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Research Paper

COMPUTATIONAL ANALYSIS OF STRESS INTENSITY FACTORS FOR A SINGLE-EDGE-NOTCH TENSION SPECIMEN BY A STRESS FUNCTION

Nandish R V^{1*}, Niranjan Pattar¹, C V Raja Reddy¹, S Paul Vizhian² and S Ramachandra³

*Corresponding Author: Nandish R V, ✉ nandish.rv@gmail.com

Cracks and flaws occur in many structures and components for several reasons. Cracks may develop during the manufacturing or later as a result of environmental conditions which can significantly degrade the structural integrity of a component under the action of applied loads. Often for the old fleet of imported aircrafts, the design data and the material data are not available. The extent of degradation of the component or system during service also has to be assessed. These issues form the basic input criteria for assessment of the damage and life extension methodology. So by determining the stress intensity factor values for different crack size in a single edge cracked specimen by theoretical and finite element method makes validation with respect to the standard problems. Since geometries with very small cracks are of particular relevance for equivalent initial flaw size, stress intensity factor values for very small cracks are of special interest, and results obtained for various crack length for the variation in load. The influence of the mesh type on the precision of the finite element method results is assessed.

Keywords: Stress intensity factor, Fracture mechanics, Single edge notch tension, Crack

INTRODUCTION

Fracture mechanics deals with the study of propagation of cracks or flaws in a structure under applied loads. It involves in analytical predictions of crack propagation and failure with experimental results. Calculating fracture

parameters such as Stress Intensity Factor (SIF) in the crack region, which can be used to estimate crack growth rate. The crack length increases with each application of some cyclic load, environmental conditions such as temperature or extensive exposure to radiation

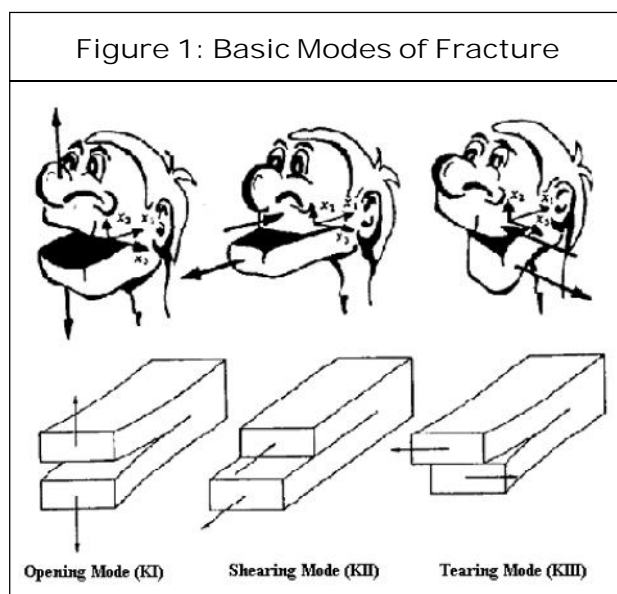
¹ Department of Mechanical Engineering, The Oxford College of Engineering, Bengaluru 560068, India.

² Department of Mechanical Engineering, University Visvesvaraya College of Engineering, Bengaluru 560001, India.

³ Scientist 'F' Grade, Gas Turbine Research Establishment, Bengaluru 560093, India.

can affect the fracture propensity of a given material. A concept of applied mechanics develops an understanding of stress and deformation fields around crack tip. It can lead to catastrophic failure of the structure. Structural design concepts traditionally use a strength-of-material approach where, as in fracture mechanics, accounts for the cracks or flaws in a structure. It includes flaw size as one variable, and fracture toughness replaces strength of material as a relevant material parameter. Sound knowledge of stress and deformation fields helps in prediction of safe life designs (Anderson, 1994).

The concepts of fracture-mechanics are widely used, but are not only limited in the fields of aerospace and mechanical engineering. The critical value of the amplitude of the stress and deformation fields characterizes the fracture toughness in stress-intensity-factor criterion, only under certain circumstances both criteria are equivalent. Depending on the relative movement of the two surfaces of the crack, three fracture modes exist as shown in Figure 1. First mode is opening or tensile,



second, shearing or sliding and third tearing or out of plane mode respectively. Fracture is generally characterized by a combination of fracture modes. Typical fracture mechanics parameters describe either the energy-release rate or the amplitude of the stress and deformation fields ahead of the crack tip. Stress-intensity factor, Energy-release rate and J-Integral are the parameters widely used in fracture mechanics analysis. Stress intensity factors and energy release rates are limited to linear elastic fracture mechanics and J-Integral is applicable to both linear elastic and nonlinear elastic-plastic materials (Rahman et al., 2011).

In the conventional failure analysis maximum stress will be below the material strength. This maximum stress approach is usually adequate when one principal stress dominates, may not be valid when the structure undergoes multi-axial loadings. Many failure criteria are based on principal stresses, strains, or strain energy. Effective strength of material and the loading conditions can determine the effectiveness of the structure. But still in reality, many more factors are to be considered like the assembly, loading, environment, maintenance, and service life. These concerns hopefully offset by a single safety factor. To analyze the relationship amongst stresses, cracks, and fracture toughness, the knowledge of fracture mechanics is required. Recent trends of fracture research include dynamic and time-dependent fracture on nonlinear materials, related to local, global, and geometry-dependent fractures. Existing major theories are with a single-parameter approach (G, K, J, or CTOD); in reality more than one

parameter is required to describe the behavior of the crack growth.

A condition for the emergence of a discontinuity in an elastic peridynamic body is resulting in a material stability condition for crack propagation. By determining whether a small discontinuity, superposed on a possibly large deformation grows over time. Stability is to be determined by the sign of the Eigen values of a tensor field that depends only on the linearised material properties. The condition for nucleation of a discontinuity in displacement can be interpreted in terms of the dynamic stability of plane waves with very short wavelength (Silling *et al.*, 2010). Micro-mechanisms of ductile fracturing are able to examine the microstructures evolution that takes place from necking to fracture initiation and growth, which leads to a better understanding of the underpinning mechanisms of the fracturing material. The ductile fracture involves void nucleation, void growth and coalescence, creating dimpled fracture surfaces. The microstructure does not change significantly during the deformation and that there are no shear bands developing in and around the fracture zones (Amini *et al.*, 2010).

The fracture toughness of viscoelastic or hyperelastic materials depends on the surface energy required to create new crack surface at the crack tip, a significant amount of energy is dissipated through viscoelastic processes in the bulk material around the crack tip and if the crack propagates very rapidly, inertia effects will come into play and contribute to the fracture toughness (Martin, 2011).

LINEAR ELASTIC FRACTURE MECHANICS

Linear elastic fracture mechanics first assumes that the material is isotropic and linear elastic. The stress field near the crack tip is calculated using the theory of elasticity. The crack will grow when the stresses near the crack tip exceed the material fracture toughness. In linear elastic fracture mechanics, most formulas are either plane stresses or plane strains problems, associated with the three basic modes of loadings on a cracked body: opening, sliding, and tearing. Linear elastic fracture mechanics is valid only when the inelastic deformation is small compared to the crack size, called small-scale yielding. If large zones of plastic deformation develop before the crack grows, elastic plastic fracture mechanics must be considered.

The linear elastic fracture mechanics analysis can be outlined based on linear elasticity theories; the stress field near a crack tip is a function of the location, the loading conditions, and the geometry of the specimen or object. In practice, the stress intensity factor K is calculated based on the stress field at the crack tip and compared against the known fracture toughness of the material.

The crack tip stress field is a function of the location of crack, loading type, and geometry.

$$\dagger_{ij}^{Tip} \equiv \dagger_{ij}^{Tip}(Location, Loading, Geometry) \equiv \dagger_{ij}^{Tip}(r, \theta, k) \dots(1)$$

where location can be represented by r and using the polar coordinate system whereas the loading and geometry terms can be grouped into a single parameter K , called the stress intensity factor.

$$K \equiv K(\dagger_{Loading, Geometry}) \quad \dots(2)$$

The fracture toughness of a material can be obtained by experiment. It is material specific.

$$\dagger_{ij}^{Toughness} = \dagger_{ij}^{Toughness}(Material) \quad \dots(3)$$

The stress intensity factor associated with the fracture toughness of the material is called the critical stress intensity factor K_c . Where K_c is material dependent.

$$K_c \equiv K_c(Material) \quad \dots(4)$$

THEORETICAL FORMULATION

Stress Intensity Factor and Crack Tip Stresses

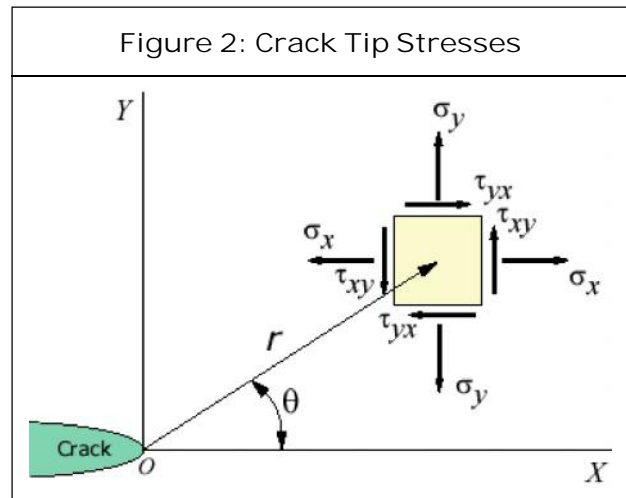
The stress fields near a crack tip of an isotropic linear elastic material can be expressed as a product of $1/\sqrt{r}$ and a function of ‘ θ ’ with a scaling factor K . Crack tips as shown in Figure 2 produce $1/\sqrt{r}$ singularity. With respect to the modes of the fracture and load acting on it, the following are the stress equations for various modes of fracture.

$$\lim_{y \rightarrow 0} \dagger_{ij}^{(I)} = \frac{K_I}{\sqrt{2fr}} f_{ij}^{(I)}(\theta) \quad \dots(5)$$

$$\lim_{y \rightarrow 0} \dagger_{ij}^{(II)} = \frac{K_{II}}{\sqrt{2fr}} f_{ij}^{(II)}(\theta) \quad \dots(6)$$

$$\lim_{y \rightarrow 0} \dagger_{ij}^{(III)} = \frac{K_{III}}{\sqrt{2fr}} f_{ij}^{(III)}(\theta) \quad \dots(7)$$

The superscripts and subscripts denote the three different modes that different loadings may be applied to a crack. Based on the linear theory the stresses at the crack tip are infinity but in reality there is always a plastic zone at the crack tip that limits the stresses to finite values. It is very difficult to model and know the actual stresses in the plastic zone and



compare them to the maximum allowable stresses of the material to determine whether a crack is going to grow or not. One can then determine the crack stability by comparing K and K_c directly. Some literature may prefer using strain energy release rate G over stress intensity factor K . These two factors are directly related by

$$G = \frac{K^2}{E} (Plane Stress) \quad \dots(8)$$

$$G = \frac{K^2}{E} (1 - \nu^2) (Plane Strain) \quad \dots(9)$$

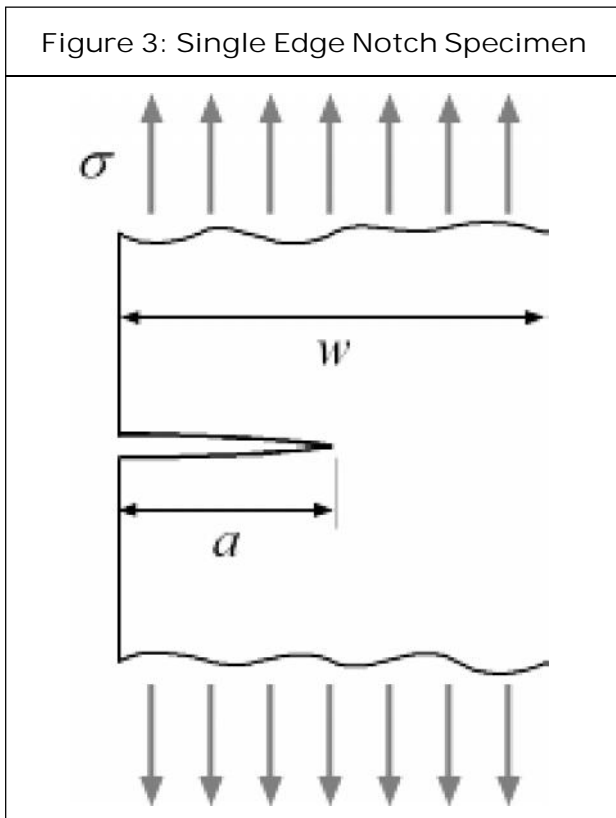
Stress and displacement fields near a crack tip of a linear elastic isotropic material are listed separately for all three modes. For linear elastic materials, the principle of superposition applies (Brown et al., 1966). A summation of each mode can be considered for the mixed-mode problems.

$$\dagger_{ij}^{(Total)} = \dagger_{ij}^{(I)} + \dagger_{ij}^{(II)} + \dagger_{ij}^{(III)} \quad \dots(10)$$

Infinite Stripe Single Edge Notch Specimen Under Tension

Stress intensity factor for the Infinite Stripe with an Edge through Cracks under Tension as shown in Figure 3 is given by (Bernard et al., 1964).

Figure 3: Single Edge Notch Specimen



$$K_I = \sqrt{r} a \left[\frac{2w \tan\left(\frac{fr}{2w}\right)}{fr} \right]^{\frac{1}{2}} \left[\frac{0.752 + 2.02 \frac{2}{w} + 0.37 \left(1 - \sin\left(\frac{fr}{2w}\right)\right)}{\cos\left(\frac{fr}{2w}\right)} \right] \dots(11)$$

This is the case has to considered as plane strain condition. The stress intensity factor is a function of a/W , where 'a' is the length of the crack and 'W' is the width of the specimen.

COMPUTATIONAL FRACTURE MECHANICS

The overall objective of fracture mechanics is the determination of the rate of change of shape of an existing crack. Crack propagates under given loading and environmental conditions with some rate and configuration. The corresponding computational requirement is to obtain the field displacement, strain rate, stress and energy from which the driving force

for crack propagation might be extracted. One can think of this classical role of computational fracture mechanics as quantifying the driving force provided by crack front fields, for crack growth. While experimental fracture mechanics traditionally quantifies the resisting force provided by the material containing the crack. However, the role of computational fracture mechanics has been expanding. Not only does it continue to encompass its classical responsibility to compute driving forces, but it is now also frequently employed to predict a material's resistance to crack growth, and even the process of crack nucleation itself (Xiaohua et al., 2010).

Finite element analysis of crack problems involving tens-of-thousands of degrees-of-freedom required mainframe computing and few minutes to complete. It is now increasingly common to apply supercomputers or large cluster computers to fracture problems involving millions of degrees-of-freedom. Simulation, the process of reproducing birth-to-death mimicry of complex physical processes, is becoming as common as analysis. The traditional process of determining is only a description of a current state. Single Edge Notch Tension (SENT) Specimen is considered with dimensions of length $L = 200$ mm, Width $W = 50$ mm and Thickness $b = 5$ mm with the crack length $a = 4, 8, 12, 16, 20, 24, 26, 30, 32, 36$ mm respectively. Specimen is of mild steel having young's modulus of 2×10^{11} N/m², poissons ratio of 0.3 and density of 7830 Kg/m³.

Validation of Theoretical Values with FEA

For the theoretical validation or benchmarking the computation results with theoretical values,

Figure 4: Stress Intensity Plot for SENT Specimen, Plane Strain Condition

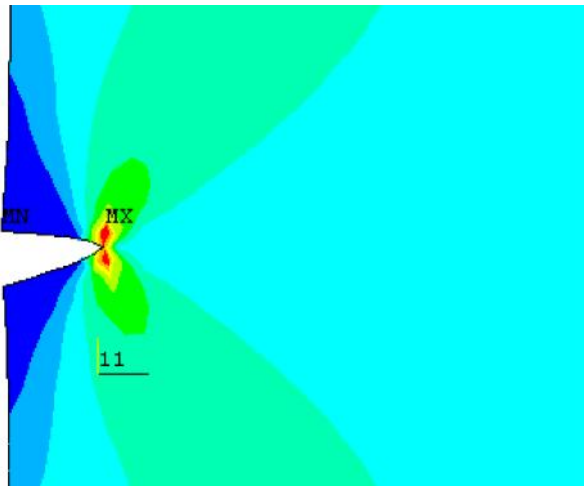
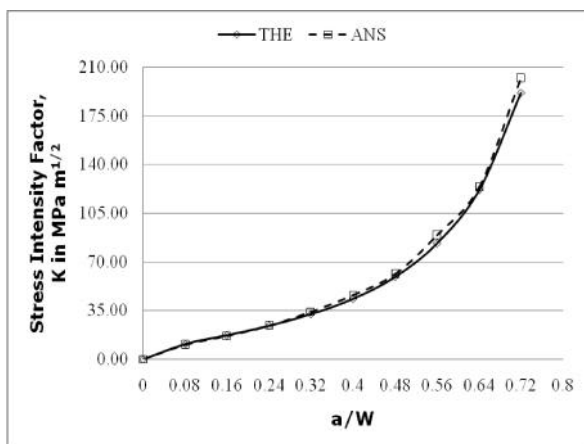


Figure 5: Comparison of Theoretical and Computation SIF for the Variation of a/W for 10 KN Load Plane Strain Condition



plane strain conditions are to be considered. Single Edge Notch Tensile specimen subjected to 10 KN load, for crack length variation of 4, 8, 12, 16, 20, 24, 28, 32, 36 mm respectively. For the variation of crack size or 'a/W' ratio the analysis is carried out for the single edge notch tension specimen and results are plotted with the theoretical values. Also stress intensity also plotted by finite element method. Figure 4 shows the stress

intensity plot for the single edge notch tension specimen subjected to 10 KN tensile load in plane strain condition and from the Figure 5 it can be observed that the stress intensity factor results are matching with the theoretical values. So it can be considered for the further analysis with real working environment.

SIF in SENT Specimen in Plane Stress Condition

Single edge notch tension specimen is considered having finite length, can be considered as subjected to plane stress condition so that computational time will be less. Stress intensity factor is found with the variation of 'a/W' ratio, for the loads of 5, 10, 15, 20, 25 KN and compared with the critical stress intensity factor for the mild steel of $140 \text{ MPa.m}^{1/2}$. Figure 6 shows the stress intensity in the single edge notch tension specimen subjected to the 10 KN load in tension in plane stress condition. From Figure 7 it can be observed that for the variation of crack size and load, stress intensity factor will increase.

Figure 6: Stress Intensity Plot for SENT Specimen with 4 mm Crack, for 10 KN Load Plane Stress Condition

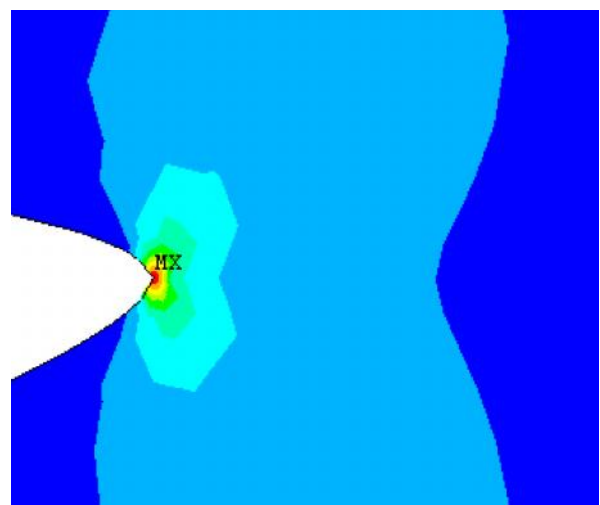
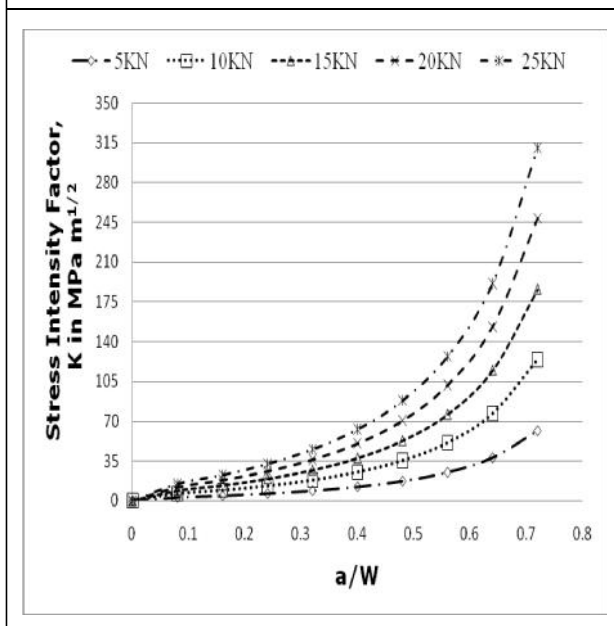



Figure 7: Stress Intensity Factor K_I v/s 'a/W' Ratio for the Variation of Load



From these plots it is easy for failure prediction in any of the component in the form of SENT specimen. This concept can be used for the replacement of the components by finding the critical crack length or 'a/w' ratio.

CONCLUSION

Finding the stress intensity factor has more significance over the traditional failure criteria, like finding the stress concentration factor or its factor of safety for the applied load. Since geometries with very small cracks are of particular relevance for equivalent initial flaw size, stress intensity factor values for very small cracks are of special interest. The stress intensity factor increases with increase in 'a/w' ratio for the varying load, and as it reaches to critical value (i.e., $140 \text{ MPa}\cdot\text{m}^{1/2}$ for steel) failure will occur. For critical value of stress intensity factor, 'a/w' ratio can be measured, intern critical crack size for the constant thickness of plate and applied load. This gives

an idea of finding the critical crack size with respect to the critical stress intensity factor of the material irrespective of the loading. 

REFERENCES

1. Amini M R and Nemat-Nasser S (2010), "Micromechanisms of Ductile Fracturing of DH-36 Steel Plates Under Impulsive Loads and Influence of Polyurea Reinforcing", *Int. J. Fract.*, Vol. 162, pp. 205-217.
2. Anderson T L (1994), *Fracture Mechanics – Fundamentals and Applications*, 2nd Edition, CRC, Boca Raton.
3. Bernard Gross, John E Srawley and William F Brown (1964), "Stress-Intensity Factors for A Single-Edge-Notch Tension Specimen by Boundary Collocation of A Stress Function", Lewis Research Center Cleueland, Ohio, National Aeronautics and Space Administration, Washington DC.
4. Brown W F and Jr Srawley J E (1966), "Plane Strain Crack Toughness Testing of High Strength Metallic Materials", ASTM STP-410.
5. Martin Kroon (2011), "Steady-State Crack Growth in Rubber-Like Solids", *Int. J. Fract.*, Vol. 169, pp. 49-60.
6. Rahman Seifi, Amir Hossein Mahmoudi and Majid Babalhavaeji (2011), "J-Integral and CMOD for External Inclined Cracks on Autofrettaged Cylinders", *Int. J. Fract.*, Vol. 169, pp. 199-212.
7. Silling S A, Weckner O, Askari E and Bobaru F (2010), "Crack Nucleation in a Peridynamic Solid", *Int. J. Fract.*, Vol. 162, pp. 219-227.

8. Xiaohua Hu, David S Wilkinson, Mukesh Jain and Raja K Mishra (2010), "A Parametric Finite Element Study and an Analytical Model of Particle Distributions on Post-Necking Deformation and Failure Mode in AA5754 Aluminum Alloy Sheets", *Int. J. Fract.*, Vol. 164, pp. 167-183.



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