also analyzed in this study. Euler-Bernoulli beam theory is used in the finite element method for constituting the element matrices. The New mark integration method is employed for forced vibration analysis. The conclusions drawn can be summarized as follows:

1. The moving load and the maximum dynamic displacements for the mid-point of the beam are not in the same phase at overcritical part. The time at which the maximum mid-point displacement is observed shifts right with increasing ~ values regardless of the boundary condition of the beam.
2. The highest dynamic displacements occur for a pinned – pinned beam. For pinned – pinned boundary conditions the dynamic magnification values are greater than those obtained for clamped – clamped and clamped – pinned beams for low and high moving load speeds. The clamped – clamped boundary conditions generally gives the lower dynamic magnification values except the middle speed region.
3. Attaching a spring to the frame at the conjunction points of beam and columns makes the frame more rigid and shifts the mode shapes of the frame structure up.

4. A longer beam implies a smaller first natural frequency for frame structure; similarly longer columns imply smaller natural frequencies.
5. With lower ~ values (~ < 1) springs are very effective for all nodes. In this interval, higher D_d values are obtained with increasing spring stiffness. In the middle and high speed region, attaching a spring to the frame is not an advisable solution due to the increasing D_d values.
6. Maximum D_d occurs after the moving load left the beam for both columns and beam of the frame structure when the ~ value is greater than some critical values.
7. Lower D_d values are observed with increasing damping ratio for a clamped-clamped beam. The occurring time of maximum dynamic displacement shifts left with increasing damping ratio.
8. Maximum D_d values are observed at smaller ~ values both for the beam of the frame and spring attached frame with increasing damping ratio.

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