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Research Paper

ANALYSIS OF TRANSIENT HEAT CONDUCTION IN DIFFERENT GEOMETRIES BY POLYNOMIAL APPROXIMATION METHOD

Devanshu Prasad[1]*

**Corresponding Author: Devanshu Prasad,* ✉ *devanshuprsd@gmail.com*

Present work deals with the analytical solution of unsteady state one-dimensional heat conduction problems. An improved lumped parameter model has been adopted to predict the variation of temperature field in a long slab and cylinder. Polynomial approximation method is used to solve the transient conduction equations for both the slab and tube geometry. A variety of models including boundary heat flux for both slabs and tube and, heat generation in both slab and tube has been analyzed. Furthermore, for both slab and cylindrical geometry, a number of guess temperature profiles have been assumed to obtain a generalized solution. Based on the analysis, a modified Biot number has been proposed that predicts the temperature variation irrespective of the geometry of the problem. In all the cases, a closed form solution is obtained between temperature, Biot number, heat source parameter and time. The result of the present analysis has been compared with earlier numerical and analytical results. A good agreement has been obtained between the present prediction and the available results.

Keywords: Lumped model, Polynomial approximation method, Transient, Conduction, Modified biot number

INTRODUCTION

Heat transfer generally takes place by three modes such as conduction, convection and radiation. Heat transmission, in majority of real situations, occurs as a result of combinations of these modes of heat transfer. Conduction is the transfer of thermal energy between neighbouring molecules in a substance due to a temperature gradient. It always takes place from a region of higher temperature to a region of lower temperature, and acts to equalize temperature differences. Conduction needs matter and does not require any bulk motion of matter. Conduction takes place in all forms of matter such as solids, liquids, gases and plasmas. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free

[1] Mechanical Engineering Department, Faculty of Engineering & Technology, Sri Sankaracharya Technical Campus (SSTC), Bhilai.

electrons. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion.

TRANSIENT HEAT CONDUCTION

Unsteady or transient heat conduction state implies a change with time, usually only of the temperature. It is fundamentally due to sudden change of conditions. Transient heat conduction occurs in cooling of I.C engines, automobile engines, heating and cooling of metal billets, cooling and freezing of food, heat treatment of metals by quenching, starting and stopping of various heat exchange units in power insulation, brick burning, vulcanization of rubber, etc. There are two distinct types of unsteady state namely periodic and non periodic. In periodic, the temperature variation with time at all points in the region is periodic. An example of periodic conduction may be the temperature variations in building during a period of twenty four hours, surface of earth during a period of twenty four hours, heat processing of regenerators, cylinder of an I.C engines, etc. In a non-periodic transient state, the temperature at any point within the system varies non-linearly with time. Heating of an ingot in furnaces, cooling of bars, blanks and metal billets in steel works, etc., are examples of non-periodic conduction.

HEAT CONDUCTION PROBLEMS

The solution of the heat conduction problems involves the functional dependence of temperature on various parameters such as space and time. Obtaining a solution means determining a temperature distribution which is consistent with conditions on the boundaries and also consistent with any specified constraints internal to the region. Jian Su (2001) and Keshavarz and Taheri (2007) have obtained this type of solution.

One Dimensional Analysis

In general, the flow of heat takes place in different spatial coordinates. In some cases the analysis is done by considering the variation of temperature in one-dimension. In a slab one dimension is considered when face dimensions in each direction along the surface are very large compared to the region thickness, with uniform boundary condition is applied to each surface. Cylindrical geometries of one-dimension have axial length very large compared to the maximum conduction region radius. At a spherical geometry to have one-dimensional analysis a uniform condition is applied to each concentric surface which bounds the region.

Steady and Unsteady Analysis

Steady State Analysis: A steady-state thermal analysis predicts the effects of steady thermal loads on a system. A system is said to attain steady state when variation of various parameters namely, temperature, pressure and density does not vary with time. A steady-state analysis also can be considered the last step of a transient thermal analysis. We can use steady-state thermal analysis to determine temperatures, thermal gradients, heat flow rates, and heat fluxes in an object which do not vary over time. A steady-state thermal analysis may be either linear, by assuming constant material properties or can be nonlinear case, with material properties varying with temperature. The thermal properties of most material do vary with temperature, so the analysis becomes non linear. Furthermore, by considering radiation effects system also become nonlinear.

Unsteady State Analysis: Before a steady state condition is reached, certain amount of time is elapsed after the heat transfer process is initiated to allow the transient conditions to disappear. For instance while determining the rate of heat flow through wall, we do not consider the period during which the furnace starts up and the temperature of the interior, as well as those of the walls, gradually increase. We usually assume that this period of transition has passed and that steady-state condition has been established.

In the temperature distribution in an electrically heated wire, we usually neglect warming- up period. Yet we know that when we turn on a toaster, it takes some time before the resistance wires attain maximum temperature, although heat generation starts instantaneously when the current begins to flow. Another type of unsteady-heat-flow problem involves with periodic variations of temperature and heat flow. Periodic heat flow occurs in internal-combustion engines, air-conditioning, instrumentation, and process control. For example the temperature inside stone buildings remains quite higher for several hours after sunset. In the morning, even though the atmosphere has already become warm, the air inside the buildings will remain comfortably cool for several hours. The reason for this phenomenon is the existence of a time lag before temperature equilibrium between the inside of the building and the outdoor temperature.

Another typical example is the periodic heat flow through the walls of engines where temperature increases only during a portion of their cycle of operation. When the engine warms up and operates in the steady state, the temperature at any point in the wall undergoes cycle variation with time. While the engine is warming up, a transient heat-flow phenomenon is considered on the cyclic variations.

One Dimensional Unsteady State Analysis: In case of unsteady analysis the temperature field depends upon time. Depending on conditions the analysis can be one-dimensional, two dimensional or three dimensional. One dimensional unsteady heat transfer is found at a solid fuel rocket nozzle, in re-entry heat shields, in reactor components, and in combustion devices. The consideration may relate to temperature limitation of materials, to heat transfer characteristics, or to the thermal stressing of materials, which may accompany changing temperature distributions.

DESCRIPTION OF ANALYTICAL METHOD AND NUMERICAL METHOD

In general, we employ either an analytical method or numerical method to solve steady or transient conduction equation valid for various dimensions (1D/2D). Numerical technique generally used is finite difference, finite element, relaxation method, etc. The most of the practical two dimensional heat problems involving irregular geometries is solved by numerical techniques. The main advantage of numerical methods is it can be applied to any two dimensional shape irrespective of its complexity or boundary condition. The numerical analysis, due to widespread use of digital computers these days, is the primary method of solving complex heat transfer problems.

The heat conduction problems depending upon the various parameters can be obtained through analytical solution. An analytical method uses Laplace equation for solving the heat conduction problems. Heat balance integral method, Hermite-type approximation method, polynomial approximation method, Wiener-Hopf technique are few examples of analytical method.

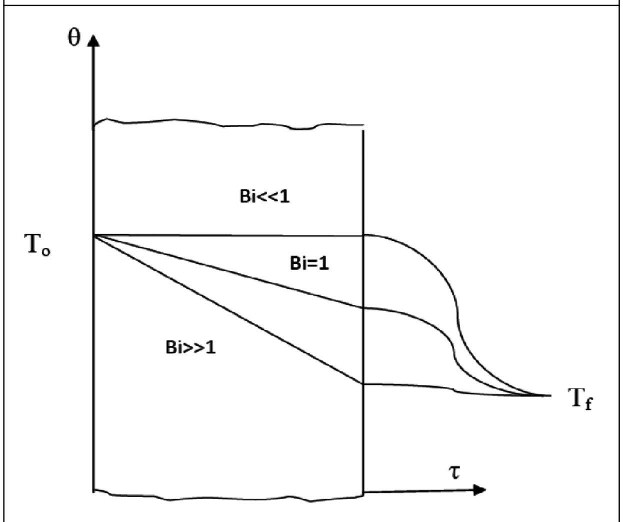
LOW BIOT NUMBER IN 1-D HEAT CONDUCTION PROBLEMS

The Biot number represents the ratio of the time scale for heat removed from the body by surface convection to the time scale for making the body temperature uniform by heat conduction. However, a simple lumped model is only valid for very low Biot numbers. In this preliminary model, solid resistance can be ignored in comparison with fluid resistance, and so the solid has a uniform temperature that is simply a function of time. The criterion for the Biot number is about 0.1, which is applicable just for either small solids or for solids with high thermal conductivity. In other words, the simple lumped model is valid for moderate to low temperature gradients. In many engineering applications, the Biot number is much higher than 0.1, and so the condition for a simple lumped model is not satisfied. Additionally, the moderate to low temperature gradient assumption is not reasonable in such applications, thus more accurate models should be adopted. Lots of investigations have been done to use or modify the lumped model. The purpose of modified lumped parameter models is to establish simple and more precise relations for higher values of Biot numbers and large temperature gradients. For example, if a model is able to predict average temperature for Biot numbers up to 10, such a model can be used for a much wider range of materials with lower thermal conductivity.

Figure 1 shows the variation of temperature with time for various values of Biot number. The figure predicts that for higher values Biot number temperature variation with respect to time is higher. When Biot number is more than one the heat transfer is higher which require more time to transfer the heat from body to outside. Thus the variation of temperature with time is negligible. Whereas as gradually the Biot number increase, the heat transfer rate decrease, and thus it results to rapid cooling.

Figure 1 predicts how at Biot number more than one the temperature variation with time is more as compared to Biot number with one and less than one.

Figure 1: Schematic of Variation of Biot Number in a Slab



SOLUTION OF HEAT CONDUCTION PROBLEMS

The objective of conduction analysis is to determine the temperature field in a body and how the temperature varies within the portion of the body. The temperature field usually depends on boundary conditions, initial condition, material properties and geometry of the body. Why one need to know temperature field. To compute the heat flux at any location, compute thermal stress, expansion, deflection, design insulation thickness, heat treatment method, these all analysis leads to know the temperature field. The solution of conduction problems involves the functional dependence of temperature on space and time coordinate.

Analytical Exact Solutions

Keshavarz and Taheri (2007) have analyzed the transient one-dimensional heat conduction of slab/rod by employing polynomial approximation method. In their paper, an improved lumped model is being implemented for a typical long slab, long cylinder and sphere. It has been shown that in comparison to a finite difference solution, the improved model is able to calculate average temperature as a function of time for higher value of Biot numbers. The comparison also presents model in better accuracy when compared with others recently developed models. The simplified relations obtained in this study can be used for engineering calculations in many conditions. He had obtained the temperature distribution as:

$$\exp\left(-\frac{B(m+1)(m+3)}{m+B+3}\tau\right)$$

Jian Su (2001) have analyzed unsteady cooling of a long slab by asymmetric heat convection within the framework of lumped parameter model. They have used improved lumped model where the heat conduction may be analyzed with larger values of Biot number. The proposed lumped models are obtained through two point Hermite approximations method. Closed form analytical solutions are obtained from the lumped models.

Higher order lumped models (HI, 1/H0, 0 approximation) is compared with a finite difference solution and predicts a significance improvement of average temperature prediction over the classical lumped model. The expression was written as:

$$\exp\left(\frac{3(B_1+B_2+2B_2+2B_1B_2)}{2(3+2B_1+2B_2+B_1B_2)}\right)$$

Correa and Cotta (1998) have directly related to the task of modelling diffusion problems. The author presented a formulation tool, aimed at reducing, as much as possible and within prescribed accuracy requirements, the number of dimensions in a certain diffusion formulation. It is shown how appropriate integration strategies can be employed to deduce mathematical formulations of improved accuracy In comparison, with the well-established classical lumping procedures. They have demonstrated heat conduction problems and examined against the Classical Lumped System Analysis (CLSA) and the exact solutions of the fully differential systems.

OBJECTIVE OF PRESENT WORK

- An effort will be made to predict the temperature field in solid by employing a polynomial approximation method.

- Effort will be made analyze more practical case such as heat generation in solid and specified heat flux at the solid surface is investigated.
- Effort will be made to obtain new functional parameters that affect the transient heat transfer process.
- It is tried to consider various geometries for the analysis.

SOLUTION PROCEDURE

Polynomial Approximation Method (PAM) is one of the simplest, and in some cases, accurate methods used to solve transient conduction problems. The method involves two steps: first, selection of the proper guess temperature profile, and second, to convert a partial differential equation into an equation. This can then be converted into an ordinary differential equation, where the dependent variable is average temperature and independent variable is time. The steps are applied on dimensionless governing equation

Assuming constant physical properties, k and α, the generalized transient heat conduction valid for slab, cylinder and sphere can be expressed as:

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^m} \frac{\partial}{\partial r}\left(r^m \frac{\partial T}{\partial r}\right)$$

where, $m = 0$ for slab, 1 and 2 for cylinder and sphere, respectively.

We have covered different heat conduction problems for the analysis. The analytical method used is polynomial approximation method. Two problems are taken for heat flux, and two for heat generation. At the last a simple slab and cylinder is considered with different profiles. The result and discussion from the above analysis has been presented in the figures and tables, illustrated in following sections. Furthermore, the present prediction is compared with the analysis of Correa and Cotta (1998), Jian Su (2001) and Keshavarz and Taheri (2007).

RESULTS AND DISCUSSION

We have tried to analyze the heat conduction behaviour for both cartesian and cylindrical geometry. Based on the previous analysis closed form solution for temperature, Biot number (B), heat source parameter (Q), and time for both slab and tube has been obtained. Figure 2 shows the variation of temperature with time for various heat source parameters for a slab. This figure contains Biot number as constant. With higher value of heat source parameter, the temperature inside the slab does not vary with time. However for lower value of heat source parameter, the temperature decreases with the increase of time.

Figure 2: Average Dimensionless Temperature vs. Dimensionless Time for Slab at B = 1

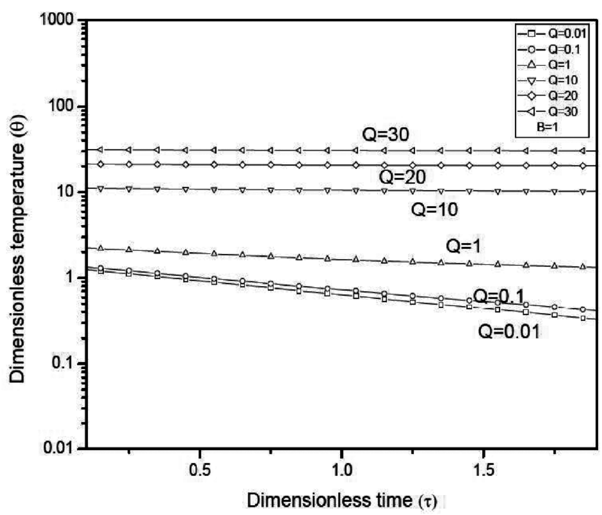
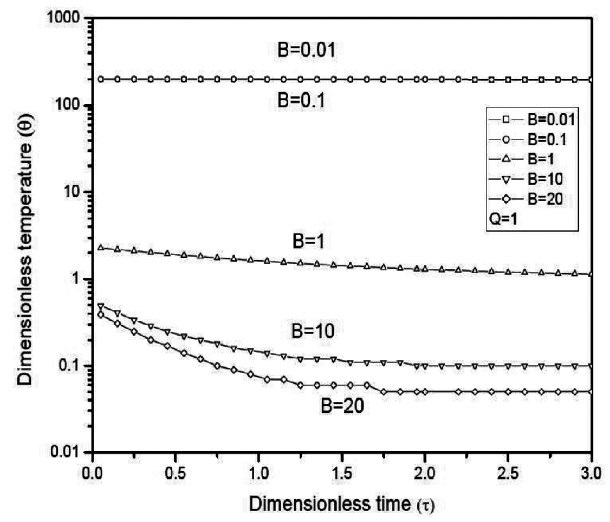


Figure 3 shows the variation of temperature with time for various Biot numbers, having heat source parameter as constant for a slab. With lower value of Biot numbers, the temperature inside the slab does not vary with time. However for higher value of Biot numbers, the temperature decreases with the increase of time.

Figure 3: Average Dimensionless Temperature vs. Dimensionless Time for Slab at *Q* = 1



Similarly Figure 4 shows the variation of temperature with time for various heat source parameters for a tube. This figure contains Biot number as constant. With higher value of heat source parameter, the temperature inside the tube does not vary with time. However at lower values of heat source parameters, the temperature decreases with increase of time.

Figure 5 shows the variation of temperature with time for various Biot numbers, having heat source parameter as constant for a tube. With lower value of Biot numbers, the temperature inside the tube does not vary with time. For higher value of Biot numbers, the temperature decreases with the increase of time.

Figure 4: Average Dimensionless Temperature vs. Dimensionless Time for Cylinder at *B* = 1

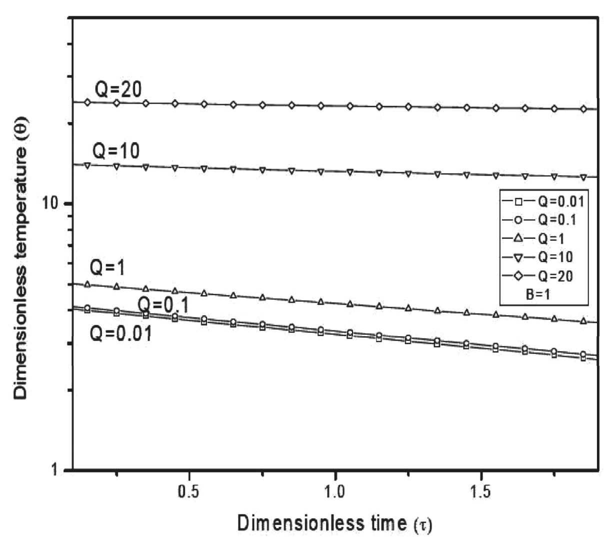
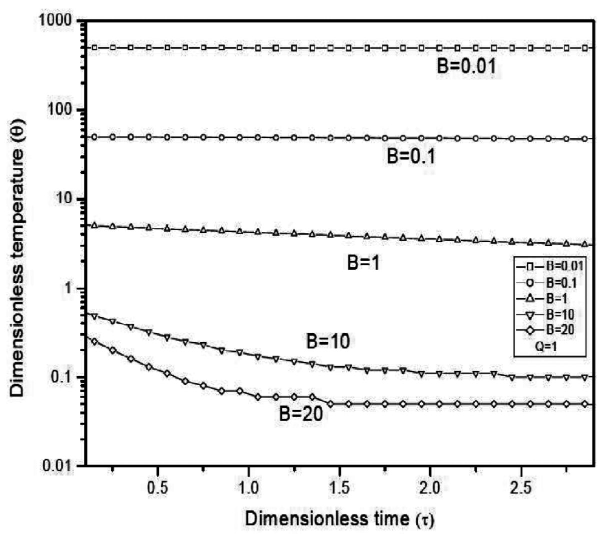
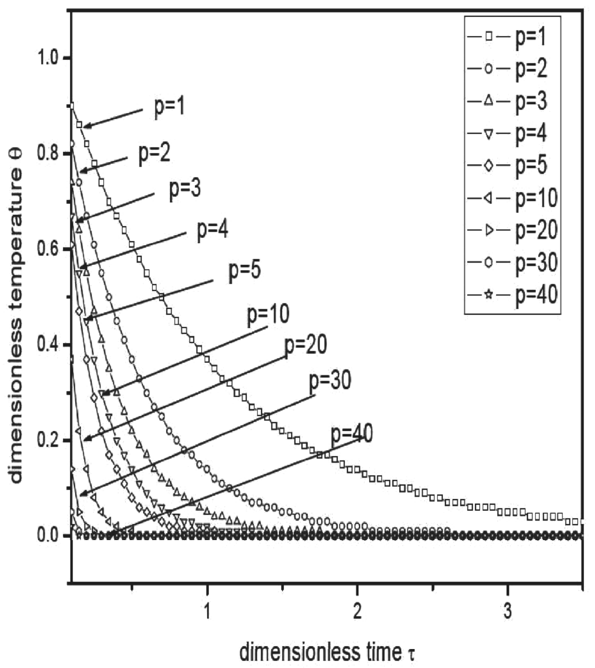


Figure 5: Average Dimensionless Temperature vs. Dimensionless Time for Cylinder at *Q* = 1



We have considered a variety of temperature profiles to see their effect on the solution. Based on the analysis a modified Biot number has been proposed, which is independent of geometry of the problem. Figure 6 shows the variety of temperature with time for different values of modified Biot number, P. It is seen that, for higher values of P represent higher values of Biot number. Therefore the heat removed from the solid to surrounding is higher at higher Biot number. This leads to sudden change in temperature for higher value of P. This trend is observed in the present prediction and is shown in Figure 6.

Figure 6: Variation of Average Temperature with Dimensionless Time, for P = 1 to 40 for a Slab



Tabulation of Results

Validation of Present Results:

Figure 7 shows the comparison of present analysis with the other available results. These include classified lumped system analysis and exact solution by Correa and Cotta (1998) of a slab. It is observed that the present prediction shows a better result compared to the Classical Lumped System Analysis (CLSA). The present prediction agrees well

Table 1: Comparison of Solutions of Average Temperature Obtained from Different Heat Conduction Problems

Average Temperature	Slab with Heat Flux	Slab with Heat Generation	Tube with Heat Flux	Tube with Heat Generation
θ	$\theta = \left(\frac{e^{-U\tau} + V}{U}\right)$ where, $U = \frac{B}{1 + B/3}$ $V = \frac{Q}{1 + B/3}$	$\theta = \left(\frac{e^{-\tau U} + V}{U}\right)$ where, $U = \frac{B}{1 + B/3}$ $V = \frac{G}{1 + B/3}$	$\theta = \left(\frac{e^{-U\tau} + V}{U}\right)$ where, $U = \frac{B}{(4 + B)/8}$ $V = \frac{Q}{(4 + B)/8}$	$\theta = \left(\frac{e^{-U\tau} + V}{U}\right)$ where, $U = \frac{2B}{1 + B/4}$ $V = \frac{G}{1 + B/4}$

Table 2: Comparison of Modified Biot Number Against Various Temperature Profiles for a Slab

S. No.	Profile	Value of P
1.	$\theta_p = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2$	$P = \frac{3B}{B + 3}$

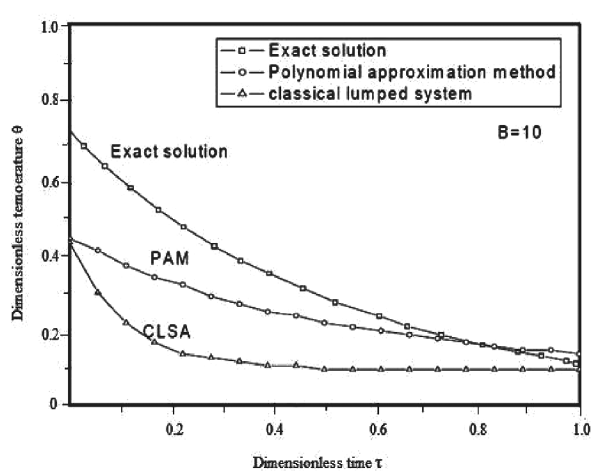
Table 2 (Cont.)

S. No.	Profile	Value of P
2.	$\theta = a_0 + a_1\left(x^2 - x\right) + a_2\left(x^3 - x^2\right)$	$P = \dfrac{13B}{13 + 12B}$
3.	$\theta = a_0 + a_1\left(x^4 - x^2\right) + a_2\left(x^3 - x\right)$	$P = \dfrac{30B}{30 + 17B}$
4.	$\theta = a_0 + a_1\left(x^4 - x^2\right) + a_2\left(x^2 - x\right)$	$P = \dfrac{30B}{30 + 13B}$
5.	$\theta = a_0 + a_1\left(x^4 - x\right) + a_2\left(x^3 - x^3\right)$	$P = \dfrac{24B}{24 + 13B}$
6.	$\theta = a_0 + a_1\left(x^4 - x^3\right) + a_2\left(x^4 - x\right)$	$P = \dfrac{20B}{20 + 21B}$

Table 3: Comparison of Modified Biot Number Against Various Temperature Profiles for a Cylinder

S. No.	Profile	Value of P
1.	$\theta_p = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2$	$P = \dfrac{8B}{B + 4}$
2.	$\theta = a_0 + a_1\left(x^2 - x\right) + a_2\, x^2$	$P = \dfrac{4B}{B + 2}$
3.	$\theta = a_0 + a_1\left(x^3 - x\right) + a_2\, x^3$	$P = \dfrac{30B}{3B + 15}$
4.	$\theta = a_0 + a_1\left(x^4 - x\right) + a_2\left(x^3 - x^2\right)$	$P = \dfrac{10B}{4B + 5}$

Figure 7: Comparison of Solutions of PAM, LSA and Exact Solution for a Slab Having Constant Internal Heat Generation



with the exact solution of Correa and Cotta (1998) at higher time. However at shorter time, the present analysis under predicts the temperature in solid compared to the exact solution. This may be due to the consideration of lumped model for the analysis.

CONCLUSION

An improved lumped parameter model is applied to the transient heat conduction in a long slab and long cylinder. Polynomial approximation method is used to predict the transient distribution temperature of the slab and tube geometry. Four different cases namely, boundary heat flux for both slab and tube and, heat generation in both slab and tube has been analyzed. Additionally different temperature profiles have been used to obtain solutions for a slab. A unique number, known as modified Biot number is, obtained from the analysis. It is seen that the modified Biot number, which is a function of Biot number, plays important role in the transfer of heat in the solid. Based on the analysis the following conclusions have been obtained.

- Initially a slab subjected to heat flux on one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a closed form solution has been obtained.

$$\theta = \left(\frac{e^{-U\tau} + v}{U} \right)$$

where $U = \dfrac{B}{1 + B/3}, V = \dfrac{Q}{1 + B/3}$

- A long cylinder subjected to heat flux on one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a solution has been obtained.

$$\theta = \left(\frac{e^{-U\tau} + v}{U} \right)$$

where $U = \dfrac{B}{(4 + B)/8}, V = \dfrac{Q}{(4 + B)/8}$

- A slab subjected to heat generation at one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a closed form solution has been obtained.

$$\theta = \left(\frac{e^{-U\tau} + v}{U} \right)$$

where $U = \dfrac{B}{\left(1 + {}^{B}\!/_{3}\right)}, V = \dfrac{G}{\left(1 + {}^{B}\!/_{3}\right)}$

- A long cylinder subjected to heat generation at one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis a closed form solution has been obtained.

$$\theta = \left(\frac{e^{-U\tau} + v}{U} \right)$$

where $U = \dfrac{2B}{\left(1 + {}^{B}\!/_{4}\right)}, V = \dfrac{G}{\left(1 + {}^{B}\!/_{4}\right)}$

- Based on the analysis a unique parameter known as modified Biot number obtained from the analysis and is shown in Tables 2 and 3. With higher value of heat source parameter, the temperature inside the slab/ tube does not vary with time. However at lower values of heat source parameters, the temperature decreases with increase of time. With lower value of Biot numbers, the temperature inside the slab/tube does not vary with time. For higher value of Biot numbers, the temperature decreases with the increase of time.

SCOPE FOR FURTHER WORK

- Polynomial approximation method can be used to obtain solution of more complex problem involving variable properties and variable heat transfer coefficients, radiation at the surface of the slab.
- Other approximation method, such as Heat Balance Integral method, Biots variation method can be used to obtain the solution for various complex heat transfer problems.

- Efforts can be made to analyze two dimensional unsteady problems by employing various approximate methods.

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