



Research Paper

NON LINEAR REGRESSION MODEL OF SURFACE ROUGHNESS

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Turning is one of the most widely used metal cutting processes. The increasing importance of turning operation is gaining new dimensions in the present industrial age in which the growing competition calls for all the efforts to be directed towards the economical manufacture of machined parts as well as surface finish is one of the most critical quality measure in mechanical products. In present work, a non linear regression analysis is adopted to establish a prediction model for surface roughness which may help to optimize machining process. Once the process parameters viz., cutting speed, feed, depth of cut, Nose Radius are given, the surface roughness can be found out experimentally following which a comparative study are made to analyze the deviation in surface roughness values from prediction model. The work piece material is EN8 which is machined by carbide inserted tool. All the experimental works have been conducted on CNC lathe. The experiments are carried out by using design of experiment. Finally the contributions are summarized in tabular form and may be used as an indicative of quality measure of machined parts.

Keywords: Design of experiment, Regression approach, Machining process optimization

INTRODUCTION

Todays manufacturing industries are very much concerned about the quality of their products. They are focused on producing high quality products in time at minimum cost. Surface Roughness (finish) is one of the crucial performance parameters that have to be controlled within suitable limits for a particular process. Therefore, prediction or monitoring

of the surface roughness of machined components has been an important area of research. Surface roughness is harder to attain and track than physical dimensions are, because relatively many factors affect surface roughness. Some of these factors can be controlled and some cannot. Controllable process parameters include feed, cutting speed, tool geometry, nose radius and tool

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setup. Other factors, such as tool, work piece and machine vibration, tool wear and degradation, and work piece and tool material variability can not be controlled as easily. Surface roughness also affects several functional attributes of parts, such as contact causing surface friction, wearing, light reflection, heat transmission, ability of distributing and holding a lubricant, coating or resisting fatigue. Therefore the desired finish surface is usually specified and the appropriate are selected to reach the required quality. Several works have been reported in the broad field of tool condition monitoring. Researchers are trying to develop a robust and accurate model that can find a correlation between the cutting parameters and the surface roughness of the machined products. The purpose of developing the non linear regression model relating the machining responses and their machining factors is to facilitate the optimization of the machining process. Using this regression model, the objective function and process constraints are formulated, and the optimization problem has been solved by using regression approach.

MATERIALS AND METHODS

Design of Experiment (DOE)

It is a structured, organized method that is used to determine the relationship between the different factors (Xs) affecting a process and the output of that process (Y) as well as to understand the impact of specific changes to the inputs of the process, and then to maximize, minimize or normalize the outcome by manipulating the input. The DOE process is divided into three main phases, which encompasses all experimental approaches.

These three phases are: 1) The Planning Phase, 2) The Conducting Phase, 3) The analyzing phase. The planning phase is when factors and levels are selected and, therefore the most important stage of experimentation. Also the correct selection factors and levels is non statistical in nature and more dependent upon product or process expertise. The second most important phase is the conducting phase, when the test results are actually collected. If experiments are well planned and conducted, the analysis is actually much easier and more likely to yield positive information about factors and levels. The analysis phase is when the positive or negative information concerning the selected factors and levels is generated based on the previous two phases. This phase is statistical in nature. The major steps to complete an effective designed experiment are listed in the following 12 steps. The planning phase includes steps 1 through 9, the conducting step 10, and the analysis phase include steps 11 and 12. The following phases are 1) Stating the problem(s) or areas(s) of concern. 2) Stating the objective(s) of the experiment. 3) Stating the quality characteristic(s) and measurement system(s). 4) Select the factors that may influence. 5) The selected quality characteristics. 6) Identify control and noise factors. 7) Select levels of factor. 8) Select the appropriate Orthogonal Array (OA) or Ors. 9) Select interactions that may influence the selected quality characteristics or go back to step 4 (iterative steps) and assign factors to OA(s) and locate interactions. 10) Conduct tests described by trials in OAs. 11) Analyze and interpret results of the experimental trials. 12) Conduct confirmation experiment.

Specifications of Surface Roughness Measuring Instrument

Make: Taylor/Hobson (Supplied from England),
 Traverse Unit: Traverse Speed: 1mm/Sec
 Measurement: Metric/Inch Preset by DIP-Switch
 Parameters: Ra, Rq, Rz (DIN), Ry and Sm.

Specification of EN8 Material

Chemical composition C-0.40%, Si-0.25%, Mn-0.80%, S-0.05% max., P-0.05% max.
 Mechanical properties: Normalized and Tempered, Tensile Strength (tons/sq.in.) – 35 (min)/40 (min), Yield Strength (tons/sq.in.) – 18 (min)/28 (min), Elongation % – 20/22, Izod Impact Value (Ft-lb) – 25 (min), Weld ability-Poor (Figure 1).

Cutting Tool Material

The tool used was cemented carbide insert type. The geometry of tool is: Rake angle 6° (+ve), 5° (+ve) clearance angle, 60° (+ve) major cutting edge angle, 60° (+ve) included angle and 0° cutting edge inclination angle.

In the above process parameters matrix keeping one parameter constant chosen

Figure 1: Experimental Setup for Turning a Nine Stepped Cylindrical Work Piece of EN8 Material on CNC Lathe Machine



Table 1: Process Parameters

Process Parameters	Levels		
R.P.M of Work Piece	400	600	800
Depth of Cut (mm) (d)	0.7	1.6	2.0
Feed (mm/rev.) (f)	0.1	0.2	0.3
Nose Radius (mm) (NR)	0.4	0.8	1.2

arbitrarily, remaining three parameters have been varied in the consecutive step turning process of the work piece to tabulate surface roughness values which has been shown in Table 2. Although parameter values have been chosen arbitrarily. This matrix pattern has been adopted to ease strong interaction among parameters to vary roughness values in the vicinity of the average value as well as to assess and predict roughness quality if occurred any change in parameter or deviation in parameter value during machining operation to an extent.

THEORY

In statistics, nonlinear regression is the problem of inference for a model

$$y = f(x, \theta) + \varepsilon \quad \dots(1)$$

Based on multidimensional x, y data, where f is some nonlinear function with respect to unknown parameters θ and ε random variable. At a minimum, we may like to obtain the parameter values associated with the best fitting curve (usually, least squares). Also, statistical inference may be needed, such as confidence intervals for parameters, or a test of whether or not the fitted model agrees well with the data. The nonlinear regression is clarified by considering the case of polynomial regression, which actually is best not treated as a case of nonlinear regression. When f takes a form such as:

$$F(x) = ax^2 + bx + c \quad \dots(2)$$

Our function f is nonlinear as a function of x but it is linear as a function of unknown parameters a , b , and c . The latter is the sense of “linear” in the context of statistical regression modeling. The appropriate computational procedures for polynomial regression are procedures of (multiple) linear regression with two predictor variables x and x^2 say. However, on occasion it is suggested that nonlinear regression is needed for fitting polynomials. Practical consequences of the misunderstanding include that a nonlinear optimization procedure may be used when the solution is actually available in closed form. Also, capabilities for linear regression are likely to be more comprehensive in some software than capabilities related to nonlinear regression.

The linear mathematical expression for surface roughness commonly used is represented by:

$$Y = \varphi(V, f, d, NR) \quad \dots(3)$$

where Y is the machining response, φ is the response function, V, f, d, NR are machining variables, i.e., Cutting speed, Feed, Depth of cut and Nose radius. When expressed in the non-linear form the above equation becomes

$$Y = CV^{n1} f^{n2} d^{n3} NR^{n4} \quad \dots(4)$$

The surface roughness equation may be formulated as:

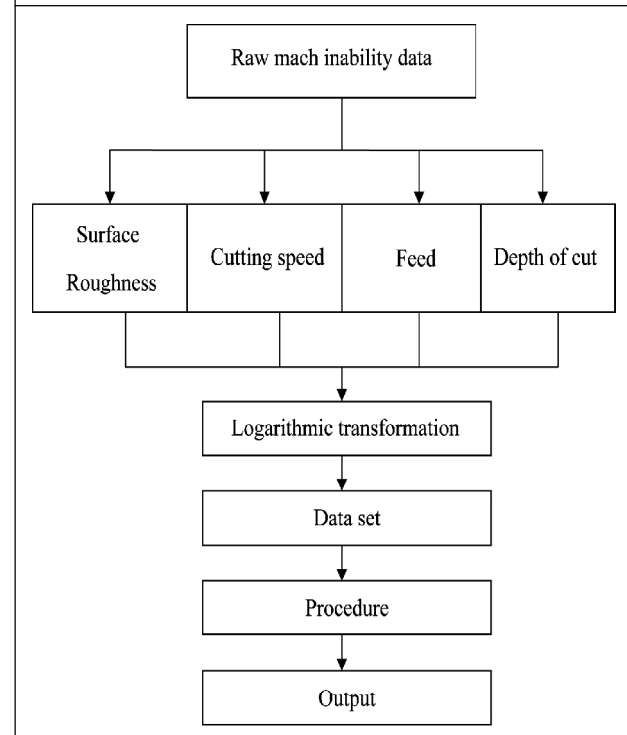
$$Ra = CV^{n1} f^{n2} d^{n3} NR^{n4} \quad \dots(5)$$

Now logarithm taken on both sides of the above equation

$$\begin{aligned} \log Ra &= \log C + n1 \log V + n2 \log f + n3 \log d \\ &+ n4 \log NR \end{aligned} \quad \dots(6)$$

Using the above mathematical expression, the objective function and process constraints are formulated, and the optimization problem then solved by using non linear regression approach is as shown in Figure 2.

Figure 2: Flow Chart for Processing Non-Linear Model in Regression Analysis



The following factors are used in the present analysis:

Regression sum of squares:

$$SS_R = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 \quad \dots(7)$$

where $\bar{y}$ = sum of dependent variable, $\hat{y}_i$ = fitted value of i^{th} dependent variable

Residual sum of squares

$$SS_{Res} = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad \dots(8)$$

where y_i = observed value of i^{th} dependent variable

Total sum of squares: Total variability in the observations of data

$$SS_T = SS_R + SS_{Res} \quad \dots(9)$$

Predictor variables

$$k = SS_R / \sigma^2 \quad \dots(10)$$

This has the same number of degree of freedom as number of regressor or predictor variables in the model where σ^2 = variance and Regression mean squares

$$MS_R = \frac{SS_R}{k} \quad \dots(11)$$

Residual mean squares

$$MS_{Res} = \frac{SS_{Res}}{n - p} \quad \dots(12)$$

where $p = k + 1$ = parameter in the regression model and n = number of observations

Test of statistic:

$$F = \frac{MS_R}{MS_{Res}} \quad \dots(13)$$

Coefficient of determination:

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_{Res}}{SS_T} \quad \dots(14)$$

Adjusted coefficient of determination:

$$\text{Adjusted } R^2 = 1 - \frac{SS_{Res}/(n - p)}{SS_T/(n - 1)} \quad \dots(15)$$

Standard error of regression:

$$SE = \sqrt{MS_{Res}} \quad \dots(16)$$

RESULTS AND DISCUSSION

The work piece consists of 9 steps, total measurements taken: 9 Ra (μm)

MINITAB Software Aided Regression Analysis

The regression equation is

$$\log Ra = 2.15 - 0.662 \log V + 0.427 \log f - 0.504 \log d - 0.102 \log NR \quad \dots(17)$$

$S = 0.2008$; $R\text{-Sq} = 39.8\%$; $R\text{-Sq (Adj.)} = 36.6\%$

Where R denotes an observation with a large standardized residual.

Where DF denotes degree of freedom of regression and residual error.

The data of analysis of variance of the roughness model shown in the surface roughness model is developed as:

Table 2: Experimentally Obtained Surface Roughness Values

Step Diameter (mm)	RPM	Cutting Speed (m/min.)	Feed (mm/Rev)	Depth of Cut (mm)	Nose Radius (mm)	Ra (μm)
32.06	400	39.04	0.2	0.7	0.4	10.32
32.03	600	62.27	0.2	0.7	0.4	9.16
32.11	800	88.25	0.2	0.7	0.4	8.86
30.12	400	46.63	0.1	0.7	0.4	9.72
30.05	600	73.62	0.1	0.7	0.4	9.04
28.97	800	102.98	0.1	0.7	0.4	11.86
28.67	400	54.04	0.3	0.7	0.4	13.02
28.05	600	84.93	0.3	0.7	0.4	9.44
26.03	800	118.21	0.3	0.7	0.4	12.02

Table 3: Predictor-Coefficient Relationship

Predictor	Coef.	SE Coef.	T	P
Constant	2.1451	0.2971	7.22	0.000
log V	-0.6623	0.1486	-4.46	0.000
log f	0.4267	0.1134	3.76	0.000
log d	-0.5037	0.1156	-4.36	0.000
log NR	-0.1017	0.1127	-0.90	0.370

Table 4: Analysis of Variance

Source	DF	SS	MS	F	P
Regression	4	2.02407	0.50602	12.55	0.000
Residual Error	76	3.06404	0.04032	—	—
Total	80	5.08811	—	—	—
Source	DF	Seq SS	—	—	—
log V	1	0.65624	—	—	—
log f	1	0.57009	—	—	—
log d	1	0.76489	—	—	—
log NR	1	0.03285	—	—	—

Table 5: log Ra vs. log V Relationship and Fit-Residual Observations

Observation	log V	log Ra	Fit	SE Fit	Residual	St. Residual
1	1.59	0.6300	0.9097	0.0617	-0.2797	-1.46
2	1.79	0.7900	0.7772	0.0490	0.0128	0.07
3	1.95	0.6900	0.6713	0.0505	0.0187	0.10
4	1.67	0.5700	0.7287	0.0607	-0.1587	-0.83
5	1.87	0.6100	0.5962	0.0567	0.0138	0.07
6	2.01	0.5900	0.5035	0.0628	0.0865	0.45
7	1.73	0.9600	0.8938	0.0576	0.0662	0.34
8	1.93	0.8100	0.7613	0.0549	0.0487	0.25
9	2.07	0.7800	0.6686	0.0621	0.1114	0.58

Table 6: Comparison of Actual and Calculated Surface Roughness Values

RPM	Cutting Speed (m/min)	Feed (mm/Rev)	Depth of Cut (mm)	Nose Radius (mm)	Actual Ra (μm)	Calculated Ra (μm)
400	38.86	0.2	1.6	0.4	10.32	8.39
600	61.93	0.2	1.6	0.4	9.16	7.96
800	87.55	0.2	1.6	0.4	8.86	6.15
400	46.24	0.1	1.6	0.4	9.72	7.58

Table 6 (Cont.)

RPM	Cutting Speed (m/min)	Feed (mm/Rev)	Depth of Cut (mm)	Nose Radius (mm)	Actual Ra (μm)	Calculated Ra (μm)
600	73.09	0.1	1.6	0.4	9.04	6.64
800	102.43	0.1	1.6	0.4	11.86	8.11
400	53.85	0.3	1.6	0.4	13.02	9.16
600	84.38	0.3	1.6	0.4	9.44	5.84
800	117.49	0.3	1.6	0.4	12.02	8.08

$$\begin{aligned} \log Ra &= 139.669 - 0.6623 \log V \\ &+ 0.4267 \log f - 0.5037 \log d \\ &- 0.1017 \log NR \end{aligned} \quad \dots(18)$$

The surface roughness equation formulated from the above model is:

$$Ra = 139.669 V^{-0.6623} f^{0.4267} d^{-0.5037} NR^{-0.1017} \quad \dots(19)$$

The R-square value of 38.9% indicated the variability in the surface roughness was explained by the model with factors V , f and d . based on the mathematical model, it can be concluded that the cutting speed is a dominant factor in the roughness model of finish turning in heavy machining operation.

The experimental results obtained from surface roughness values have been analyzed. The data for cutting speed, feed, depth of cut and surface roughness are recorded and analyzed (software aided) in terms of obtaining confidence or prediction interval of a regression line. Two curves (Figure 3) surrounding the best-fit line define either the 95% confidence interval or 95% prediction interval of the regression line. The dashed lines (Figure 4) demarcate the confidence interval are curved. This does not mean that the confidence interval includes the possibility of curves as well as straight lines. Rather, the

Figure 3: Prediction Intervals

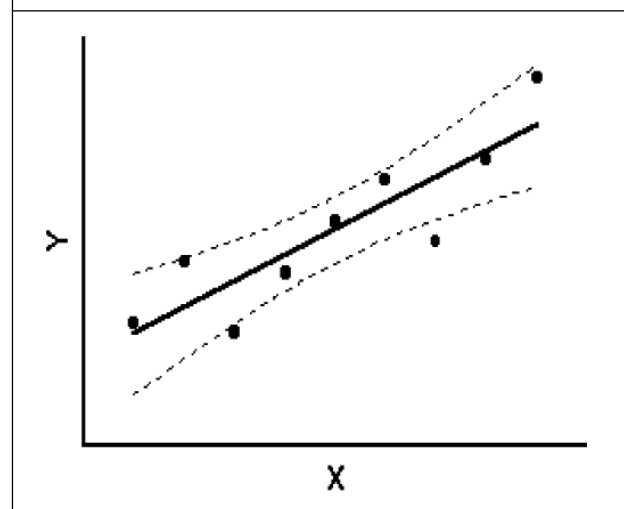
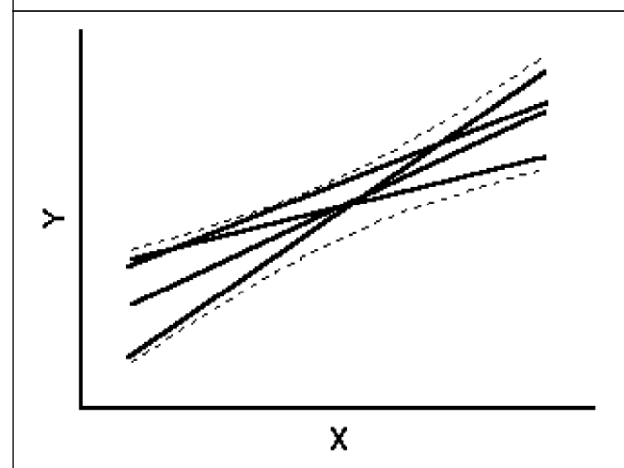
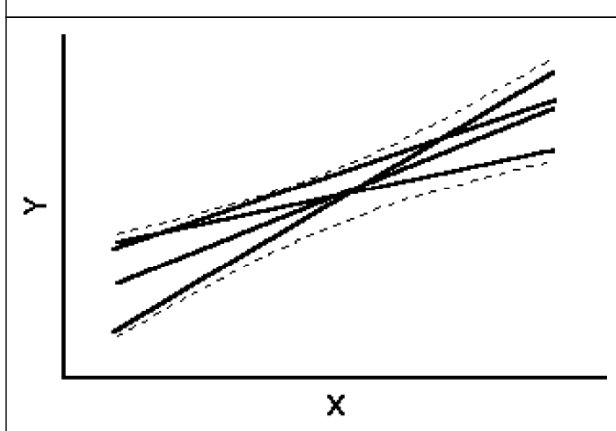


Figure 4: Confidence Graph



curved lines are the boundaries of all possible straight lines. The Figure 5 shows four possible linear regression lines (solid) that lie within the confidence interval (dashed). Given the

Figure 5: Possible Regression Graph

assumptions of linear regression, we can be 95% confident that the two curved confidence bands enclose the true best-fit linear regression line, leaving a 5% chance that the true line is outside those boundaries. Many data points will be outside the 95% confidence interval boundary. The confidence interval is 95% sure to contain the best-fit regression line. This is not the same as saying it will contain 95% of the data points. The 95% prediction interval is the area in which you expect 95% of all data points to fall. In contrast, the 95% confidence interval is the area that has a 95% chance of containing the true regression line. This graph shows both prediction and confidence intervals (the curves defining the prediction intervals are further from the regression line).

CONCLUSION

This research manuscript mainly has developed a methodology for the prediction of surface roughness in turning operation using non linear regression analysis. A good number of experiments have been conducted on EN8 work-piece material using carbide cutting tool by Design of Experiment (DOE). Strong interactions have been established among the

machining parameters for optimization a selected set of parameters/or optimization techniques as well as showing the effect of surface roughness with design parameters. The predicted surface roughness from the present non linear regression analysis model is very close to the roughness values from machined surface measured experimentally, thus showing the efficacy of regression analysis for predicting surface roughness during turning operation with machining parameters. ●

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