



Research Paper

# OPTIMIZING OF MAKESPAN IN JOB SHOP SCHEDULING PROBLEM: A COMBINED NEW APPROACH

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Scheduling is the most important issue to be solved in the real time environment. One such emerging problem in the scheduling is the job shop scheduling problem, applied in various fields of engineering. The Job Shop Scheduling Problem (JSP) is one of the hardest combinatorial optimization problems. The performance of schedules released to a shop floor may greatly be affected by unexpected disruptions. The main objective of the JSP is to find a schedule of operations that can minimize the maximum completion time (called makespan) that is the completed time of carrying total operations out in the schedule of n jobs and m machines. Recently many works have been reported to reduce the makespan time in JSP. No Scheduling technique has guaranteed optimality. This paper aims at providing a well optimized scheduling technique; minimize the makespan, process time and the number of iterations. This paper proposes a Genetic algorithm with Unordered Subsequence Exchange cross-over (USXX) and Hybrid approach called a PSO-GA. This algorithm is a stochastic procedure that uses a population of solution, called particles, which move in search space. Using the special cross over technique USXX the most of the benchmark results are compared and obtain the results near to optimal value of the benchmark problems. The hybrid approach produced the better computational time compare to the GA. This approach is also applied to maximize net present value. Multiple runs of both algorithms are performed and the results are averaged in order to achieve meaningful comparisons. These finding are very promising and demonstrate the applicability of this hybrid approach for this existing problem.

Keywords: Job Shop Scheduling Problem (JSP), Makespan, Genetic algorithm, Particle swarm optimization

## INTRODUCTON

The JOB-SHOP scheduling problem (JSSP) is a very important practical problem in both

fields of production management and combinatorial optimization. Very important to bring out the efficient methods for solving the

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JSSP have significant effects on profitability and product quality. The JSSP has drawn the attention of researchers for last three decades, but because scheduling problem varies widely according to specific production tasks and most of them are strongly nondeterministic polynomial time hard problems, some test problems of moderate size are still unsolved. In fact, only small-size instances of the problems can be solved within a reasonable computational time by exact optimization algorithms such as branch and bound (Heilmann, 2003; and Akkan and Karabati, 2004) and dynamic programming (Potts and Van Wassenhove, 1987; and Lorigeon, 2002), including the benchmark instance 10 x 10 of Fisher and Thompson, which was proposed in 1963 and only solved 20 years later. Problems of dimension 15 x 15 are still considered to be beyond the reach of today's exact methods. These methods mainly include dispatching priority rules (Jensen *et al.*, 1995; Klein, 2000; and Canbolat and Gundogar, 2004), shifting bottleneck approach (Pezzella and Merelli, 2000; and Huang and Yin, 2004), Lagrangian relaxation (Kaskavelis and Caramanis, 1998; and Chen and Luh, 2003), and tabu search (Ponnambalam *et al.*, 2000; Watson *et al.*, 2003; and Geyik and Cedimoglu, 2004) and have made considerable achievement. In recently much attention has been devoted to metaheuristics with the emergence of new techniques from the field of artificial intelligence such as Genetic Algorithm (GA) (Hajri *et al.*, 2000; Gang and Wu, 2004; Amirthagadeswaran and Arunachalam, 2006; and Liu *et al.*, 2006), simulated annealing (Low *et al.*, 2004), ant colony optimization, Particle Swarm Optimization (PSO), artificial neural network,

Artificial Immune System (AIS). These metaheuristics can be regarded as problem independent approaches and are well suited to solve complex problems that may be difficult to solve by traditional technique. Moreover, they are capable of producing high-quality solutions with reasonable computational effort. Among the metaheuristics algorithms, GA has been used with increasing frequency to address scheduling problems and may not remain to have much room for improvement. However, the use of PSO and GA for the solution of scheduling problems has been scarce, specifically the PSO. In addition, many research results of the JSSP show that it is difficult to obtain a good enough solution only by single search scheme. Motivated by these perspectives, we propose a novel hybrid intelligent algorithm for the JSSP based on PSO and GA in this paper. Both PSO and GA are evolutionary computation techniques based on swarm intelligence. They exhibit implicit parallelism and contain certain redundancy and historical information of past solutions. Therefore, the proposed hybrid algorithm also effectively exploits the capabilities of distributed and parallel computing of swarm intelligence approaches.

## OVERVIEW OF JSSP

Job shop Scheduling Problem is a well known constraints satisfaction problem in the field. In a JSP we have a finite set of  $N$  jobs, where  $N = \{1, \dots, n\}$ , that have to process in a set  $M$  of machines, where  $M = \{1, \dots, m\}$ . Each and every job should divide into a series of  $m$  operations  $O_{ik}$ , where subscript  $k$  indicates the machine  $M_k$  on which the operation has to be processed. The technological order of machines, i.e., process routing for job is

predefined. We consider the static job shop scheduling case every job is a chain of operations and every operation has to be processed on a given machine for a given time. The task is to find the completion time of the very last operation is minimal.

Table 1: The Notations of Conceptual Model

| Notations  | Description                          |
|------------|--------------------------------------|
| $M$        | Number of machines                   |
| $N$        | Number of jobs                       |
| $O_{ij}$   | $j^{\text{th}}$ operation of job $i$ |
| $T_{ij}$   | Processing time of $O_{ij}$          |
| $F_{ij}$   | Finish time of $O_{ij}$              |
| $C_i$      | Completion time of job $i$           |
| $N_i$      | Number of operation of job $i$       |
| $C_{\max}$ | Maximum completion time of all job   |
| $O_{ik}$   | The start time of $j$ on machine $k$ |

Minimization of makespan:

$$f(x) = C_{\max}(O_{jk} + T_{ij}) \quad \dots(1)$$

w.r.t

$$F_{ij} \leq T_{ij+1} \quad i = 1, \dots, n_i \quad \dots(2)$$

$$Y_{mij} \leq a_{mij} \quad m = 1, \dots, M \quad \dots(3)$$

Constraint (1) specifies the makespan completion time and (2) indicates the Precedence Constraints among the operation is executed. Constraint (3) indicates that can be assigned to just one machine from among the given machines.

### Example Problem

This problem contains 3 machines ( $m$ ), 3 jobs ( $n$ ), Process routing, processing time also given. If the operation sequence is given it will work based On the Precedence constraints and capacity constraints and satisfies the

constraints of the problem also. Let we see how it works.

### Permutation Representation

$$J_1 = \{M_3, M_1, M_2\}$$

$$J_2 = \{M_2, M_3, M_1\}$$

$$J_3 = \{M_2, M_1, M_3\}$$

### Operation Sequence

$$M_1 = (J_3, J_2, J_1)$$

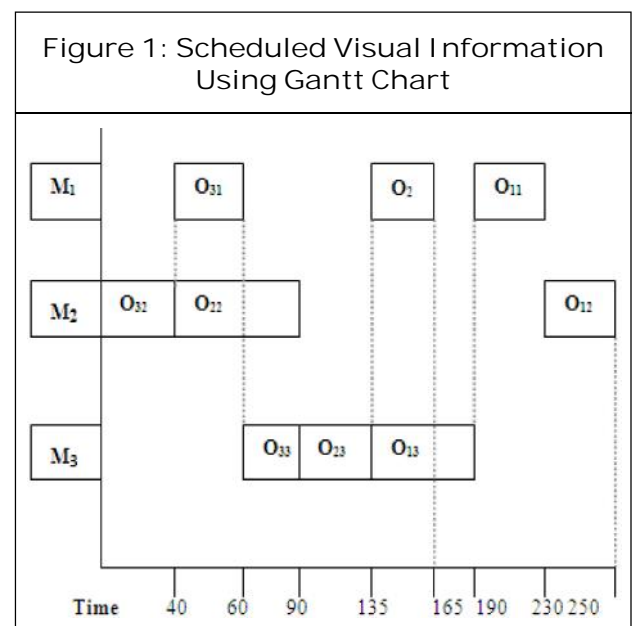
$$M_2 = (J_3, J_2, J_1)$$

$$M_3 = (J_3, J_2, J_1)$$

Table 2: Opeartion Processing Times

| Jobs  | Machines |       |       |
|-------|----------|-------|-------|
|       | $M_1$    | $M_2$ | $M_3$ |
| $J_1$ | 40       | 20    | 55    |
| $J_2$ | 30       | 50    | 45    |
| $J_3$ | 20       | 40    | 30    |

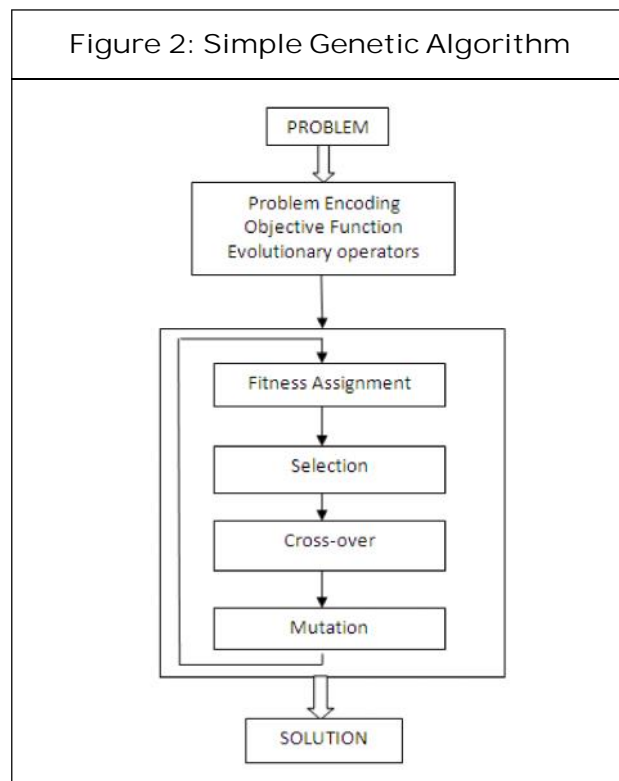
The above inputs schedule information and makespan time is calculated using the Gantt chart. For the above operation sequence or schedule and machine allocation or



permutation, with operation time, the makespan time is 250 calculated through Gantt chart. The scheduled information in Figure 1. Clearly showed satisfies the precedence and capacity constraints.

### GENETIC ALGORITHM FOR JSSP

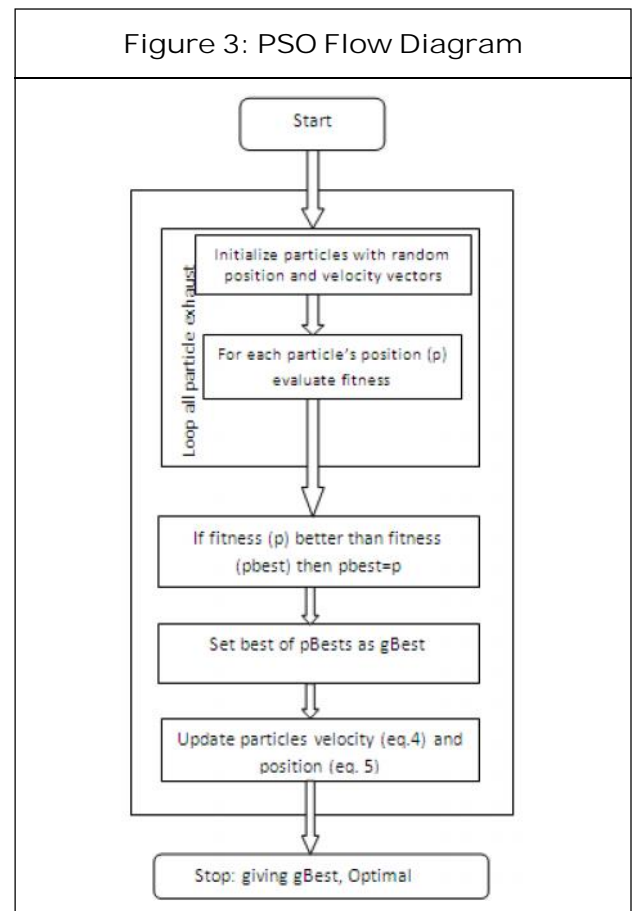
The Genetic Algorithm is a Meta heuristic technique, which may be used to solve maximization optimization problem. The genetic algorithm works based on natural populations evolve according to the principle of natural selection that is survival of the fittest, first clearly stated by Charles Darwin in the Origin of Species. It starts with initial solution called Populations and it is filled with chromosome. Each element in chromosome is called gene. Job is represented by each gene in chromosome and the job sequence in a schedule based on the position of the gene.



GA uses Crossover and Mutation operation to generate a new Population. A typical Genetic algorithm illustrated in Figure 2.

### PARTICLE SWAM OPTIMIZATION

Particle Swarm Optimization (PSO) is a population based search algorithm inspired by bird flocking and fish schooling originally designed and introduced by Kennedy and Eberhart in 1995. In contrast to evolutionary computation paradigms such as genetic algorithm, a swarm is similar to a population while a particle is similar to an individual. The fly through a multidimensional search space in which position of each particle is adjusted to its own experience and the experience of its neighbors. PSO System combines local



search methods through self experience with global search methods through neighboring experience, attempting to balance exploration and exploitation. The general flow diagram for PSO illustrated in Figure 3.

The mathematical model for calculating velocity vector and updating position as follows,

$$V_{td} = W * V_{td} + C_1 * r_1 (P_{td} - X_{td}) + C_2 * r_2 (P_{td} - X_{td}) \quad \dots(4)$$

$$X_{td} = x_{td} + V_{td} \quad \dots(5)$$

where  $W$  is the inertial weight. It is used to control the amount of the previous velocity between the global exploration abilities of the swarm.  $C_1$  and  $C_2$  are two positive constants, and they represent the weight of the acceleration terms that pull each particle toward  $P_{td}$  and  $P_{gd}$  positions,  $r_1$ ,  $r_2$  are two random functions in the range  $[0, 1]$ .

For Equation (4), in traditional PSO, Update of the velocity consists of the following three parts.  $W * V_{td}$  is referred to as “thrust” part which represents the influence of the last velocity towards the current velocity.  $C_1 * r_1 (P_{td} - X_{td})$  is the cognitive part, which represents the self thinking by itself.  $C_2 * r_2 (P_{td} - X_{td})$  is the social part, which represents the cooperation among particles. The traditional PSO resolves the simple optimization concept that individuals are evolved by cooperation and competition among the individuals to accomplish a common goal. In large search process, each particle of the swarm shares the known information globally and benefits from discoveries and previous experiences of all other social group.

The value stream map for the present state is constructed as shown in Figure 3. A first

design of VSM is realized according to the original data and the layout, identifying the key times of each assembly station. This design represents the starting point of improvement. Next, the map of the parts flow is shown to verify the materials movement between the stations, calculating the productive and unproductive times, stocks and metrics that will help to characterize the process and marking some targets of progress. Metrics used are DtD and LR (Equations 1 and 2).

The present state is based on the housing of the core components and the assembly of the engine is a push system. The majority of the raw materials is sourced from local industries and is stored in the engine storey department and during the assembly process the raw materials are brought from the stores and are washed, cleaned and is separated among the assembly line. Since the number of raw materials needed for the assembly process is unknown there is either an over stock.

## PROPOSED SCHEME

### Genetic Approach

The proposed scheme objective is to solve a job shop scheduling problem to minimize the makespan time. In order to solve a JSSP artificial intelligence technique Genetic Algorithm (GA) is used. The genetic algorithm is a probabilistic Meta heuristic technique, which is used to solve optimization problems. They are based on the genetic process of chromosome. Over many generations, natural population evolves according to the principles of natural selection that is survival of the fittest. It starts with the initial solution called population and it is filled with chromosome.

Each element in chromosome is called gene. Job is represented by each gene in chromosome and the job sequence in a schedule based on the position of the gene. In our proposed algorithm unordered subsequence exchange crossover (USXX) and shift change Mutation is used.

### Objective Function

The main objective of the JSSP is to find a schedule of operations that can minimize the maximum completion time called makespan that is the completed time of carrying total operations out in the schedule for n jobs and m machines. The objective function takes the input as the number of jobs, number of operations, chromosome, operation time sequence and machine sequence of the corresponding operation sequence.

$$C_{max} = \max_{1 \leq i \leq n} C_i \rightarrow \min \quad \dots(6)$$

### Fitness Function

The objective is to minimize the makespan, so the following fitness function is applied,

$$f(x) = \frac{1}{makespan(x)} \quad \dots(7)$$

### Chromosome Representation and Encoding Scheme

The searching space is created in  $n \times m$  dimensions space for n jobs on m machines job shop scheduling problem. The problem solution is represented as a chromosome in Genetic Algorithm. The position of a chromosome consists of  $n \times m$  dimensions and is represented with  $n \times m$  real numbers. Chromosome is initialized based on jobs and machines. Then assign the rank with respect to smaller value in the chromosome and vice versa. Decoding by using the formula,

$$(R_i \bmod n) + 1 \quad \dots(8)$$

where,

$R_i$  is integer number by ranking

$n$ , is number of jobs

From the Equation (8) proposed scheme will obtain the operation sequence. The following Table 2 shows representation of chromosome and generating the operation sequence.

|                         |          |          |          |          |          |          |
|-------------------------|----------|----------|----------|----------|----------|----------|
| Chromosome              | 4.5      | 1.3      | 2.3      | 8.2      | 4.1      | 3.5      |
| Give the Rank ( $R_i$ ) | 5        | 1        | 2        | 6        | 4        | 3        |
| $(R_i \bmod n) + 1$     | 3        | 2        | 3        | 1        | 2        | 1        |
| Operation Sequence      | $O_{31}$ | $O_{21}$ | $O_{32}$ | $O_{11}$ | $O_{22}$ | $O_{11}$ |

From the Table 2 operation sequence defines 3<sup>rd</sup> job 1<sup>st</sup> operation, 2<sup>nd</sup> job 1<sup>st</sup> operation, 3<sup>rd</sup> job 2<sup>nd</sup> operation, 1<sup>st</sup> job 1<sup>st</sup> operation, 2<sup>nd</sup> job 2<sup>nd</sup> operation, 1<sup>st</sup> job 2<sup>nd</sup> operation. This operation Processed based on Process Routing that is Precedence constraints based. That means allocated jobs are proceed as per planned without changing the job sequence.

### Algorithm Steps

**Step 1:** Initialize the number of chromosomes by generating  $n \times m$  real numbers for each chromosome.

**Step 2:** Assign the operation time sequence and Machine sequence for selected chromosomes.

**Step 3:** Find the makespan value for each and every chromosome using the objective function and also find the minimum makespan value among different values.

**Step 4:** Select  $N/2$  chromosomes using the Roulette-Wheel selection from different chromosomes.

**Step 5:** Applying the Unordered subsequence exchange crossover and shift change Mutation to generate the new chromosomes.

**Step 6:** Find the makespan values for newly generated chromosomes using the objective function.

**Step 7:** Choose the best chromosomes which have the minimum makespan values from newly generated and also from old chromosomes.

**Step 8:** Find the minimum makespan value among different chromosomes.

**Step 9:** Terminates if the maximum number of iteration is reached or optimal value is obtained.

#### Proposed Hybrid PSO-GA Algorithm

The idea of the proposed hybrid algorithm, called PSO-GA. The PSO possesses high search efficiency by combining local search and global search. By reasonably hybridizing these two methodologies, a hybrid PSO-GA algorithm model could be proposed to omit the concrete velocity-displacement updating method in traditional PSO for the JSS problem. The Proposed hybrid algorithm includes two phases: (1) the initial solutions are randomly generated as per Table 2. But assume like chromosome are now particles. (2) the PSO algorithm combined with GA algorithm is run. The General outline of the hybrid algorithm is summarized as follows:

**Step 1:** Generate initial population. If any of the generated schedules is infeasible,

reconstruct them until all the initial solutions are feasible.

**Step 2:** Evaluate each particles objective value in the swarm, and compare them, then set the pbest position with a copy of particle itself and gbest position the particle with the lowest fitness in the swarm.

**Step 3:** If the termination criterion which is usually a sufficiently good fitness or a specified number of generations are not met, then go to step 4; otherwise go to step 7.

**Step 4:** Set iteration = iteration + 1

**Step 5:** Apply Unordered Subsequence Exchange cross-over and Shift change Mutation.

**Step 6:** Convert current particles to a JSSP scheduling solution; then go to step 2.

**Step 7:** Computational results.

#### Unordered Subsequence Exchange Cross-Over

Unordered subsequence exchange crossover creates a new children's even the subsequence of parent 1 is not in the same order in parent 2. The algorithm for USXX is as follows.

**Step 1:** Select the two parents from the different chromosomes initialized with the sequence of all operation. Say P1 and P2.

**Step 2:** Generate two children from P1 and P2 name it as C1 and C2.

**Step 3:** Select the gene from P1 and same gene selected in P2 but it should unordered position in P2.

**Step 4:** Crossover started from P1 that is P2 unselected genes are move to the P1 in

unselected position. So C1 is generated. Likewise to generate C2 crossover from P2 to P1.

### Shift Change Mutation

Shift change mutation is implemented by shift the same job index in every place into the same direction so that new sequence is generated.

## EXPERIMENTAL RESULTS

The Performance of the proposed Genetic algorithm and Hybrid PSO-GA for the JSSP is examined by using some 12 test problems taken from the OR-Library. Numerical experiments are performed in Java Language on a Workstation with Intel (R) Core (TM) i5 2.50 GHz Processor and 2 GB memory. Table 4 summarizes the results of the experiments. The content of the table include the name of the test problem (instance). The size of the problem, the Best Known Value (BKV). The value of the best solution found by using Genetic Algorithm and Hybrid PSO-GA. Table 3 summarizes the parameters of the GA and PSO-GA.

From the above tabulated results it is shown that for benchmarks LA01, LA05, LA13 the Hybrid PSO-GA yields better results than GA with unordered subsequence exchange cross-over by reducing makespan.

| Parameter Name        | Value |
|-----------------------|-------|
| Swarm Size            | 20    |
| number of Particles   | 100   |
| C1 and C2             | 2.1   |
| Crossover Probability | 0.7   |
| Mutation Probability  | 0.2   |
| Number of Iterations  | 15000 |

Table 5: Comparison of Results Between the GA and PSO-GA

| Instance | Size   | Best Known Value | GA          | Proposed PSO |
|----------|--------|------------------|-------------|--------------|
| FT06     | 6 x 6  | 55               | 55          | 55           |
| LA01     | 10 x 5 | 666              | <b>739</b>  | <b>666</b>   |
| LA05     | 10 x 5 | 593              | <b>600</b>  | <b>593</b>   |
| LA06     | 15 x 5 | 926              | 926         | 926          |
| LA07     | 15 x 5 | 890              | 890         | 890          |
| LA08     | 15 x 5 | 863              | 863         | 863          |
| LA09     | 15 x 5 | 951              | 951         | 951          |
| LA10     | 15 x 5 | 958              | 958         | 958          |
| LA11     | 20 x 5 | 1222             | 1222        | 1222         |
| LA12     | 20 x 5 | 1039             | 1039        | 1039         |
| LA13     | 20 x 5 | 1150             | <b>1160</b> | <b>1150</b>  |
| LA14     | 20 x 5 | 1292             | 1292        | 1292         |

## CONCLUSION

in this paper, according to characteristics of static JSSP, a GA with Unordered subsequence exchange cross over and a Hybrid approach called PSO-GA is proposed to solve the JSSP. Two proposed algorithm are compared and evaluated. The best result is obtained by using Hybrid approach. In future the proposed algorithm can solve a Job Shop Scheduling problem in dynamic environment with multi-criteria objective. Other hybridization heuristic methods are to be used to solve the Job-Shop Scheduling Problem. 🌀

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