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Research Paper

DYNAMIC MODELING AND PATH PLANNING OF FLEXIBLE-LINK MANIPULATORS

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In this paper, dynamic modeling and path planning of flexible-link manipulators are presented. Each link of flexible manipulator is modeled by finite number of elements, and the displacement of element is formulated based on nodal coordinates and shape functions of beam element. Then, the kinetic and potential energy of the system is developed using the displacement in the reference coordinate systems. Then, by employing the Lagrange principle, the nonlinear dynamic model of the system is derived and validated.

Keywords: Flexible-link, Manipulator, Dynamic modeling, Finite element method

INTRODUCTION

Flexible-link manipulators exhibit many advantages over their traditional rigid ones: they have light weight, their motors are smaller, and their production is frugal. Because of these novel features, the application of flexible manipulators are exceedingly developed during last decades, and they have been achieved an important role in many fields of science such as surgical operation (Kumar *et al.*, 2000; and Liao *et al.*, 2008), nuclear application (Jansen *et al.*, 1991; and Perrot *et al.*, 2003), and aero space structures (Satoko and Kazuya, 2010; and Zhong, 2011). Thus, the dynamic modeling and analysis of such system is important and treated by some authors: Book (1999) analyzed the dynamic behavior of flexible manipulators based on recursive lagrangian method. Moreover, a Newton-Euler approach is presented in Rakhsha and Goldenberg (1985) to model the dynamic of a flexible robot. Meghdari and Fahimi (2001) used an analytical method to decouple the dynamic equations of elastic manipulators. Furthermore, a lumped model of a planer flexible manipulator is presented in Megahed and Hamza (2004). Singh (Megahed and Hamza, 2004) used an extended Hamilton's principle to derive the equation of motion of the flexible manipulator. Korayem and Rahimi (2011) and Korayem *et al.* (2012 and 2013) presented the dynamic

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modeling of flexible manipulator systems, based on assumed mode method. In their method, the flexible behavior of the system is modeled via eigen functions multiplied by modal coordinate of the system.

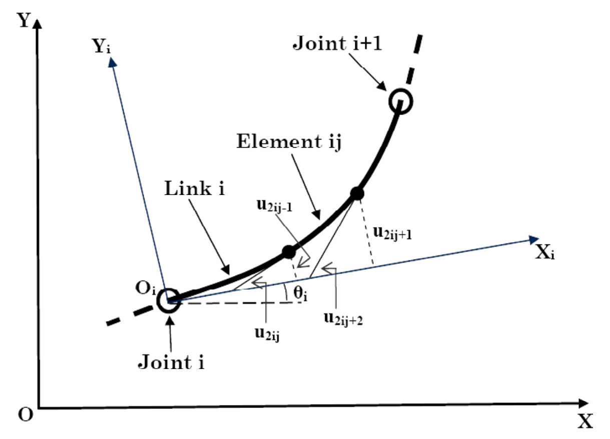
In this paper, the mathematical analysis and dynamic modeling of flexible manipulator is presented based on finite element method. Each link of the system is modeled by finite number of elements, and the displacement vector of each point of the robot is formulated in the reference coordinate by means of finite formulation of beam element. Then, the kinetic and potential energies of the system are presented, and the dynamic model of the system is derived using Lagrange principle. Finally, simulation results are presented.

FINITE ELEMENT FORMULATION FOR MATHEMATICAL MODEL OF THE SYSTEM

To present the mathematical and dynamic model of the flexible manipulators, the system with m number of links, each link is divided to n_i elements with length of l_{ij}. As the total displacement of each point of the flexible manipulator can be presented as $\vec{r}_{ij}$. According to Figure 1, the reference coordinate system is shown by *OXY*, and the local coordinate system attached to i^{th} link is assumed as $O_iX_iY_i$. The parameters of the flexible manipulator are shown as follows: *ij* is j^{th} element of i^{th} link, $\vec{r}_{ij}$ is displacement vector of element, $\vec{r}_{o_i}$ is displacement vector of ith joint, θ_i represents angular displacement of ith joint, n_i is number of elements of i^{th} link, L_i is length of i^{th} link, m_i is mass per length of ith link, g is gravitational constant of earth, l_{ij} is length of j^{th} element of ith link, E_i is elasticity modulus of i^{th} link, I_i is moment of inertia of i^{th} link, T_0^i represents rotation matrix between local and global coordinate system. Figure 1 shows a flexible-link manipulator.

To present the total displacement vector of ij^{th} element of the system in the global coordinate system, this vector is assumed as a summation of displacement of O_i, and the deflection of the link in the local coordinate $O_iX_iY_i$:

Figure 1: The Flexible Manipulator



$$\vec{r}_{ij} = \vec{r}_{ij,r} + \vec{r}_{ij,f} = \vec{r}_{O_i} + T_0^i \begin{bmatrix} (j-1)l_i + x_{ij} \\ y_{ij} \end{bmatrix} \qquad ...(1)$$

Where y_{ij} is the deflection of element due to flexibility of system in the local coordinates. By implementation of finite element method, this displacement is presumed a summation of Hermitian shape function multiplied to nodal coordinate of the element (Zienkiewicz *et al.*, 2005):

$$y_{ij}(x_{ij}, t) = \sum_{k=1}^{4} \phi_k(x_{ij})\, u_{2ij-2+k} \qquad ...(2)$$

Where ϕ_k shows the shape function and $u_{2ij-2+k}$ are the nodal coordinate of the systems, and are given as (Zienkiewicz *et al.*, 2005):

$$\phi_1(x_{ij}) = 1 - 3\frac{x_{ij}^2}{l_{ij}^2} + 2\frac{x_{ij}^3}{l_{ij}^3} \quad ...(3)$$

$$\phi_2(x_{ij}) = x_{ij} - 2\frac{x_{ij}^2}{l_{ij}} + \frac{x_{ij}^3}{l_{ij}^2} \quad ...(4)$$

$$\phi_3(x_{ij}) = 3\frac{x_{ij}^2}{l_{ij}^2} - 2\frac{x_{ij}^3}{l_{ij}^3} \quad ...(5)$$

$$\phi_4(x_{ij}) = -\frac{x_{ij}^2}{l_{ij}} + \frac{x_{ij}^3}{l_{ij}^2} \quad ...(6)$$

As the displacement vector of element is formulated, the kinetic energy of the element is stated as follows:

$$T_{ij} = \frac{1}{2}\int_0^{l_i} m_i \left[\frac{\partial \vec{r}_{ij}}{\partial t}^T . \frac{\partial \vec{r}_{ij}}{\partial t}\right] dx_{ij} \quad 0 < x_{ij} < l_{ij}$$

...(7)

If, the vectors $\vec{z}_{ij} = [\theta_i \quad u_{2ij-1} \quad u_{2ij} \quad u_{2ij+1} \quad u_{2ij+2}]^T$ and $\vec{\psi}_{ij} = [u_{2ij-1} \quad u_{2ij} \quad u_{2ij+1} \quad u_{2ij+2}]^T$ are defined, the Equation (7) can be rewritten as:

$$T_{ij} = \frac{1}{2}\dot{\vec{z}}_{ij}^T M_{ij} \dot{\vec{z}}_{ij}$$

$$M_{ij}(l,p) = \int_0^{l_i} m_i \left[\frac{\partial \vec{r}_{ij}}{\partial z_{ij_l}}\right]^T . \left[\frac{\partial \vec{r}_{ij}}{\partial z_{ij_p}}\right] dx_{ij} \quad ...(8)$$

Beside, the potential energy of the element is shown as V_{ij}, and is a summation of gravitational potential energy V_{ij_g}, and elastic potential energy V_{ij_e}:

$$V_{ij} = V_{gij} + V_{eij} \quad ...(8)$$

The gravitational potential energy is given as:

$$V_{gij} = \int_0^{l_i} m_i g [0 \quad 1] \vec{r}_{ij}\, dx_{ij} \quad ...(9)$$

And the elastic energy of the system is:

$$V_{eij} = \frac{1}{2}\int_0^{l_i} EI_i (\frac{\partial^2 y_{ij}}{\partial x_{ij}^2}) dx_{ij} = \frac{1}{2}\vec{\psi}_{ij}^T K_{ij} \vec{\psi}_{ij} \quad ...(10)$$

Where the stiffness matrix K_{ij} is presented as:

$$K_{ij} = \frac{EI_i}{l_i^3}\begin{bmatrix} 12 & 6l_i & -12 & 6l_i \\ 6l_i & 4l_i^2 & -6l_i & 2l_i^2 \\ -12 & -6l_i & 12 & -6l_i \\ 6l_i & 2l_i^2 & -6l_i & 4l_i^2 \end{bmatrix} \quad ...(11)$$

Now, the generalized coordinate vector is defined as $\vec{q}$, and the total kinetic and potential energy of the system can be written as:

$$T(\vec{q},\dot{\vec{q}}) = \sum_{i=1}^{m}\sum_{j=1}^{n_i} T_{ij} \quad V(\vec{q}) = \sum_{i=1}^{m}\sum_{j=1}^{n_i} V_{ij} \quad ...(12)$$

Then, the Lagrange function is introduced as $L(\vec{q},\dot{\vec{q}}) = T - V$, and the Lagrange' principle is developed: The principle of Lagrange for dynamic systems is expressed as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j \quad ...(13)$$

Where q_j represents the generalized coordinates, Q_j is the generalized external force. Thus, by implementation of Lagrange principle, the nonlinear dynamic equations of the system are summarized as follows:

$$M\ddot{\vec{q}} + f(\vec{q},\dot{\vec{q}}) = B\vec{\tau} \quad ...(13)$$

As in Equation (13) is presented, the nonlinear dynamic model of the system is developed, and no linearization is done. Thus, the nonlinear terms affect the dynamic of the system.

It must be noticed that for each link of the flexible manipulator, the first node is coincided on the joint of the link. Thus, these nodal coordinates are zero:

$$u_{i1}(t)=0\ ,\ u_{i2}(t)=0 \qquad ...(14)$$

DYNAMIC MODEL OF A SINGLE LINK MANIPULATOR

For a single-link flexible manipulator, as the link modeled by one element, the generalized coordinate vector of the system is $\vec{q}=[\theta_1 \ \ u_3 \ \ u_4]$, where θ_1 is angular displacement of the robot joint, *u*3 and *u*4 are the elastic deflection and slope of the end point of the flexible manipulator. Moreover, the rotation matrix of the system is:

$$T_0^1=\begin{bmatrix}\cos(\theta_1) & -\sin(\theta_1)\\ \sin(\theta_1) & \cos(\theta_1)\end{bmatrix} \qquad ...(15)$$

And the total displacement of any point of the robot is:

$$\vec{r}_1=\begin{bmatrix} x_{11}\cos(\theta_1)-\sin(\theta_1)\left((\frac{3x_{11}^2}{L1^2}-\frac{2x_{11}^3}{L1^3})u_3+(\frac{-x_{11}^2}{L1}+\frac{x_{11}^3}{L1^2})u_4\right) \\ x_{11}\sin(\theta_1)+\cos(\theta_1)\left((\frac{3x_{11}^2}{L1^2}-\frac{2x_{11}^3}{L1^3})u_3+(\frac{-x_{11}^2}{L1}+\frac{x_{11}^3}{L1^2})u_4\right)\end{bmatrix} \qquad ...(16)$$

So, the kinetic and potential energies of the single-link flexible manipulator are stated as:

$$T=\frac{1}{420}m_1L_1\left(2L_1^2\dot{\theta}_1^2u_4^2+147L_1\dot{\theta}_1\dot{u}_3-21L_1^2\dot{\theta}_1\dot{u}_4 +78\dot{\theta}_1^2u_3^2+2L_1^2\dot{u}_4^2-22L_1\dot{\theta}_1^2u_3u_4+78\dot{u}_3^2 +70L_1^2\dot{\theta}_1^2-22L_1\dot{u}_3\dot{u}_4\right) \qquad ...(17)$$

$$V_g=\frac{1}{12}m_1gL_1\left(6u_3\cos(\theta_1)-L_1u_4\cos(\theta_1)+6L_1\sin(\theta_1)\right) \qquad ...(18)$$

$$V_e=\frac{2EI_1}{L_1^3}\left(3u_3^2-3L_1u_3u_4+L_1^2u_4^2\right) \qquad ...(19)$$

Thus, the dynamic equation of the system can be derived, using Lagrange principle.

To simulate the dynamic behavior of the system, the parameters are given as: *m*1 = 5 kg, *L*1 = 1 m, *I*1 = 5e-9, *E* = 20e9 pa, *g* = 9.81. The simulation results are as follows:

Figure 2: Angular Displacement of Flexible Manipulator

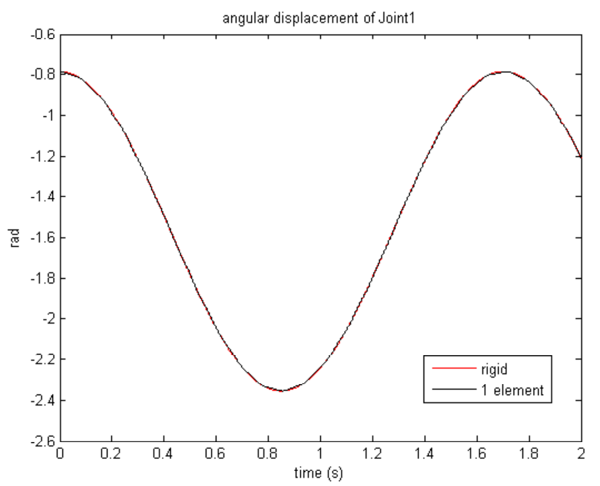
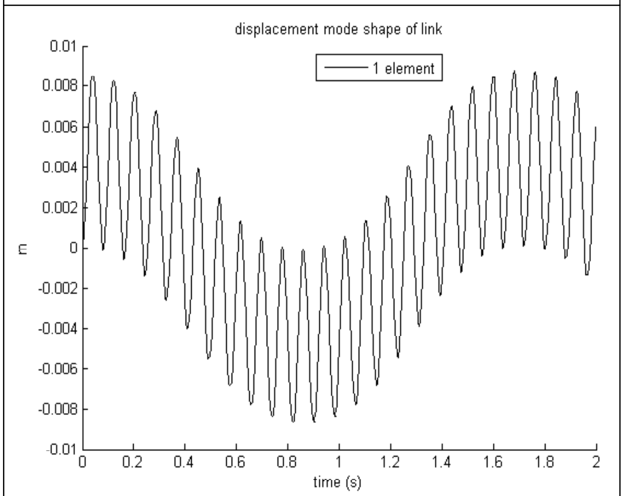


Figure 3: Displacement Mode Shape of the Flexible Manipulator



CONCLUSION

In this paper, the nonlinear dynamic analysis of the flexible manipulators has been studied using finite element method. The total displacement vector of the system has been formulated using Hermitian shape function. Hence, the total displacement of the elastic arm in reference coordinate system has been presented, the Lagrange principle has been used to derive the nonlinear dynamic motion of the elastic manipulator. Finally, the proposed method has been employed to derive the dynamic equations of a single-link manipulator, and some simulations are done.

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