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Research Paper

OPTIMIZATION OF COST MAINTENANCE AND REPLACEMENT FOR ONE-UNIT SYSTEM RELIABILITY MODEL WITH POST REPAIR

Sanjay Gupta[1]* and Suresh Kumar Gupta[2]

*Corresponding Author: Sanjay Gupta, ✉ space1_gupta@yahoo.co.in

In the global and competitive environment the System Maintenance is not an easy task. The system is designed for optimal mix, i.e., maximum impact at minimum cost and it increases the maintainability of the machines. In this study, optimizing the cost of reliability model for one-unit system having post repair, preventive maintenance and replacement has been presented. Mathematical Expressions to be work out with reliability measures along with the cost of maintenance for one-unit system of post repair with the help of regenerative point technique. In this paper, maintenance is defined in two ways, i.e., Preventive maintenance and Corrective maintenance. The cost function is developed as taking consideration for cost per unit time for preventive and corrective maintenance. Analytical study is to be presented with diagrammatic and graphical presentation with cut-off points for various rates/costs for optimization purpose.

Keywords: Optimization technique Cost of preventive maintenance, Inspection, Repair, Hardware reliability engineering, Regenerative point technique, Post inspection, Post repair, Failure, Measures of the system effectiveness

INTRODUCTION

System Components wear out the candidature for Preventive maintenance of machine to get the optimum reliability measurement. Here attempted to be carrying out for maintenance of system under replacement of component for optimization of total cost and Cost incurred for repair a component is less than the cost to replace a component when it fails, it makes sense to maintain the component preventively. This is very sensitive issue for reliability measure under optimization. The optimum preventive maintenance time can be found using the exponential/weibull distribution for measuring the reliability of distributed component of system which is commonly

[1] Department of Mechanical Engineering, UIET, M.D. University, Rohtak 124001, Harayana, India.

[2] Department of Applied Science,Vaish College of Engineering, Rohtak 124001, Harayana, India.

known as cost per unit time to maintain the component. We all know the profit to be calculated on the basis of differences of total revenue and cost incurred in replacement of component. The paper tried to optimize the cost incurred in component replacement of a system during post repair when we go through all the process of repairing policy. Graphical interpretation of post repair is to be discussed with failure rate for optimizing the cost. The availability and profit to be analyzed for post repair with restricting the failure rate.

One unit system under different failure and repair possibilities have been studied in the field of reliability by a large number of authors (Rander *et al.*, 1997; Tuteja *et al.*, 2001; Taneja *et al.*, 2004a and 2004b; Rizwan *et al.*, 2005; Said *et al.*, 2005; Haggag, 2009; Gupta and Gupta, 2013; and Hoang, 2006) and Said *et al.* (2005) have anaiysed Profit analysis of a two unit cold standby system with preventive maintenance and random change in units but they have not consider post repair with preventive maintenance. Now in this paper we are tried the present the optimum cost involved in one-unit System Reliability with Post Repair for Preventive measure as well as repair. A single repair facility is used under repair for failed unit. After the repair, the unit is sent for inspection to decide whether the repair is satisfactory or not. In case the repair is found unsatisfactory then unit is again sent for post repair. The post repair is needed only when the repair of the failed unit is found unsatisfactory when go through inspection. Expressions for reliability measures are obtained by using regenerative point technique. This paper is completed in two sections.

Assumptions and Notation

The assumptions for the proposed model are given below:

- In one-unit system, unit is operative initially.
- The system becomes inoperable on the failure of the unit in one-unit system.
- All the random variables are independent.
- The failure times are assumed to be exponentially.
- The failures are self announcing and switching is perfect and instantaneous.
- If the repair of the unit is not feasible, it is replaced by new one.

Model Formulation

Transition Probabilities and Mean Sojourn Times

The state transition diagram is shown as in Figure 1. States 1, 2, 3, 4 and 5 are failed states. The epochs of entry into states 0, 1, 2, 3, 4 and 5 are regeneration points and thus all the states are regenerative states.

The transition probabilities are given by

$$q_{01}(t) = {}^{a}e\text{-}^{a t}; \; q_{10}(t) = pg(t); \; q_{12} = qg(t);$$

$$q_{23}(t) = ah(t); \; q_{24}(t) = bh(t); \; q_{25}(t) = ch(t);$$

$$q_{30}(t) = g_1(t); \; q_{40}(t) = g_2(t); \; q_{50}(t) = g_3(t); \qquad \ldots(1)$$

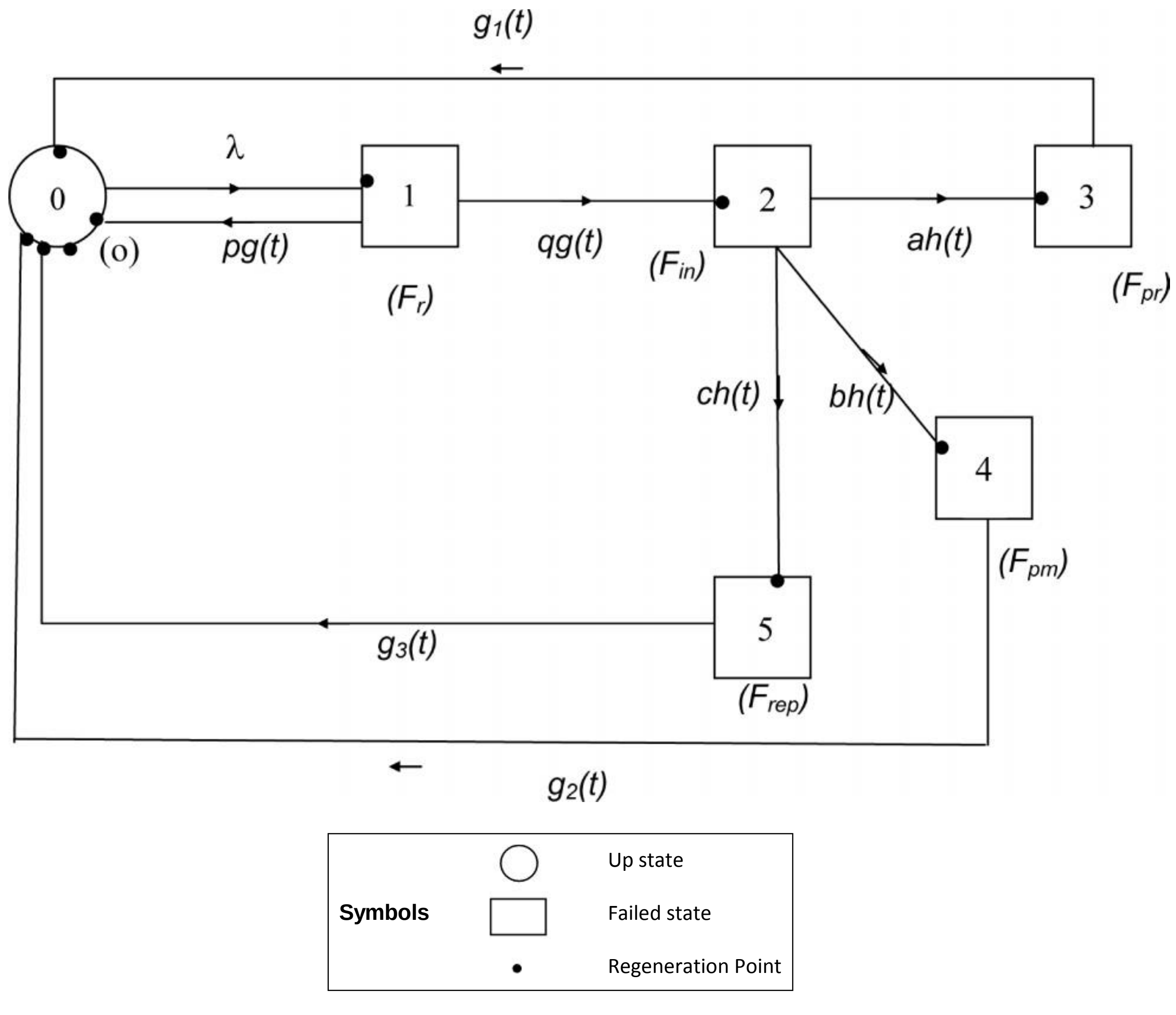
The non-zero elements $p_{ij} = \lim_{s \to 0} q_{ij}{}^*(s)$ are :

$$p_{01} = 1, \; p_{10} = p, \; p_{12} = q,$$

$$p_{23} = a, \; p_{24} = b, \; p_{25} = c,$$

$$p_{30} = 1, \; p_{40} = 1, \; p_{50} = 1 \qquad \ldots(2)$$

Figure 1: State Transition Diagram of Reliability Model



By these probabilities, it can be verified that:

$$p_{10} + p_{12} = 1,\ p_{23} + p_{24} + p_{25} = 1 \qquad \ldots(3)$$

Also $«_i$, the mean sojourn time in state *i* are:

$$«_0 = \frac{1}{a},\ «_1 = \int_0^\infty \overline{G}(t)dt = g^{*\prime}(0),\ «_2 = \int_0^\infty \overline{H}(t)dt = h^{*\prime}(0)$$

$$«_3 = \int_0^\infty \overline{G_1}(t)dt = g_1^{*\prime}(0),\ «_4 = \int_0^\infty \overline{G_2}(t)dt = g_2^{*\prime}(0)$$

$$«_5 = \int_0^\infty \overline{G_3}(t)dt = g_3^{*\prime}(0) \qquad \ldots(4)$$

The unconditional mean time taken by the system to transit for any state *j* when it has taken from epoch of entrance into regenerative state *i* is mathematically stated as:

$$m_{ij} \int_0^\infty t dQ_{ij}(t) = -\left[\frac{d}{ds} q_{ij}^{*}(s)\right] \qquad \ldots(5)$$

Thus,

$$m_{01} = «_0,\ m_{10} + m_{12} = «_1,\ m_{23} + m_{24} + m_{25} = «_2$$

$$m_{30} = «_3,\ m_{40} = «_4,\ m_{50} = «_5 \qquad \ldots(6)$$

Mean Time to System Failure

By probabilistic arguments, we obtain the following recursive relation for ¤$_i(t)$:

$$\text{¤}_0(t) = Q_{01}(t) \qquad \text{…(7)}$$

Taking Laplace-Stieltjes Transforms (L.S.T.) of above relation and solving for ¤$_0^{**}(s)$, the mean time to system failure when the system starts from the state '0' is given by

$$T_0 = \lim_{s\to 0} \frac{1-\text{¤}_0^{**}(s)}{s} = \text{«}_0 \qquad \text{…(8)}$$

Availability Analysis

Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}(t) \text{©} A_1(t)$$

$$A_1(t) = q_{10}(t) \text{©} A_0(t) + q_{12}(t) \text{©} A_2(t)$$

$$A_2(t) = q_{23}(t) \text{©} A_3(t) + q_{24}(t) \text{©} A_4(t) + q_{25}(t) \text{©} A_5(t)$$

$$A_3(t) = q_{30}(t) \text{©} A_0(t)$$

$$A_4(t) = q_{40}(t) \text{©} A_0(t)$$

$$A_5(t) = q_{50}(t) \text{©} A_0(t) \qquad \text{…(9)}$$

where,

$$M_0(t) = e^{-\text{a}t} \qquad \text{…(10)}$$

Taking Laplace Transforms (L.T.) of the above equations and solving for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \qquad \text{…(11)}$$

where,

$$N_1(s) = M_0^*(s)$$

$$D_1(s) = 1 - q_{01}^*(s)\,[q_{10}^*(s) + q_{12}^*(s)\,\{q_{23}^*(s)\,q_{30}^*(s) + q_{24}^*(s)\,q_{40}^*(s) + q_{25}^*(s)\,q_{50}^*(s)\}] \qquad \text{…(12)}$$

In steady state, the availability of the system is given by

$$A_0 = \lim_{s\to 0} sA_0^*(s) = N_1/D_1 \qquad \text{…(13)}$$

where,

$$N_1 = \text{«}_0$$

and

$$D_1 = \text{«}_0 + \text{«}_1 + q(\text{«}_2 + a\text{«}_3 + b\text{«}_4 + c\text{«}_5) \qquad \text{…(14)}$$

Busy Period Analysis of the Repairman (Repair and Post Repair Time Only)

By probabilistic arguments, we have the following recursive relation for $B_i(t)$:

$$B_0(t) = q_{01}(t) \text{©} B_1(t)$$

$$B_1(t) = W_1(t) + q_{10}(t) \text{©} B_0(t) + q_{12}(t) \text{©} B_2(t)$$

$$B_2(t) = q_{23}(t) \text{©} B_3(t) + q_{24}(t) \text{©} B_4(t) + q_{25}(t) \text{©} B_5(t)$$

$$B_3(t) = q_{30}(t) \text{©} B_0(t)$$

$$B_4(t) = q_{40}(t) \text{©} B_0(t)$$

$$B_5(t) = q_{50}(t) \text{©} B_0(t) \qquad \text{…(15)}$$

where,

$$W_1(t) = \overline{G}(t) \qquad \text{…(16)}$$

Taking Laplace Transforms of the above equations and solving them for $B_0^*(s)$, we get

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)} \qquad \text{…(17)}$$

where,

$$N_2(s) = W_1^*(s)\,q_{01}^*(s) \qquad \text{…(18)}$$

In steady-state, the total fraction of time which the system is under repair of the ordinary repairman, is given by

$$B_0 = \lim_{s \to 0} sB_0 * (s) = N_2 / D_1 \quad \ldots(19)$$

where,

$$N_2 = «_1 \quad \ldots(20)$$

and D_1 is already specified.

Busy Period Analysis of the Repairman (Inspection Time Only)

By probabilistic arguments, we have the following recursive relations for $IT_i(t)$:

$$IT_0(t) = q_{01}(t) \copyright IT_1(t)$$

$$IT_1(t) = q_{10}(t) \copyright IT_0(t) + q_{12}(t) \copyright IT_2(t)$$

$$IT_2(t) = W_2(t) + q_{23}(t) \copyright IT_3(t) + q_{24}(t) \copyright IT_4(t) + q_{25}(t) \copyright IT_5(t)$$

$$IT_3(t) = q_{30}(t) \copyright IT_0(t)$$

$$IT_4(t) = q_{40}(t) \copyright IT_0(t)$$

$$IT_5(t) = q_{50}(t) \copyright IT_0(t) \quad \ldots(21)$$

where,

$$W_2(t) = \overline{H}(t) \quad \ldots(22)$$

Taking Laplace Transforms (L.T.) of the above equations and solving them for $IT_0{*}(s)$, we get

$$IT_0 * (s) = \frac{N_3(s)}{D_1(s)} \quad \ldots(23)$$

where,

$$N_3(s) = W_2{*}(s)\, q_{10}{*}(s)\, q_{12}{*}(s) \quad \ldots(24)$$

In steady state, the total fraction of the discussion time of the expert repairman, is given by

$$IT_0 = \lim_{s \to \infty} sIT_0 * (s) = \frac{N_3}{D_1} \quad \ldots(25)$$

where,

$$N_3 = «_2 q \quad \ldots(26)$$

and D_1 is already specified.

Expected Number of Visits by the Repairman

By probabilistic arguments, we have the following recursive relations :

$$V_0(t) = Q_{01}(t) \circledS [1 + V_1(t)]$$

$$V_1(t) = Q_{10}(t) \circledS V_0(t) + Q_{12}(t) \circledS V_2(t)$$

$$V_2(t) = Q_{23}(t) \circledS V_3(t) + Q_{24}(t) \circledS V_4(t) + Q_{25}(t) \circledS V_5(t)$$

$$V_3(t) = Q_{30}(t) \circledS V_0(t)$$

$$V_4(t) = Q_{40}(t) \circledS V_0(t)$$

$$V_5(t) = Q_{50}(t) \circledS V_0(t) \quad \ldots(27)$$

Taking Laplace Stieltjes Transforms (L.S.T.) of the above equations and solving them for $V_0{**}(s)$, we get

$$V_0 ** (s) = \frac{N_4(s)}{D_1(s)} \quad \ldots(28)$$

where,

$$N_4(s) = Q_{01}{**}(s) \quad \ldots(29)$$

In steady-state, the total number of visits by the ordinary repairman per unit time is given by

$$V_0 = \lim_{t \to \infty} [V_0(t)/t] = \lim_{s \to 0} [sV_0 ** (s)] = N_4 / D_1 \quad \ldots(30)$$

where,

$$N_4 = 1 \quad \ldots(31)$$

and D_1 is already specified.

Expected Number of Preventive Maintenance

By probabilistic arguments, we have the following recursive relations:

$$PM_0(t) = Q_{01}(t) \circledS PM_1(t)$$

$PM_1(t) = Q_{10}(t) \circledS PM_0(t) + Q_{12}(t) \circledS [1 + PM_2(t)]$

$PM_2(t) = Q_{23}(t) \circledS PM_3(t) + Q_{24}(t) \circledS PM_4(t) + Q25(t) \circledS PM_5(t)$

$PM_3(t) = Q_{30}(t) \circledS PM_0(t)$

$PM_4(t) = Q_{40}(t) \circledS PM_0(t)$

$PM_5(t) = Q_{50}(t) \circledS PM_0(t)$ …(32)

Taking L.S.T.of the above equations and solving them for $PM_0{}^{**}(s)$, we get

$$PM_0{}^{**}(s) = \frac{N_5(s)}{D_1(s)} \quad \text{…(33)}$$

where,

$N_5(s) = Q_{01}{}^{**}(s)\, Q_{12}{}^{*}(s)$ …(34)

In steady-state, the total number of preventive maintenance per unit time is given by

$$PM_0 = \lim_{t\to\infty}[PM_0(t)/t] = \lim_{s\to 0}[sPM_0{}^{**}(s)] = N_5 / D_1$$

…(35)

where,

$N_5 = q$ …(36)

and D_1 is already specified.

Busy Period Analysis of Replacement Time Only

By probabilistic arguments, we have the following recursive relations:

$B_0^R(t) = q_{01}(t) \copyright B_1^R(t)$

$B_1^R(t) = q_{10}(t) \copyright B_0^R(t) + q_{12}(t) \copyright B_2^R(t)$

$B_2^R(t) = q_{23}(t) \copyright B_3^R(\mathrm{t}) + q_{24}(t) \copyright B_4^R(t) + q_{25}(t) \copyright B_5^R(t)$

$B_3^R(t) = q_{30}(t) \copyright B_0^R(t)$

$B_4^R(t) = q_{40}(t) \copyright B_0^R(t)$

$B_5^R(t) = q_{50}(t) \copyright B_0^R(\mathrm{t})$ …(37)

Taking Laplace Transforms of the above equations and solving them for $B_0^R{}^*(s)$, we get

$$B_0^R{}^*(s) = \frac{N_6(s)}{D_1(s)} \quad \text{…(38)}$$

where,

$N_6(s) = «_6 d$ …(39)

and D_1 is already specified.

Expected Number of Replacement

By probabilistic arguments, we have the following recursive relations:

$RP_0(t) = Q_{01}(t) \circledS RP_1(t)$

$RP_1(t) = Q_{10}(t) \circledS RP_0(t) + Q_{12}(t) \circledS RP_2(t)$

$RP_2(t) = Q_{23}(t) \circledS RP_3(t) + Q_{24}(t) \circledS RP_4(t) + Q_{25}(t) \circledS RP_5(t)$

$RP_3(t) = Q_{30}(t) \circledS RP_0(t)$

$RP_4(t) = Q_{40}(t) \circledS RP_0(t)$

$RP_5(t) = Q_{50}(t) \circledS RP_0(t)$ …(40)

Taking L.S.T. of the above equations and solving them for $RP_0{}^{**}(s)$, we get

$$RP_0{}^{**}(s) = \frac{N_7(s)}{D_1(s)} \quad \text{…(41)}$$

where,

$N_7(s) = Q_{01}{}^{**}(s)\, Q_{12}{}^{**}(s)$ …(42)

In steady-state, the total number of expected replacement per unit time is given by

$$RP_0 = \lim_{t\to\infty}[RP_0{}^{**}(t)/t] = \lim_{s\to 0}[sRP_0{}^{**}(s)] = N_7 / D_1$$

…(43)

where,

$$N_7 = d \quad \text{...(44)}$$

and D_1 is already specified.

COST OPTIMIZATION

Preventive and Corrective Maintenance

Preventive Maintenance (PM) and Predictive Maintenance (PDM) system optimizes Cost Analysis in a organizations. But in many cases they are static systems. Once installed little attention is given to optimizing them so they deliver the greatest reliability at least cost. The term PM as used here includes condition assessment (PDM) tasks. One way to help identify those machines that may have the wrong degree of PM is to compare the cost of corrective maintenance with the cost of preventive maintenance. As we know the traditional U-curve showing the variation of costs with amount of PM usually has a minimum in the total cost minimum is not necessarily exactly at that point but for most realistic curve is close. Most of the machines will usually be close to the one-to-one ratio, those that are not are the ones we are looking for. Start with the machines having the biggest ratio and smallest ratio. At one extreme, a few machines will have as much as 20 to 30 times as much spent on them for corrective maintenance. It is likely that these machines probably need additional PM to reduce the total cost of maintenance and improve their reliability. If these are critical machines, then you may want to consider doing a full reliability centered maintenance analysis for them.

Cost Model for Preventive and Corrective Maintenance

Consider a non-repairable component with a failure rate behavior that is independent of the age of the system in which it is installed. The component has a life described by a weibull distribution with = 3 and ¦ = 150 days. It will cost Rs. 80 each time the component is replaced after it fail when corrective maintenance start while it cost Rs. 20 to replace the component before it fails (preventive maintenance). It is the our responsibility to maintain the reliability of the engine to determine the optimum preventive maintenance time for component. The Optimum solution to this problem will be to choose the PM time that minimizes the cost per unit time. The equation describing cost per unit time is as follows:

$$CPUT = \frac{C_p R(t) + C_U [1 - R(t)]}{\int_0^t R(s) ds} \quad \text{...(45)}$$

where,

$CPUT$ = Cost per unit time

C_p = Cost of a planned (Preventive) replacement

C_U = Cost of unplanned (corrective) replacement

$R(t)$ = Reliability Function for the component

t = Preventive Maintenance time

Note that the costs used for this model can be due to a variety of causes, which include things such as monetary cost to replace the component, cost of diminished company reputation and cost of lawsuits associated with failures. The numerator of this expression represents the average cost for a single replacement. It is the cost of preventive and corrective maintenance actions weighted by the probabilities that the component will survive or not survive the PM interval. The denominator of this expression represents the average time until a single component is replaced. The expression in the denominator is not easy to explain as it is written. However, if the denominator is integrated by parts, the cost per unit time equation becomes:

$$CPUT = \frac{C_p R(t) + C_U [1 - R(t)]}{tR(t) + \int_0^t sf(s)ds} \quad \ldots(46)$$

where $f(t)$ is the probability density function (pdf) for the component. The first term is the replacement time multiplied by the probability that the component will survive until the scheduled maintenance. The second term is the expected value of the pdf on the interval from to the replacement time. In other words, it represent the expected failure time of the component that fail before the scheduled maintenance time, weighted by the percentage of the derivative of the CPUT equation setting this derivative to zero, and solving for (t).

One approach to solving this problem is to use the optimum replacement report template in Weibull. First, the components failure rate behavior must be defined in a Standard Folio. The analyst could use a Folio Containing component failure times and calculate the parameters of the pdf for the component, but for the purpose of this example, we will assume that the parameters describing the failure distribution of the component are known from a previous analysis.

Profit Analysis with Particular Case

The expected total profit incurred to the system in steady-state is given by:

Profit = Total Revenue – Total Cost

$$P = C_0 A_0 - C_1 B_0 - C_2 IT_0 - C_3 V_0 - C_4 PM_0$$

$$- C_5 B_0^R - C_6 RP_0 \quad \ldots(47)$$

where,

C_0 = Revenue per unit up time of the system

C_1 = Cost per unit time for which the repairman is busy in repair

C_2 = Cost per unit time for which the repairman is busy in inspection

C_3 = Cost per visit of the repairman

C_4 = Cost per preventive maintenance

C_5 = Cost per unit time for replacement

C_6 = Cost per visit of the repairman for replacement

Particular Case

For graphical interpretation, the following particular case is considered:

$$g(t) = \sim e^{-\sim t};\ g_1(t) = \sim_1 e^{-\sim_1 t}$$

$$g_2(t) = \sim_2 e^{-\sim_2 t};\ g_3(t) = \sim_3 e^{-\sim_3 t} \quad \ldots(48)$$

$$h(t) = \ e^{- \ t}$$

where $\sim$, $\sim_1$, $\sim_2$, $\sim_3$ and are the parameter.

Thus, we can easily obtain the following :

$p_{01} = p_{30} = p_{40} = p_{50} = 1$, $p_{10} = p$, $p_{12} = q$, $p_{23} = a$, $p_{24} = b$, $p_{25} = c$

$$«_0 = \frac{1}{a},\ «_1 = \frac{1}{\sim},\ «_2 = \frac{1}{\ },$$

$$«_3 = \frac{1}{\sim_1},\ «_4 = \frac{1}{\sim_2},\ «_5 = \frac{1}{\sim_3}, \quad \ldots(49)$$

Using the above equations (8), (13), (9), (25), (30), (35), (38), (43), (48) and (49) we can have the expressions for M.T.S.F., availability and profit for this particular case discussed in conclusion.

Mathematical Model and Optimization Technique

The Mathematical Model of Reliability is formulated (Hoang, 2006) with the help of equation (47) and certain constraint for Problem 1 and equation (50) for Problem 2. With equation (47) and assumed the initial condition according to connivance repairman under certain constraints of budget and component for formulating problem 1 is formulated and minimize it. Another attempted to be carry on minimize the total replacement cost subject to Reliability requirement like Preventive Maintenance, Post Repair and inspection of the system component for problem 2. Simulations both the Mathematical problems can be solved by software and manual calculation. The steps of maxmin concept are used for manual calculation. For this Initially separate partial derivatives to be taken with respect to revenue (C_0) various variable costs (C_1-C_6) ,and usual variable who participated in cost minimization equate to be zero. Then calculate the sensitive point after solving the complex equation and check out the initial point for judging the optimum value for preventive maintenance. For justification of the above model, next section will give the brief idea for cost-profit analysis.

The Model is formulated as:

$$\textit{Minimization } RP_0**(s) = \frac{N_7(s)}{D_1(s)}$$

Subject to:

$$PM_0**(s) = \frac{N_5(s)}{D_1(s)},$$

$$IT_0*(s) = \frac{N_3(s)}{D_1(s)} \textit{ and}$$

$$V_0**(s) = \frac{N_4(s)}{D_1(s)} \qquad \text{...(50)}$$

Graphical Presentation and Numerical Examples

Using the equations (8), (13), (9), (25), (30), (35), (38), (43), (48) and (49) and some others have been fixed as:

p = 0.5, q = 0.5, a = 0.2, b = 0.7, c = 0.1,   = 10, ~ = 0.25, ~$_1$ = 0.4, ~$_2$ = 0.35, ~$_3$ = 0.2, ª = 0.056. Now with the help of above expressions for M.T.S.F., availability and profit we obtained the values of various measures of system effectiveness are as:

Mean time to system failure (MTSF) = 150

Availability (A_0) = 0.95321

Busy period of repairman (B_0) = 0.01896

Expected Inspection time (IT_0) = 0.000243

Expected number of visits by the repairman (V_0) = 0.004865

Expected number of Preventive maintenance (PM_0) = 0.00276

Busy period of replacement (B_0^R) = 0.008751

Expected number of replacements by repairman (RP_0) = 0.000869

Graphical Interpretation

For the graphical interpretation, the mentioned particular case is considered. Figures 2 and 3 shows the behavior of MTSF and availability respectively with respect to failure rate (ª). It is clear from the graph that the MTSF and the availability both get decrease with increase in the values of failure rate.

Figure 2: MTSF vs Failure Rate

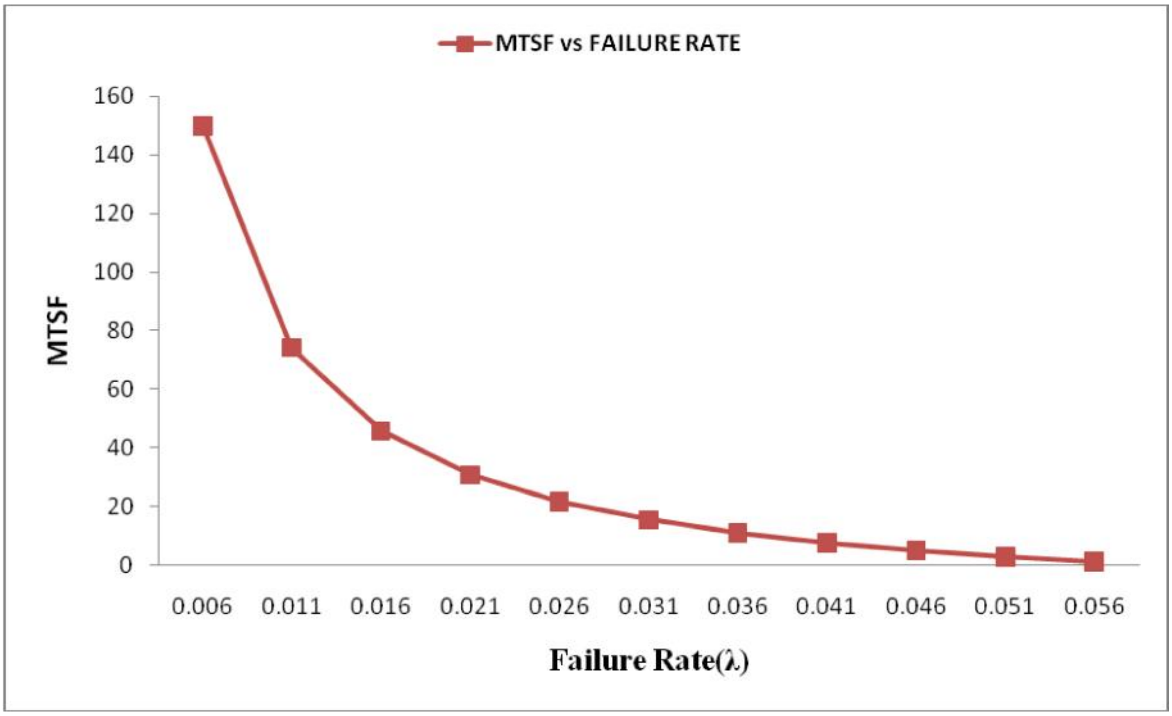


Figure 3: Availability vs Failure Rate

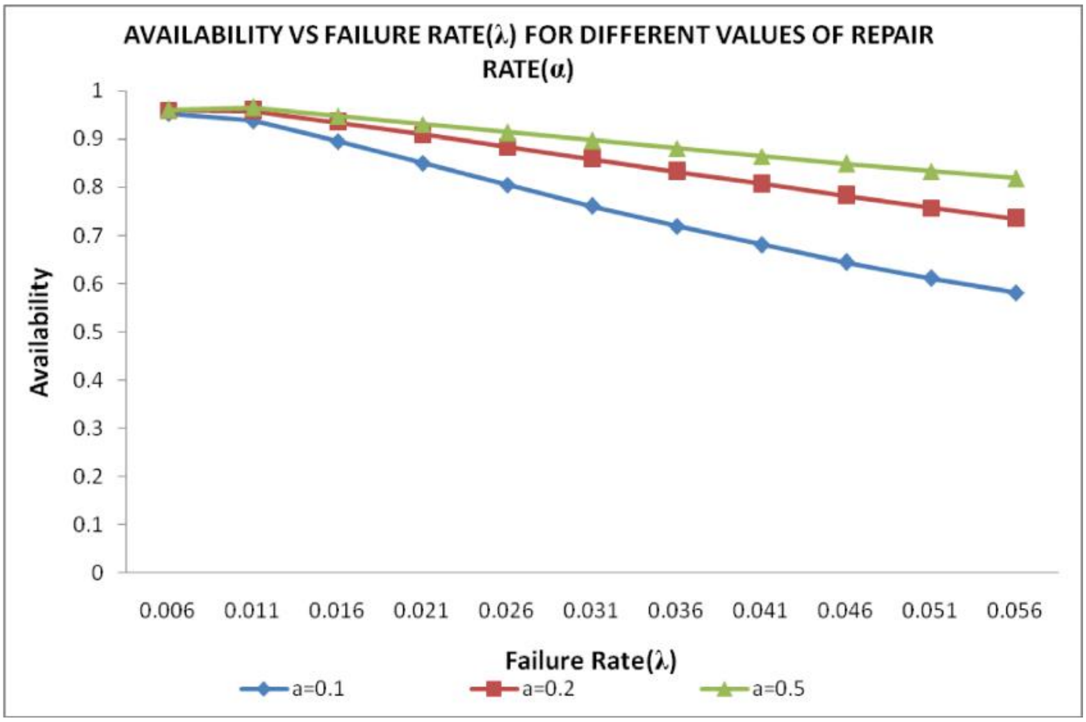


Figure 4 reveals the pattern of the profit with respect to failure rate (ª) for different values of repair rate (~). The profit decreases with the increase in the values of failure rate (ª) and is higher for higher values of repair rate (~). Following can also be observed from the graph:

1. For ~ = 0.1, P_2 > or = or < 0 according as ª < or = or > 0.0528. So, the system is profitable only if failure rate is lesser than 0.0528.
2. For ~ = 0.2, P_2 is > or = or < 0 according as ª < or = or > 0.075. So, the system is profitable only if failure rate is lesser than 0.075.
3. For ~ = 0.5, P_2 is > or = or < 0 according as ª < or = or > 0.083. So, the system is profitable only if failure rate is lesser than 0.083.

The above sensitivity of failure rate are not seen in diagram because failure rate is restricted till 0.056. So, the companies using such systems can be suggested to purchase only those system which do not have failure rates greater than those discussed in points (1) to (3) above in this particular case.

Figure 5 shows the behaviour of the profit with respect to revenue per unit time (C_0) for different values of cost (C_2). The profit increases with the increase in the values of revenue (C_0) and becomes lower for higher values of C_2. Following conclusions are drawn:

1. For C_2 = 500, P_2 > or = or < 0 according as C_0 > or = or < 505. So C_0 should be greater than 505.
2. For C_2 = 650, P_2 > or = or < 0 according as C_0 > or = or < 544. So, for this case C_0 should be greater than 544.

Figure 4: Profit vs Failure Rate

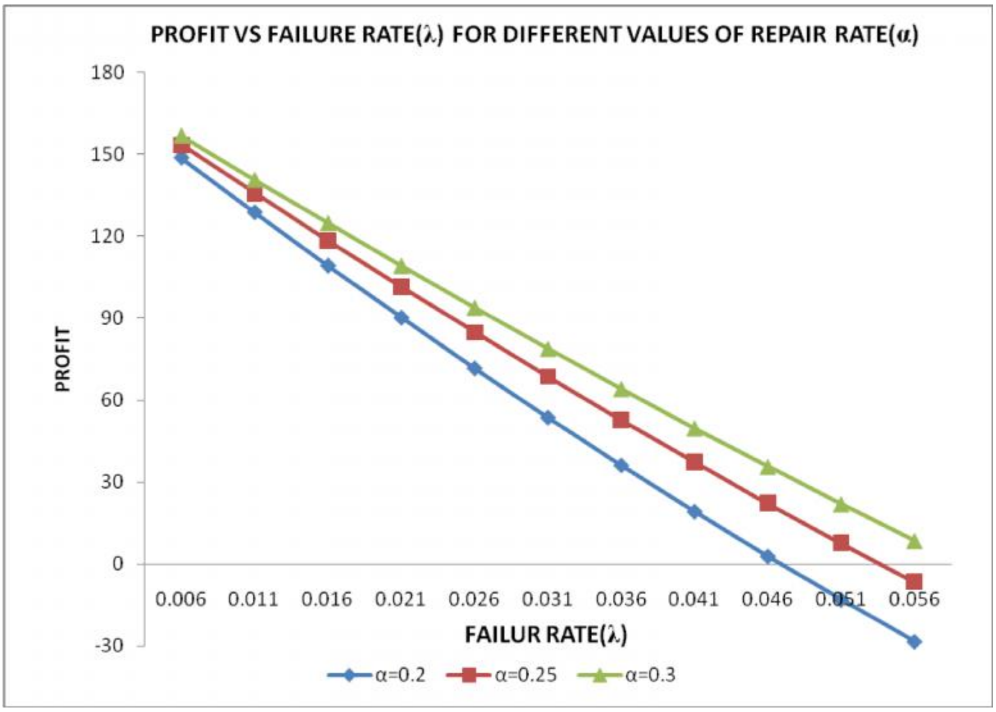
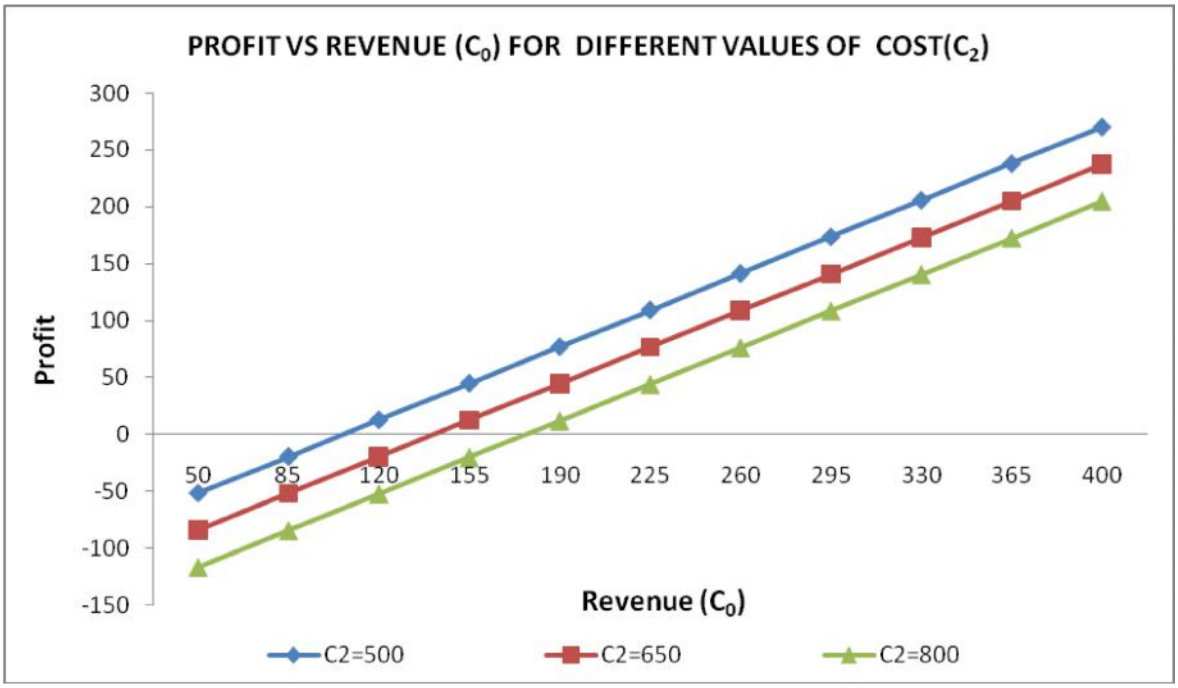


Figure 5: Profit vs Revenue



3. For $C_2 = 800$, P_2 > or = or < 0 according as C_0 > or = or < 581. Therefore, for this case C_0 should be greater than 581.

Figure 6 shows the behaviour of profit with respect to probability (*p*) for different values of probability (*a*). Profit increases as p increases and becomes higher for higher values of repair rate (~). Following conclusions can be drawn:

1. For $a = 0.2$, P_2 > or = or < 0 according as *p* > or = or < 0.62. Therefore *p* should be greater than 0.697.
2. For $a = 0.4$, P_2 > or = or < 0 according as *p* > or = or < 0.64. Therefore, system is profitable if $p > 0.688$.
3. For $a = 0.6$, P_2 > or = or < 0 according as *p* > or = or < 0.68. Therefore, system is profitable if $p > 0.678$.

It is also observed that the three curves converge as $p \to 1$ which implies that profit comes out to be same as $p \to 1$ irrespective of the values of probability (*a*).

Figure 7 reflects the profit (P_2) pattern of replacement policy as per different replacement rate ( ). The sensitivity of profit lies between 0.6 to 0.5 when cost (C_6) as exactly 600 as per replacement rate. The profit (P_2) decreases when cost (C_6) increases and the value of replacement increases than profit (P_2) also increases

Observation 1

Figure 8 interpreted the effect of post repair policy for different failure rate. When repair to be performed than failure rate may decreases as compare to without post repair. Availability of the system will be increased after post repair however failure rate decreases. For different failure rate, system is available till various durations.

Figure 6: Profit vs Probability

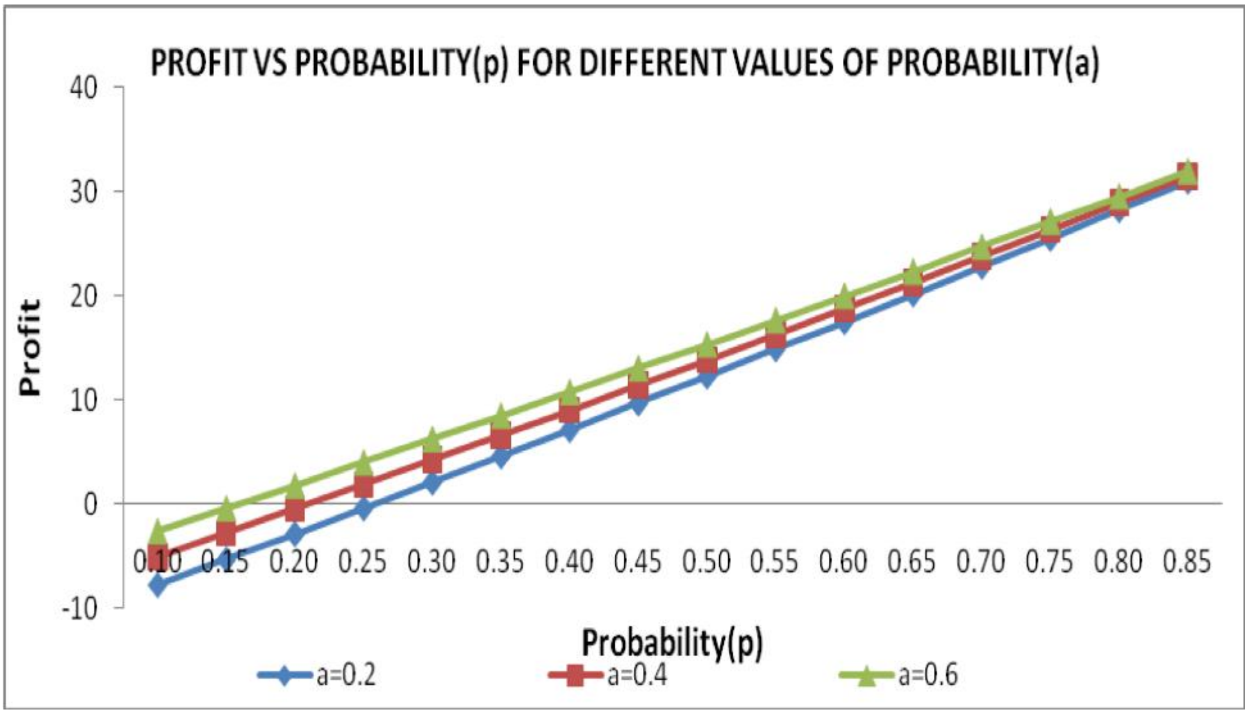


Figure 7: Profit vs Cost (C_6)

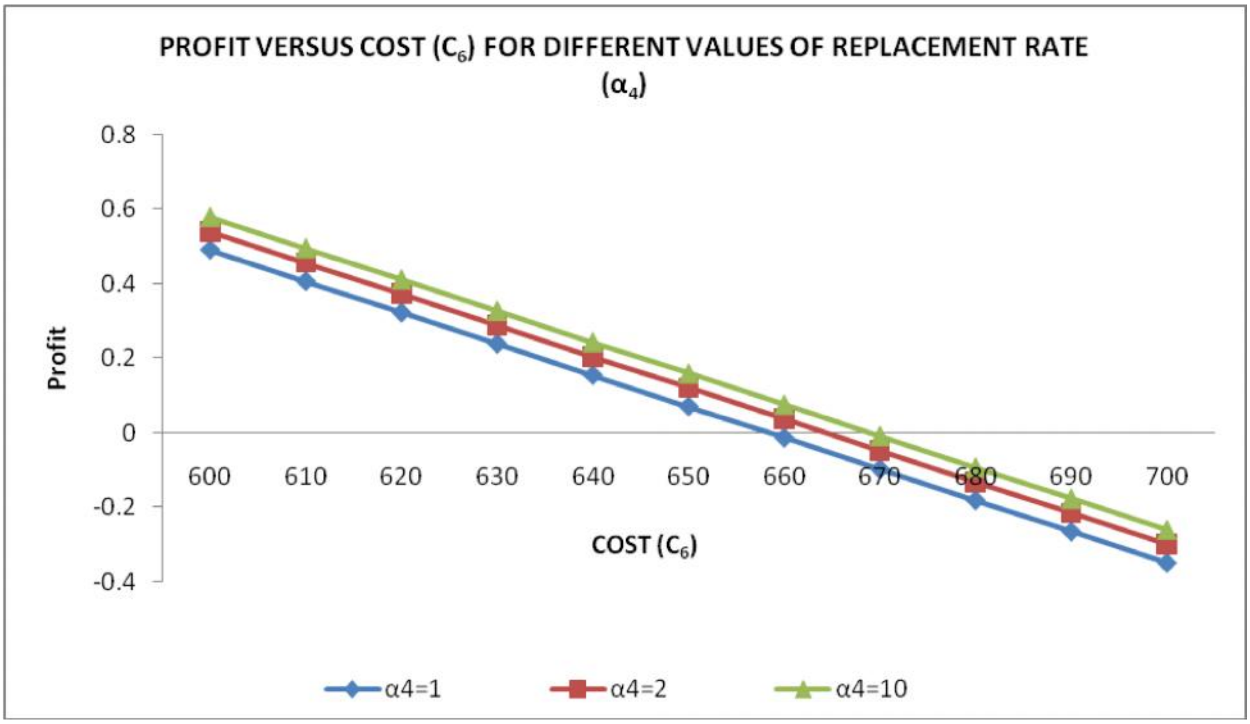


Figure 8: Availability vs Failure Rate (ª) for Different Values of Post Repair Rate(~̇)

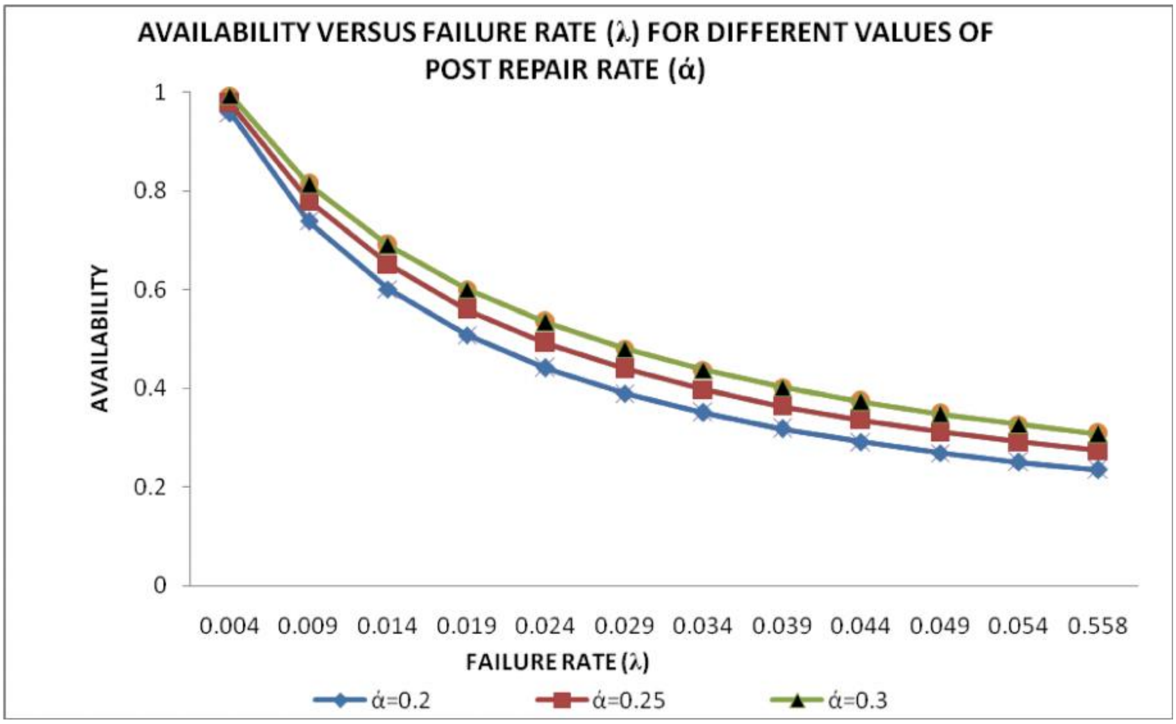
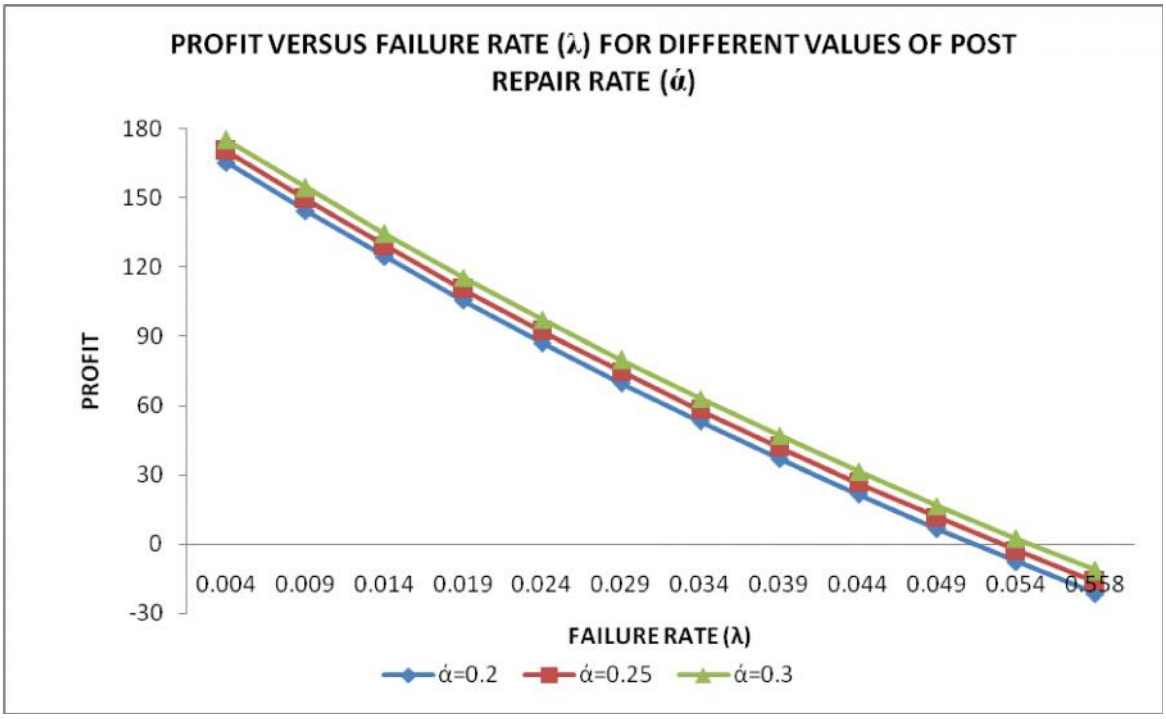


Figure 9: Profit vs Failure Rate (ª) for Different Values of Post Repair Rate(~̇)



Observation 2

Figure 9 gives the relationship between various failure rate and post repair rate as we know when repair rate increases then failure rate varies accordingly. After the performance of post repair, failure rate will be decreased as compare to without post repair. Ultimately profit for maintenance cost will be increased as per observation.

As per observation, graphs are plotted on estimated value of availability, profit and failure rate.

CONCLUSION

In this paper, we developed the expressions for the Mean Time to System Failure (MTSF), system availability, busy period and profit analysis for the system and performed graphical study to see the behavior of the failure rates and repair rates parameters on system performance. The brief concept of optimization also used for minimizing the cost of maintenance by repairman. It is observed that from graphical study, system performance increases with repair rates and decreases with failure rates. We are not able to solve the mathematical problems for judge the optimum cost in replacement policy due to lack of tools and time which is the limitation of this work. The sensitivity of optimum cost will be also worked out in near future.

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APPENDIX

Notations		
a	:	Constant failure rate of the unit
p	:	Probability that repairman is able to repair the failed unit
q	:	1-p, i.e., the probability that the repairman is unable to repair the failed unit
a	:	Probability of post repair
b	:	Probability of preventive maintenance
c	:	Probability of replacement
~	:	Repair rate
$\sim_4$	:	Replacement rate
$\sim_1, \sim_2, \sim_3,$	:	Are the usual parameter
$h(t)$, $H(t)$	:	p.d.f., c.d.f. of the inspection time
g(t), $G(t)$	:	p.d.f., c.d.f. of repair time of the repairman
$g_1(t)$, $G_1(t)$	:	p.d.f., c.d.f. of the post repair time
$g_2(t)$, $G_2(t)$	:	p.d.f., c.d.f. of the preventive maintenance time
$g_3(t)$, $G_3(t)$	:	p.d.f., c.d.f. of the replacement time
		Symbols for the State of the System
o	:	Operative
F_r	:	Failed unit under repair of the repairman
F_{in}	:	Failed unit under inspection
F_{pr}	:	Failed unit under post repair
F_{pm}	:	Failed unit under preventive maintenance
F_{rep}	:	Failed unit under replacement