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Research Paper

TRANSIENT ANALYSIS OF VEHICLE PASSIVE SUSPENSION SYSTEM

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The study emphasizes transient characteristic of vehicle passive suspension system. The one-fourth suspension system called quarter car suspension system which includes sprung and un-sprung masses having components to support axle, wheel and body of vehicle. The analysis is carried out by simulation for various speeds of vehicle. Modal analysis is carried out to get frequencies of the modeled system and then harmonic response characteristics are obtained for the sprung and un-sprung masses at the various frequency ranges. The transient analysis helps to obtain the behavior of system for time varying excitations. In order to facilitate the computation from time domain to frequency domain, the steps of evaluation are programmed using MATLAB software. The transient response for different vehicle speeds for both sprung and un-sprung masses of system at the various time intervals are plotted. The effect of components of suspension system on vehicle performance is analyzed. The presented analysis sets a benchmark for evaluation of performance characteristics of active suspension system.

Keywords: Passive, Quarter suspension, Sprung mass, Un-sprung mass, Modal, Transient

INTRODUCTION

Suspension systems are designed to cushion the impact of road irregularities. The suspension system is responsible for driving comfort and safety as the suspension carries the vehicle-body and transmits all forces between body and road. Driving comfort and handling qualities of the vehicle are greatly affected by the functioning of the suspension system. The roles of a suspension system are to support the vehicle weight, to isolate the vehicle body from road disturbances, and to maintain the traction force between the tire and the road surface. The purpose of suspension system is to improve the ride comfort, road handling and stability of vehicles. The ride characteristics of passenger vehicles can be characterized by considering quarter-car model. This method has been widely used to investigate the performance of passive suspension system. The work represented here tries to analyze the effect of suspension

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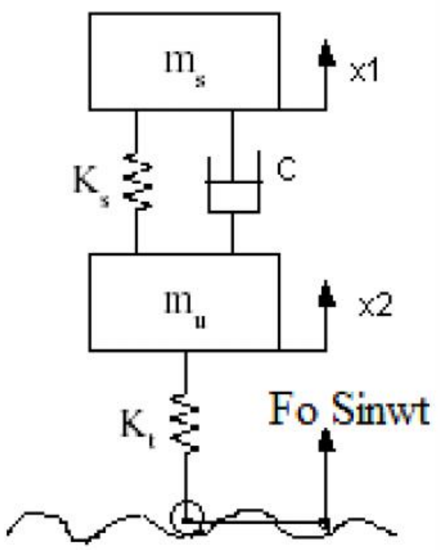
system on vehicle performance. The Vehicle performance with respect to suspension stroke is represented by the mean square relative displacement between sprung mass and un-sprung mass (Lu Sun, 2002). The objective of vehicle suspensions is to minimise the vertical forces transmitted to the passenger, and to maximise the tyre-to-road contact for handling and safety (Lakehal-Ayat *et al.*, 2002).

MATHEMATICAL MODEL

Consider the mathematical model suspension system that includes sprung mass (Ms), which is supported by the spring with spring stiffness (Ks) and a damper with damping coefficient C and un-sprung mass (Mu) with spring with spring stiffness (Kt) (Giuclea *et al.*, 2004). Assuming that it is excited by a harmonic force F_o Sinµ*t*

Figure 1: Quarter Suspension System



The equation of motion for suspension system is

$$M_s(x_1'') + C[x_1' - x_2'] + K_s[x_1 - x_2] = F_o e^{st}$$

$$M_u(x_2'') + C[x_2' - x_1'] + K_s[x_2 - x_1] + K_t.x_2 = 0$$

From the Laplace Transformation, We have

$$[M_s x_1(s)S^2] + C[Sx_1(s) - Sx_2(s)] + K_s[x_1(s) - x_2(s)]$$

$$= F(s)$$

$$x_1(s)[M_sS^2 + CS + K_s] + x_2(s)[-(K_s + CS)] = F(s)$$

After the simplification, we get

$$x_2(s)/F(s) = [CS + K_s]/(M_sM_uS^4 + S^3)\ [M_s + M_u]C + S^2[M_uK_s + M_sK_s + M_sK_t] + S[CK_t] + (K_sK_t)$$

Let us consider the vehicle parameters as

M_s = 1000 Kg, M_u = 100 Kg, K_s = 56850 N/m, K_t = 380000 N/m, C = 4523.936 N-S/m (Giuclea *et al.*, 2004)

F = 1000 N and assuming the speed of vehicle v = 40 kmph. Put these values in above equation then

$$x_2(s) = (4523936S + 56850000)/(9623610)S^4 + (4.882 \times 10^7)S^3 + (4.3412 \times 10^9)S^2 + (1.7191 \times 10^9)S + (2.1603 \times 10^{10})$$

For unit step, unit ramp and unit parabolic response

$F(s) = 1/S$, $1/S^2$ and $1/S^3$. Putting these values in the equation, we get the Output of the Transformation function.

$$x_1(s) = (M_uS^2 + CS + K_s + K_t)F(s)/(M_sM_u)S^4 + S^3(M_s + M_u)C + S^2[(M_uK_s) + (M_sK_s) + (M_sK_t)] + S(CK_t) + (K_sK_t)$$

For unit step response

$$x_1(s) = (981000)S^2 + (4523936)S + 436850000)/(9623610)S^5 + (4.882 \times 10^7)S^4 + (4.3412 \times 10^9)S^3 + (1.7191 \times 10^9)S^2 + (2.1603\text{x}1010)S$$

For unit ramp response

$x_1(s) = (981000)S^2 + (4523936)S + (436850000)/(9623610)S^6 + (4.882 \times 10^7)S^5 + (4.3412 \times 10^9)S^4 + (1.7191 \times 10^9)S^3 + (2.1603 \times 10^{10})S^2$

For unit parabolic response

$x_1(s) = (981000)S^2 + (4523936)S + (436850000)/(9623610)S^7 + (4.882 \times 10^7)S^6 + (4.3412 \times 10^9)S^5 + (1.7191 \times 10^9)S^4 + (2.1603 \times 10^{10})S^3$

RESULTS AND DISCUSSION

The MATLAB programming software is used to compute and plot either real or complex functions with respect to variables. The program is made using library function.

Determination of Frequencies

In order to determine the frequencies and to evaluate the first and second modes of vibration of the Quarter suspension system, the steps of evaluation are programmed using MATLAB 6.5 software. The following MATLAB program is fed in command window.

```
% to find roots
% ms = 1000;
% mu = 100;
% ks = 56850;
% kt = 380000;
% c = 4523.936;
syms w;
s= I*w;
%ans1 = solve('(ms*mu*s^4) + ((ms + mu)*c*s^3) + ((mu*ks + ms*ks + ms*kt)*s^2) + (c*kt*s) + (ks*kt)')
ans1 = solve('(10^5*s^4) + (4976329.6*s^3) + (442.535*10^6*s^2) + (1.719*10^9*s) + (2.1603*10^10)')
ans2 = abs (ans1)
ans2 = ans2/(2*3.142);
```

Solution

ans1 =

[–23.122376957262881811523568756271 –60.635751358559400506964340097901*I]

[–23.122376957262881811523568756271 +60.635751358559400506964340097901*I]

[–1.7592710427371181884764312437290 –6.9427799581096222006349153790075*I]

[1.7592710427371181884764312437290+ 6.9427799581096222006349153790075*I]

ans2 =

Frequencies in radians per sec

[64.894827675021987601112874562865]

[64.894827675021987601112874562865]

[7.1622083290380484806217628854572]

[7.1622083290380484806217628854572]

Frequencies in Hz

[10.326993582912474156765256932346]

[10.326993582912474156765256932346]

[1.1397530759131203820212862643948]

[1.1397530759131203820212862643948]

First Mode of Vibration

```
s =1.14;
delta = (10^5*s^4) + (4976329.6*s^3) + (442.535*10^6*s^2) + (1.719*10^9*s) + (2.1603*10^10)
delta = 2.4145e + 010
delta1 = 442.13e + 006
```

A1 = delta1/delta = 1.83e-002 radians

delta2 = 62e+006

B1 = delta2/delta = 2.56e-003 radians

Second Mode of Vibration

s = 10.33;

delta = (10^5*s^4) + (4976329.6*s^3) + (442.535*10^6*s^2) + (1.719*10^9*s) + (2.1603*10^10)

delta = 9.3207e+010

delta 1 = 494.25e+006

A2 = delta 1/delta = 5.302e-003 radians

delta 2 = 103.58e+006

B2 = delta 2/delta = 1.1113e-003 radians

Response of Sprung Mass

Transient dynamic analysis in time-history post processor is a technique used to determine the dynamic response of a structure under the action of any general time-dependent loads. It is to determine the time-varying displacements, strains, stresses, and forces in a structure as it responds to any combination of static, transient, and harmonic loads. Time scale of the loading is such that the inertia or damping effects are considered.

Unit Step Response for 1 Second

% unit step response

grid

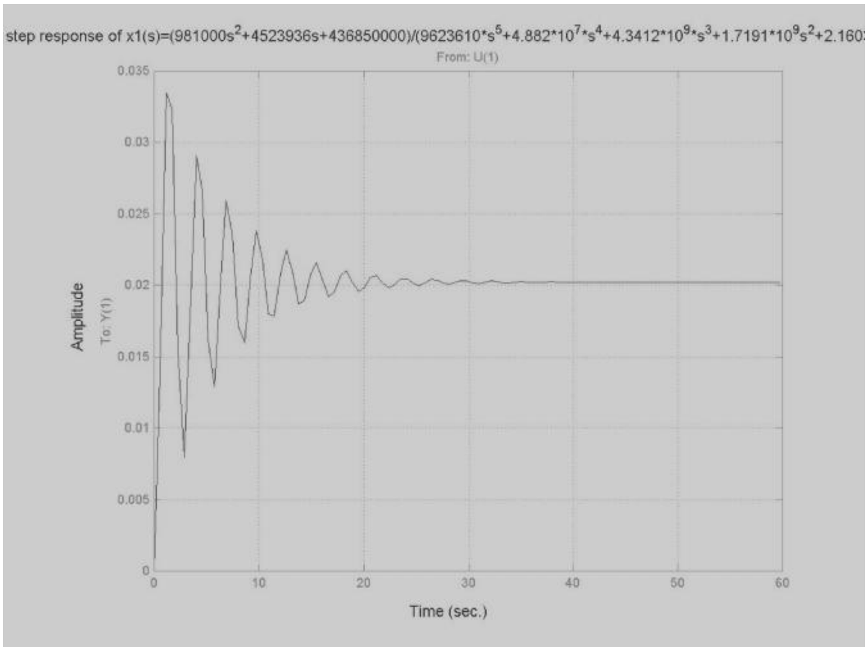
num = [0009810004523936436850000];

den = [9623610488200004341200000 17191000002160300000000];

impulse (num, den);

title ('unit step response of x1(s) = (981000s^2 + 4523936s + 436850000)/ (9623610*s^5+4.882*10^7*s^4+4.3412*10^9*s^3) + (1.7191*10^9s^2+2.1603*10^10s')

Figure 2: Transient and Steady State Response



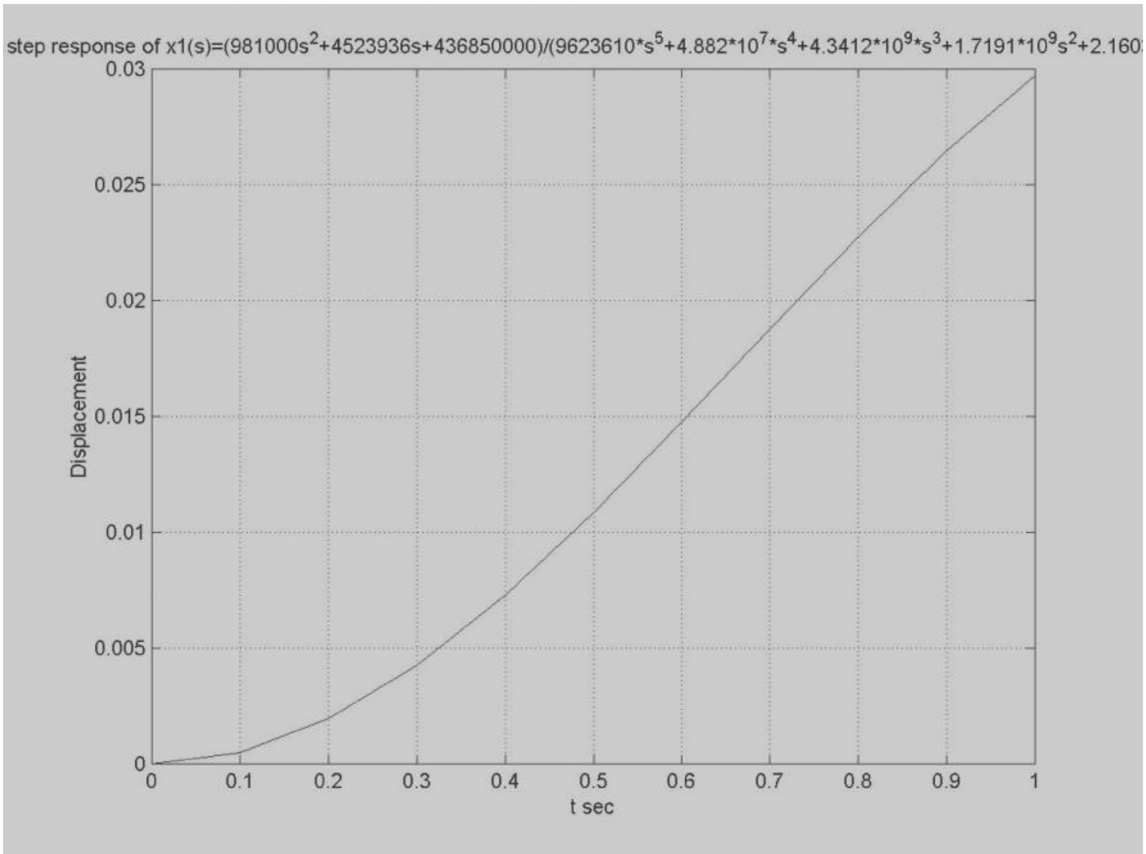
Unit Step Response for Time Interval 0 to 1 Second

```
% unit step response
t = 0:.1:1;
num = [0009810004523936436850000];
den = [96236104882000043412000001719100000216030000000];
c = impulse(num, den, t);
plot (t, c);
grid
xlabel ('t sec');
ylabel ('Displacement');
title ('unit step response of x1(s) = (981000s^2 + 4523936s + 436850000)/(9623610*s^5 + 4.882*10^7*s^4 + 4.3412*10^9*s^3 + 1.7191*10^9s^2 + 2.1603*10^10s')
```

Unit Ramp Response

```
% unit ramp response
t = 0:.1:1;
num = [00009810000452393643685 0000];
den = [962361048820000434120000017191000002160300000000];
c = impulse( num, den, t );
plot (t, c );
grid
```

Figure 3: Step Response Characteristics



xlabel ('t sec');

ylabel ('Displacement');

title ('unit ramp response of x1(s) = (981000s^2 + 4523936s + 436850000)/(9623610*s^6 + 48820000*s^5 + 4.3412*10^9*s^4 + 1.7191*10^9*s^3 + 2.1603*10^10*s^2')

Unit Parabolic Response

% unit parabolic response

t = 0:.1:1;

num = [000009810004523936436850000];

den = [962361048820000434120000017191000002160300000000];

c = impulse (num, den, t);

plot (t, c);

grid

xlabel ('t sec');

ylabel ('Displacement');

title ('unit parabolic response of x1(s) = (981000s^2 + 4523936s + 436850000)/(9623610*s^7 + 44.882*10^7*s^6 + 4.3412*10^9*s^5 + 1.7191*10^9*s^4 + 2.1603*10^10*s^3')

Response of Un-Sprung Mass

Unit Step Response

% unit step response

Figure 4: Ramp Response Characteristics

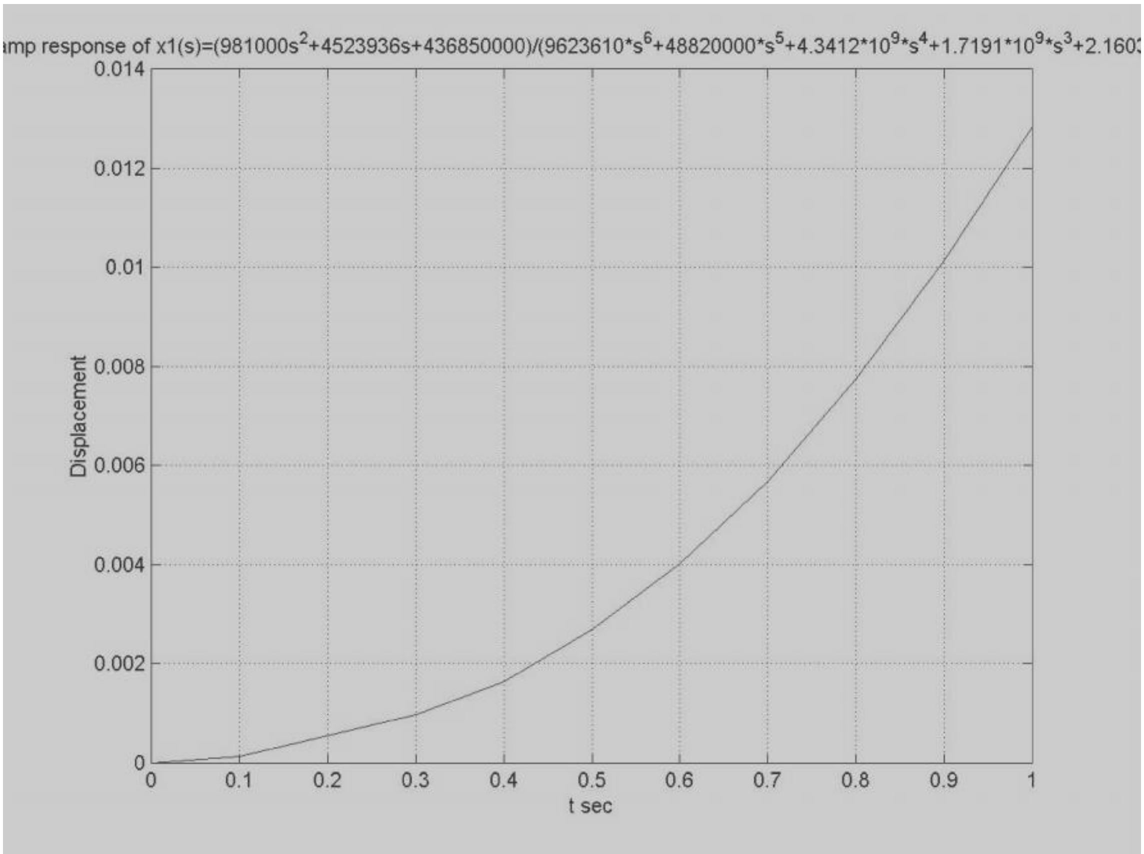
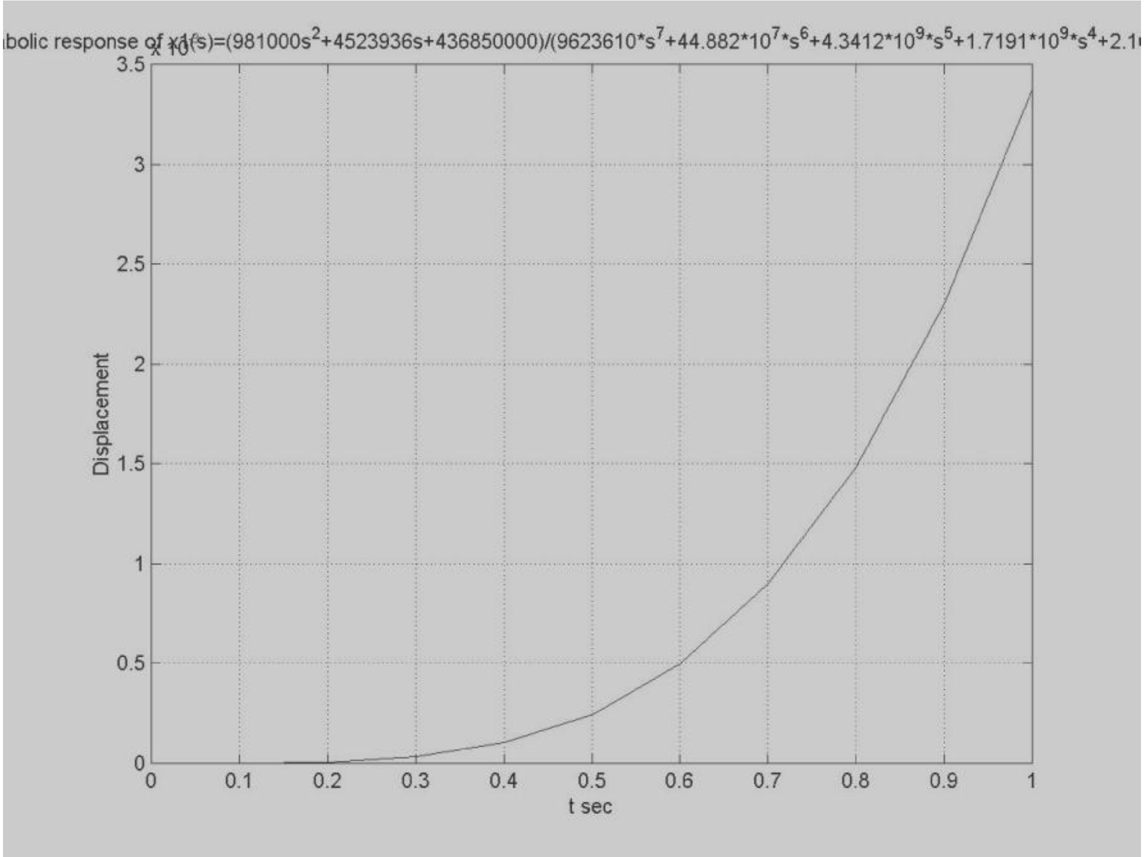


Figure 5: Parabolic Response Characteristics



t = 0:.1:1;

num = [0000452393656850000];

den = [962361048820000434120000 0171910000021603000000];

c = impulse (num, den, t);

plot (t, c);

grid

xlabel ('t sec');

ylabel ('Displacement');

title ('unit impulse response of x2(s) = (4523936s + 56850000)/(9623610*s^5 + 4.882*10^7*s^4 + 4.3412*10^9*s^3 + 1.7191*10^9s^2+2.1603*10^10s')

Unit Ramp Response

% unit ramp response

t = 0:.1:1;

num = [00000452393656850000];

den = [962361048820000434120000 0171910000021603000000000];

c = impulse (num, den, t);

plot (t, c);

grid

xlabel ('t sec');

ylabel ('Displacement');

title ('unit ramp response of x2(s) = (4523936s + 56850000)/(9623610*s^6 +

Figure 6: Step Response Characteristics

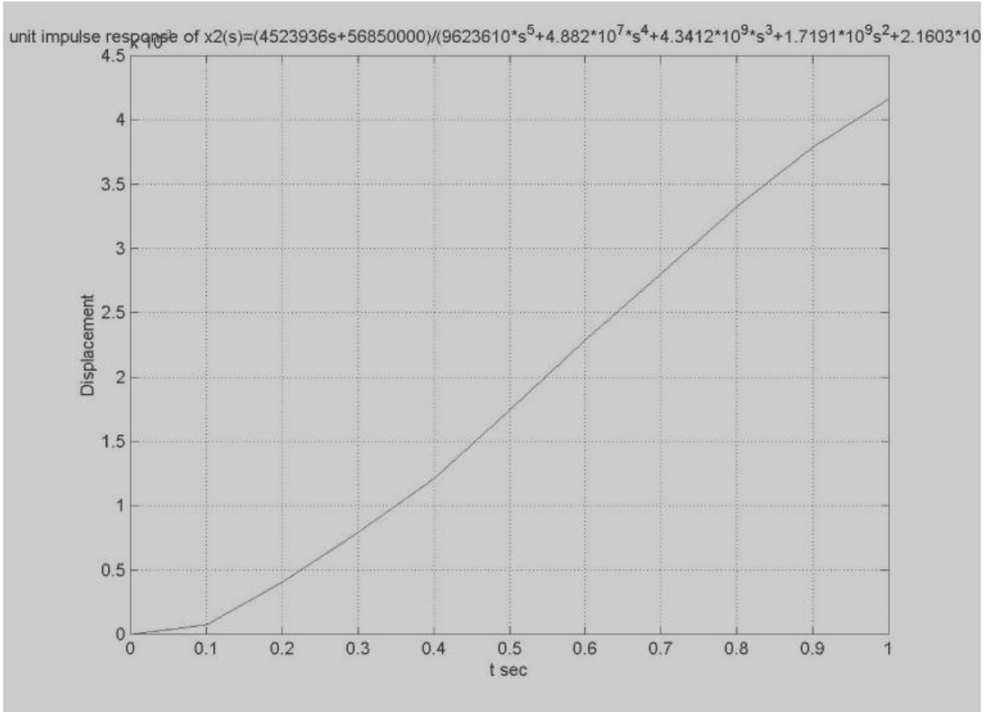
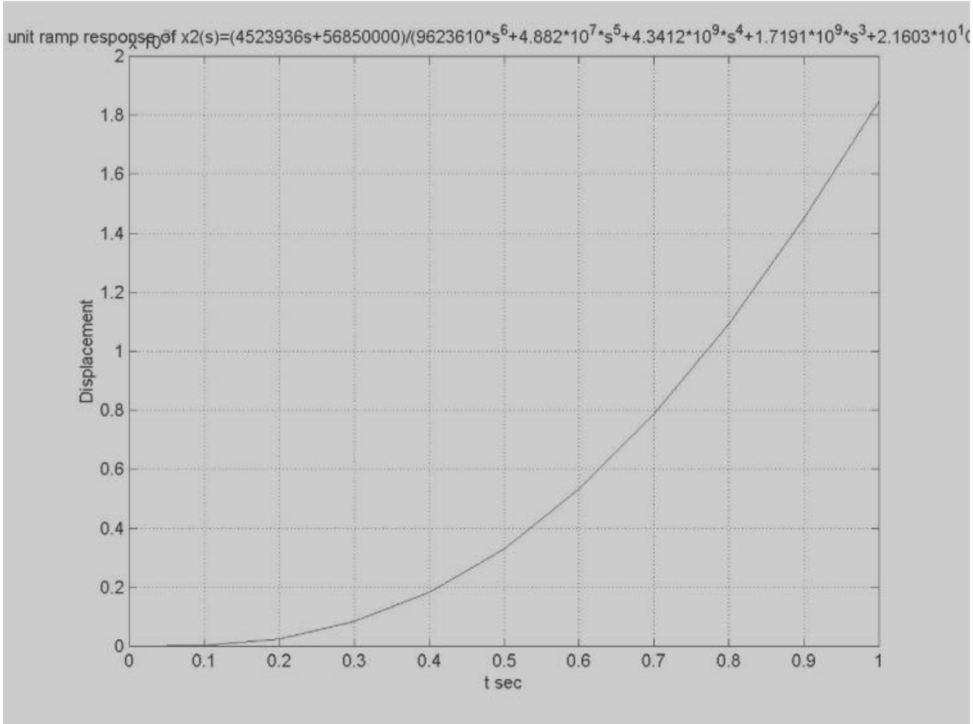


Figure 7: Ramp Response Characteristics



4.882*10^7*s^5 + 4.3412*10^9*s^4 + 1.7191*10^9*s^3+2.1603*10^10*s^2')

Unit Parabolic Response.

```
% unit parabolic response
t = 0:.1:1;
num = [000000452393656850000];
den = [962361048820000434120000017191000000216030000000000];
c = impulse (num, den, t);
plot (t, c);
grid
xlabel ('t sec');
ylabel ('Displacement');
title ('unit parabolic response of x2(s) = (4523936s + 56850000) / (9623610*s^7 + 4.882*10^7*s^6 + 4.3412*10^9*s^5 + 1.7191*10^9*s^4 + 2.1603*10^10*s^3')
```

Solutions by ANSYS Finite Element Software

Modal Analysis

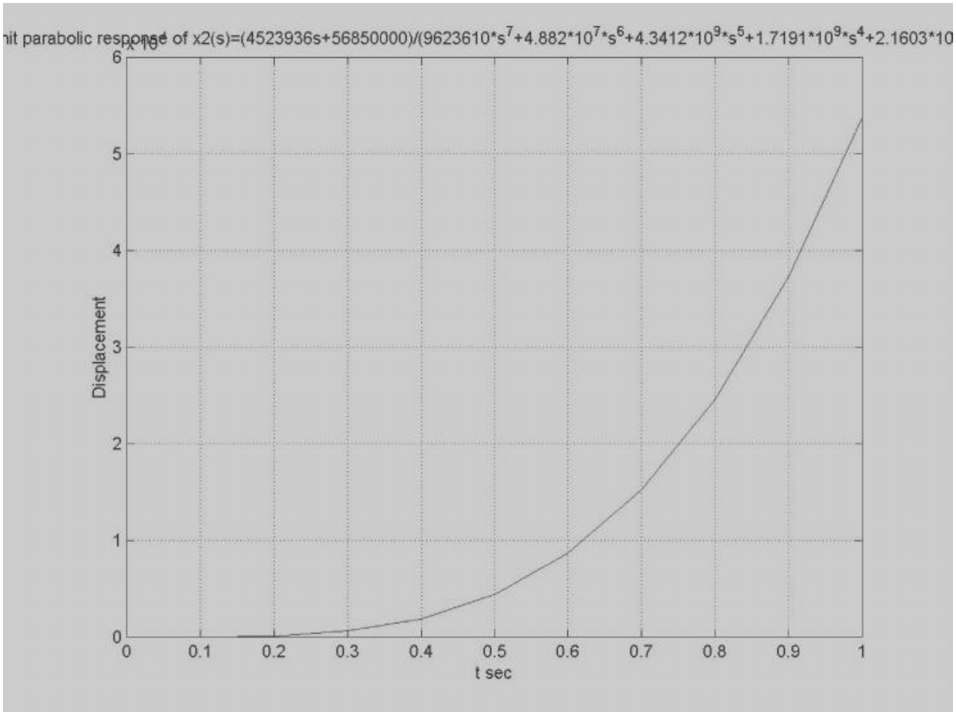
The Modal analysis is carried out on the quarter suspension system in order to determine the two frequencies of the suspension system. The natural frequencies of the modeled system which are optimal. If the system overcomes these limits then it will face harmful effects of forced vibration like resonance, fatigue and other effects of forced vibration. The modal frequencies are given below.

Set	Freq	Load step	Sub step	Cumulative
1)	1.1183 Hz	1	1	1
2)	10.528 Hz	1	2	2

Harmonic Analysis

The harmonic analysis is carried out on the quarter suspension system in order to get the

Figure 8: Parabolic Response Characteristics



variation in displacement of the components of the system with respect to the frequency range. This analysis is preceded for the different vehicle speed like 40, 50, 60 and 80 kmph because the damping factor will varies with the different speeds of the vehicle.

The following graphs show the variation of displacements of the sprung and un-sprung masses with respect to the frequency range.

Vehicle speed (v) = 40 kmph

Figure 9: Displacement verses Frequency of Sprung Mass

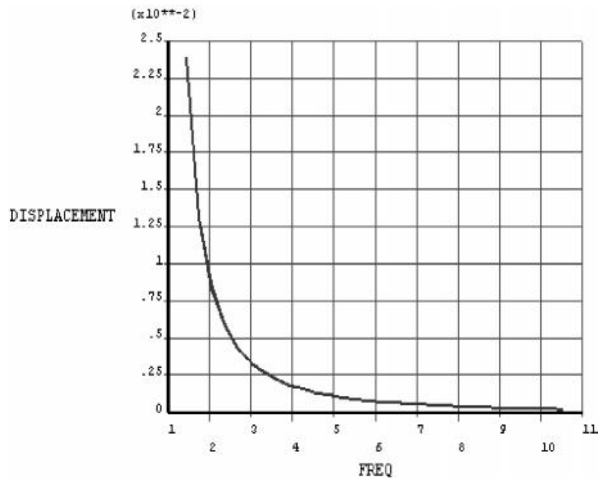
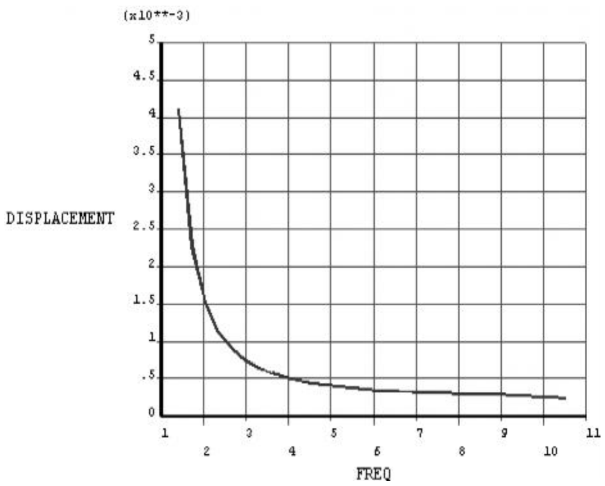


Figure 10: Displacement v/s Frequency of Un-sprung Mass



Vehicle speed (v) = 50 kmph

Figure 11: Displacement v/s Frequency of Sprung Mass

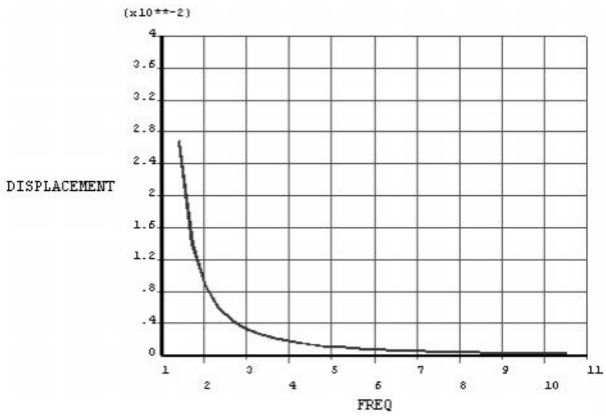
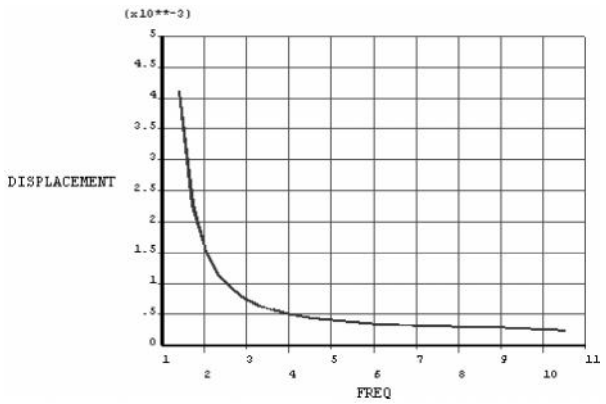


Figure 12: Displacement v/s Frequency



Vehicle speed (v) = 60 kmph

Figure 13: Displacement v/s Frequency of Sprung Mass

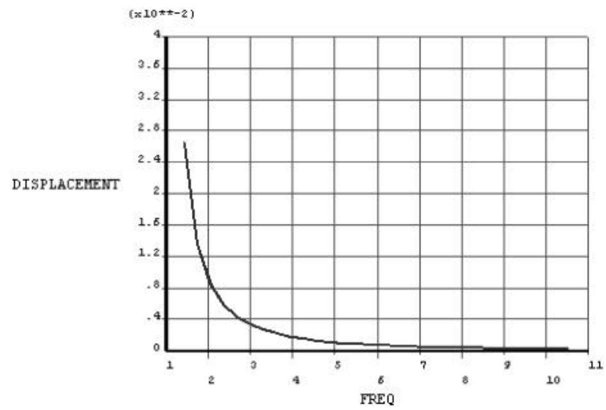
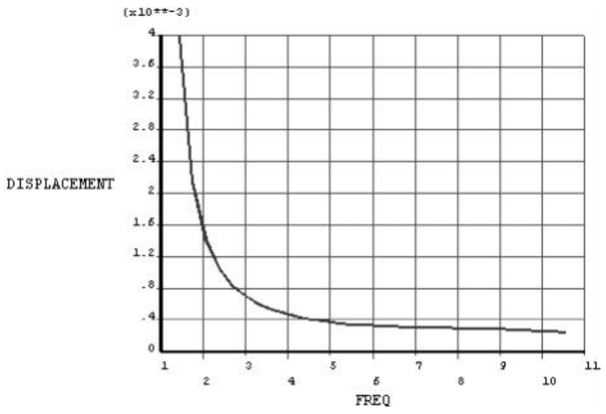


Figure 14: Displacement v/s Frequency of Un-sprung Mass



Vehicle speed (v) = 80 kmph

Figure 15: Displacement v/s Frequency of Sprung Mass

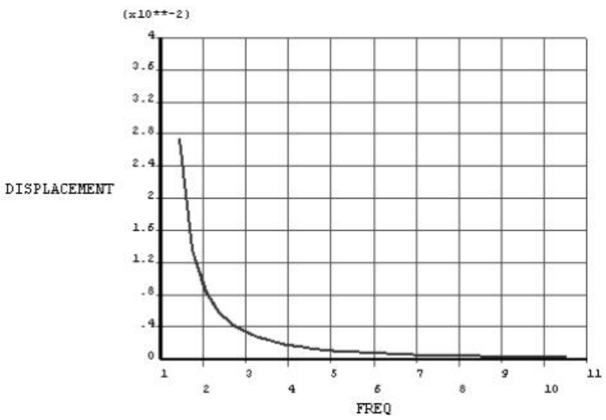
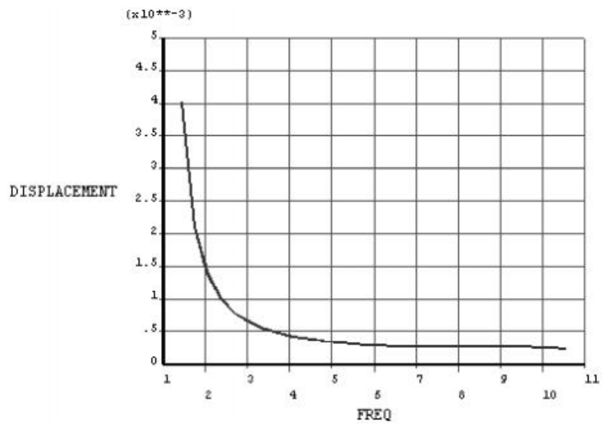


Figure 16: Displacement v/s Frequency of Un-sprung Mass



Transient Analysis

Transient analysis of the same sprung and un-sprung masses of the quarter vehicle suspension system is carried out by the method of transient analysis known as "Reduced transient dynamic analysis" to get the displacements at various time interval with different vehicle speeds.

The following graphs represent the variation in the displacement of the suspension components with respect to the time interval.

Vehicle speed (v) = 40 kmph

Figure 17: Displacement v/s Frequency of Sprung Mass

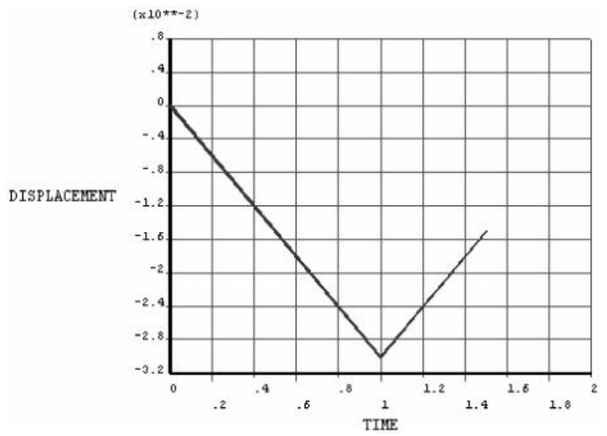
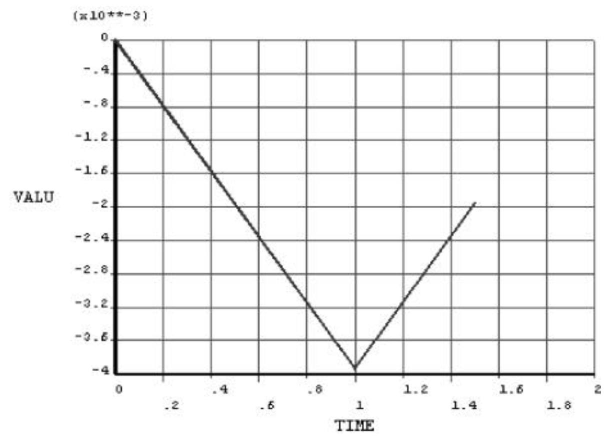


Figure 18: Displacement v/s Frequency of Un-sprung Mass



Vehicle speed (v) = 50 kmph

Figure 19: Displacement v/s Frequency of Sprung Mass

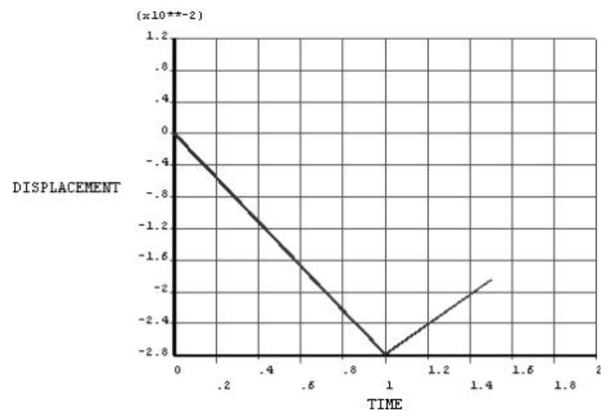
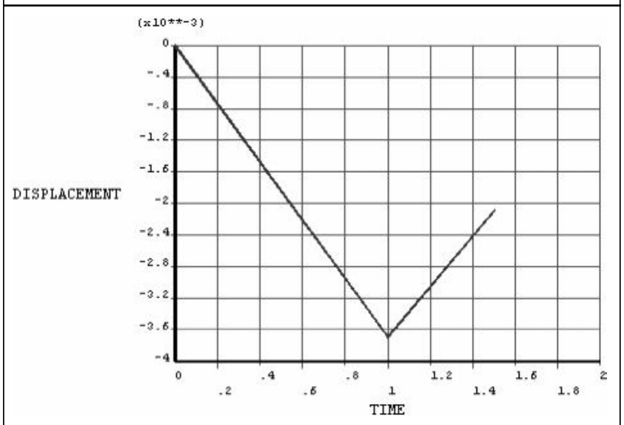


Figure 20: Displacement v/s Frequency of Un-sprung Mass



Vehicle speed (v) = 60 kmph

Figure 21: Displacement v/s Frequency of Sprung Mass

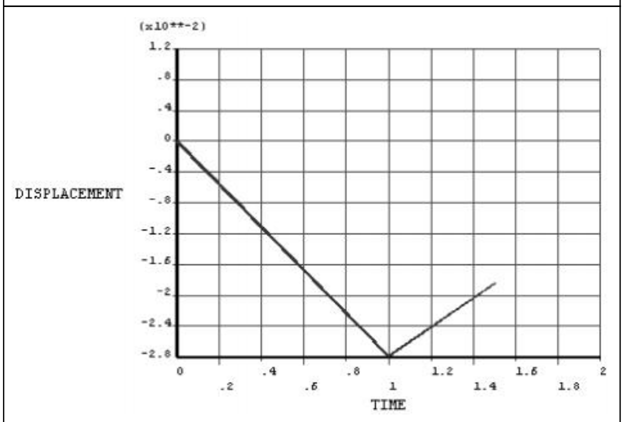
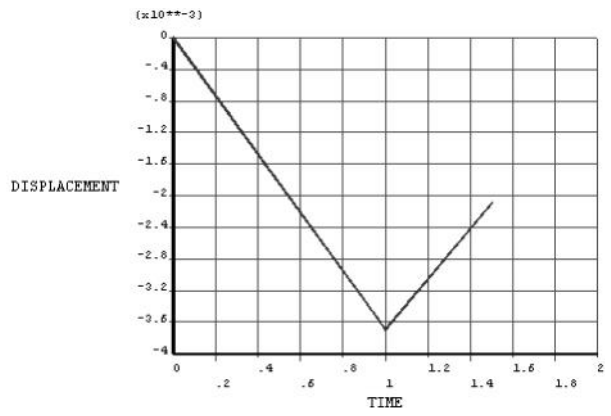


Figure 22: Displacement v/s Frequency of Un-Sprung Mass



Vehicle speed (v) = 80 kmph

Figure 23: Displacement v/s Frequency of Sprung Mass

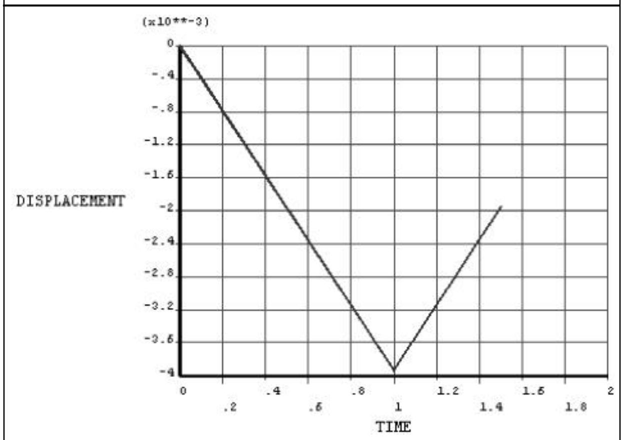
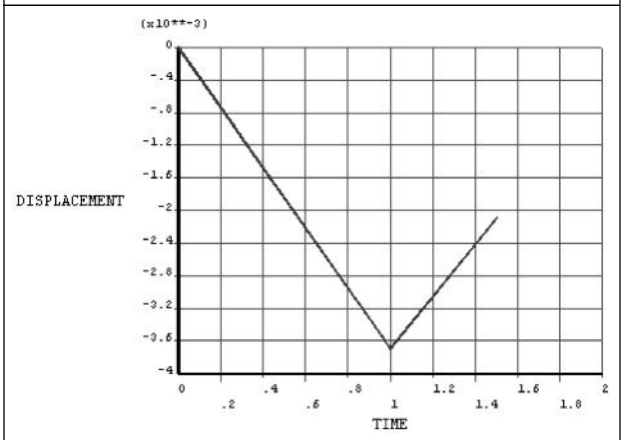


Figure 24: Displacement v/s Frequency of Un-sprung Mass



Comparison of Results by Exact Method and Finite Element Method

Comparative study is being made in between analytical and ANSYS software results for modal, harmonic and transient analysis. The ANSYS results are shown for Power Spectral Density (PSD) analysis. The results are shown below for frequencies and displacements.

Frequency

The following table gives comparison of frequencies by Exact and ANSYS software.

Table 1: Comparison of Frequencies

	Target	ANSYS	Percentage Error
F_1, H_z	1.13975	1.1183	1.88
F_2, H_z	10.32699	10.528	–1.946

Displacements at the Transient Analysis

The following table gives comparison of displacements by Exact and ANSYS software.

Table 2: Comparison of Displacements

	Target	ANSYS	Percentage Error
Sprung Mass (Ms)	0.03	0.03	1.00
Un-sprung Mass (Mu)	0.00415	0.004	3.61

Power Spectral Density (PSD)

Frequencies by Exact and ANSYS are compared and PSD displacements of the suspension components are mentioned below.

Table 3: Power Spectral Density

	Target	ANSYS	Percentage Error
Sprung Mass (Displacement Standard Deviation)	–	0.0315	–
Un-sprung Mass (Displacement Standard Deviation)	–	0.00415	–

SCOPE FOR FUTURE WORK

In case of the transient analysis by taking less than one second time, it is possible to get the more accuracy of the output. The dynamic behavior of modeled suspension system effectively evaluated by using more load steps with respect to automatic time stepping and by making variation in the load with time stepping by giving very less time interval in between the each load steps, the dynamic behavior of the system can be viewed very closely. In order to make the more effective study of the system (non-linear behavior), it is required to work on other types of spectral analysis like Single Point Response Spectrum (SPRS), Multipoint Response Spectrum (MPRS) and Dynamic Design Analysis Method (DDAM).

The non-linear buckling analysis can be done on the system to determine the buckling loads, critical loads at which the system become unstable. It is possible to get the buckled mode shapes—the characteristic shape associated with a system buckled stability response.

If COMBINATION40 is used instead of COMBINATION14, it is possible to get output statistics of the components at the end of this sub step for use in the next sub step which is used get effective transient analysis of the system.

In COMBINATION14, nodal displacements are included in the nodal solution but the COMBINATION40 has nodal displacements and rotations are included in nodal solutions.

CONCLUSION

In the present study, an effort has been made to get the transient and harmonic responses of the components like sprung and un-sprung masses of the quarter vehicle suspension system. The analysis includes for various frequency ranges of the suspension components and also for the particular time interval. Here, by using both analytical and finite element analysis is carried out to get characteristics of the sprung and un-sprung masses of the suspension system. The equations of motion the two frequencies of the systems are evaluated and computed by using the MATLab Software. The results of transient and harmonic analysis are tabulated and the results are plotted.

For a particular speed of the vehicle 40kmph is taken for instance to determine frequencies of the system, Transient and harmonic responses of the sprung and un-sprung masses are evaluated for the time interval of 1 sec.

Finally the results which are tabulated both by analytical and analysis software were found to be accurate for the various speeds of the vehicle.

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