Int. J. Mech. Eng. & Rob. Res. 2013

International Journal of Mechanical Engineering and Robotics Research

ISSN 2278 – 0149 www.ijmerr.com
Vol. 2, No. 2, April 2013

Research Paper

A MODIFIED DYNAMIC MODEL OF THE HUMAN LOWER LIMB DURING COMPLETE GAIT CYCLE

S M Nacy[1]*, S S Hassan[1] and M Y Hanna[1]

Corresponding Author: **S M Nacy, ✉ nacys2@asme.org*

A modified method for identifying dynamic model of the human lower limb during complete gait cycle was developed in this paper. The model was based on two dimensional modeling of the human lower limb; the equations of motion were derived using Euler-Lagrange method and energy approach. The lower limb is simulated as a three link robotic manipulator (thigh, shank and foot), the force plate reaction forces were performed as external forces acted on the contact point between foot and the ground. The foot is simulated as two right angle triangles where the point of contact and the angle of contact of the foot were varied as a function of time from heel strike point to the toe off point of the foot. Therefore in this analysis we have the variation of four angles namely thigh, knee, ankle and foot. Also we considered the variation of the foot length and the angle between the line of action and the vertical line from ankle joint with the variation of the reaction forces from heel strike to toe off points along the gait cycle.

Keywords: Dynamic model, Human lower limb, Euler-lagrange method

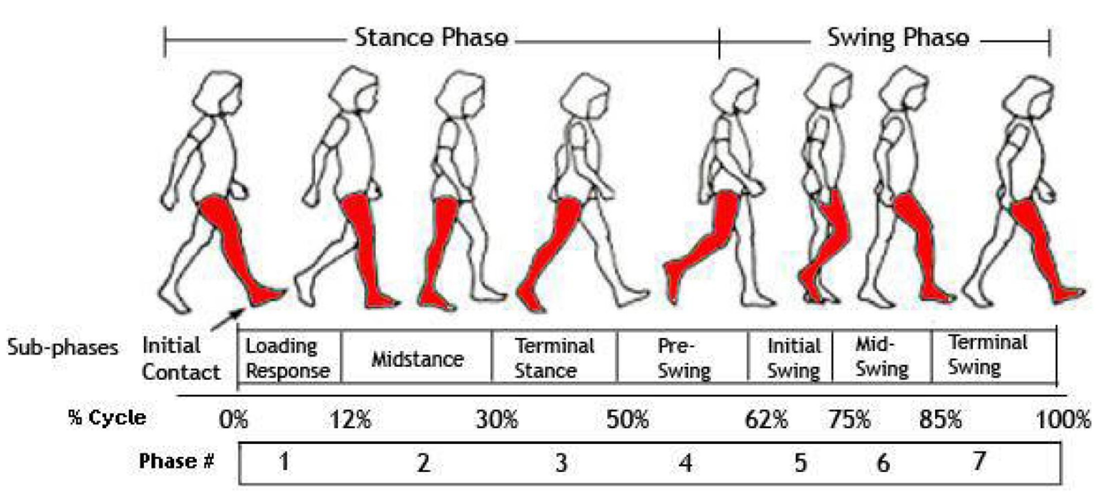
INTRODUCTION

The gait cycle is defined as the time interval between two successive occurrences of one of the repetitive events of walking (Michael, 2007). The gait cycle is divided into the stance phase (the foot is on the ground), and the swing phase (the foot is moving forward through the air), as shown in Figure 1.

Gait analysis is the study of the biomechanics of human movement aimed at quantifying factors governing the functionality of lower extremities; also the simulation of the human lower limb motion during complete gait cycle is very useful for analysis of the forces and moments acted on the human lower limb joints (Obinata *et al.*, 2004; Shabnam *et al.*, 2008; and Han and Wang, 2011). During walking, a normal healthy individual executes the most stable gait patterns, so it makes sense to capture those gait patterns and use them for the analysis of the gait cycle which is required to obtain the above target. However, in spite of the several models developed to

[1] University of Baghdad, Iraq.

Figure 1: Gait Phases

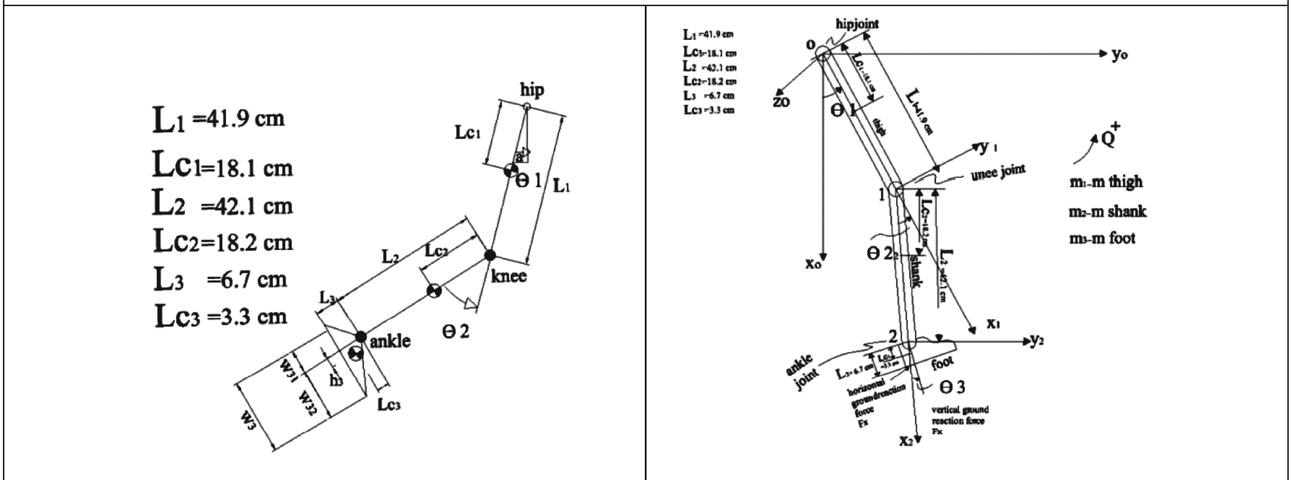


increase our understanding of normal gait, few gait models have been provided and most of them have been limited to swing phase (Luis *et al.*, 2009; Marko and Fabio, 2009; Jian *et al.*, 2010; and Han and Wang, 2011). Mathematical modeling can be effectively used to study the kinematics, dynamics and other characteristics of the human lower limb (Faramand *et al.*, 2006; Roozbeh, 2008; and Hong-Liu *et al.*, 2009). The purpose of the present study is to employ the mathematical modeling approach to analyze the kinematics and dynamics of human lower limb during complete gait cycle including both swing and stance phases and also taking into consideration the variation of the ground reaction forces and their points of contact with the foot which include many variables.

MATHEMATICAL ANALYSES

A human body can be modeled as a serial manipulator with rigid links; therefore, the

Figure 2: Model of the Lower Limb



(a) Model of the Lower Limb with Three Degree of Freedom and the Foot Model

(b) Model of the Lower Limb as Three Link Manipulator equations of motion can be obtained. In this study, the human lower limb is assumed as a three rigid link manipulator (i.e., the thigh link, the shank link and the foot link) as illustrated in Figure 2. In order to build the equations of motion, the Euler Lagrange equations are used and the energy approach was adopted, in addition to that, the equations were specialized by applying the Denavit Hatenberg (DH) convention (Mark *et al.*, 1992).

First of all, the manipulator have three joints which are the hip joint, the knee joint and the ankle joint, these three joints make three angles (θ_1, θ_2, θ_3). These three angles make the three generalized coordinates (q_1, q_2, q_3). The positive direction for these angles is viewed in Figure 2b (Andrew *et al.*, 2005; and Marko and Fabio, 2009).

The (DH) convention is applied to the manipulator and the obtained (DH) parameters are shown in Table 1.

Table 1: (DH) Parameters

Link	a_i	α_i	d_i	$?_i$
1	L_1	0	0	$?_1$
2	L_2	0	0	$?_2$
3	L_3	0	0	$?_3$

The manipulator dynamic equation of motion using matrix form is:

$D(q)q'' + c(q\ .\ q')q' + g(q) = \tau_0$ that is:

$$\begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix} \begin{pmatrix} q_1'' \\ q_2'' \\ q_3'' \end{pmatrix} + \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix} \begin{pmatrix} q_1' \\ q_2' \\ q_3' \end{pmatrix}$$

$$+ g(q) = \begin{pmatrix} \tau_{01} \\ \tau_{02} \\ \tau_{03} \end{pmatrix} \quad ...(1)$$

In order to calculate the value of $D(q)$, $C(q,\ q')$ and $g(q)$, we need to know the variation of the three angles (i.e., hip, knee and ankle angles), the human joint lower limb angles were obtained from Ref. (Michael, 2007), in these curves the variation of lower limb joint through complete gait cycle were viewed.

The inertia matrix and their second derivative of the generalized coordinates come from two parts of the kinetic energy of the three link manipulator, which are the transitional part and the rotational part of the kinetic energy.

$$(K.E)_T = 1/2m_1 V_{c1}{}^T V_{c1} + 1/2m_2 V_{c2}{}^T V_{c2} + 1/2m_3 V_{c3}{}^T V_{c3} \quad ...(2)$$

$$(K.E)_R = 1/2q'^T(JW_{c1}{}^T R_1 I_1 R_1{}^T JW_{c1} + JW_{c2}{}^T R_2 I_2 R_2{}^T JW_{c2} + JW_{c3}{}^T R_3 I_3 R_3{}^T JW_{c3})q'$$

The obtained $D(q)$ matrix is:

$$\begin{pmatrix} \begin{array}{l} I_1+I_2+I_3+m_1L_{c1}^2 \\ m_2(L_1^2+Lc_2^2+2L_1Lc_2C_2) \\ m_3(L_1^2+L_2^2+Lc_3^2+2Ll_2Lc_3C_3 \\ +2L_1L_{c3}C_{23}+2L_1L_2C_2) \end{array} & \begin{array}{l} I_2+I_3+m_2(Lc_2^2+L_1Lc_2C_2) \\ m_3(L_2^2+Lc_3^2+2L_2Lc_3C_3 \\ +L_1Lc_3C_{23}+L_1L_2C_2) \end{array} & \begin{array}{l} I_3+m_3(Lc_3^2+L_2Lc_3C_3 \\ +L_1Lc_3C_{23}) \end{array} \\ \\ \begin{array}{l} I_2+I_3+m_2(Lc_2^2+L_1Lc_2C_2) \\ m3(L_2^2+Lc_3^2+2L_2L_{c3}C_3+ \\ L_1Lc_3C_{23}+L_1L_2C_2) \end{array} & \begin{array}{l} I_2+I_3+m_2Lc_2^2 \\ m_3(Lc_3^2+L_2^2+2L_2Lc_3C_3) \end{array} & I_3+m_3(Lc_3^2+L_2L_{c3}C_3) \\ \\ \begin{array}{l} I_3+m_3(Lc_3^2+L_2L_{c3}C_3 \\ +L_1Lc_3C_{23}) \end{array} & I_3+m_3(Lc_3^2+L_2L_{c3}C_3) & I_3+m_3(Lc_3^2) \end{pmatrix}$$

The christoffel symbols $C(q\ .\ q')$ which are defined as:

$$C_{kj} = \sum_{i=1}^{n} c_{ijk}(q)q_i'$$

$$= \sum_{i=1}^{n} 1/2\,\partial d_{kj}/\partial q_i + \partial d_{ki}/\partial q_j - \partial d_{ij}/\partial q_k \quad ...(4)$$

The obtained $C(q\ .\ q')$ matrix is:

$$\begin{pmatrix} \begin{array}{l} (-m_2L_1L_{c2}S_2-m_3L_1L_2S_2 \\ -m_3L_1Lc_3S_{23})\,q_2' \\ +(-m_3L_1Lc_3S_{23} \\ -m_3L_2L_{c3}S_3)\,q_3' \end{array} & \begin{array}{l} (-m_2L_1L_{c2}S_2-m_3L_1L_{c3}S_{23} \\ -m_3L_1L_2S_2)\,q_1'+(-m_2L_1L_{c2}S_2 \\ -m_3L_1L_{c3}S_{23}-m_3L_1L_2S_2)\,q_2' \\ +(-m_3L_2Lc_3S_3-m_3L_1L_{c3}S_{23})\,q_3' \end{array} & \begin{array}{l} (-m_3L_2L_{c3}S_3-m_3L_1Lc_3S_{23})\,q_1' \\ +(-m_3L_2L_{c3}S_3-m_3L_1Lc_3S_{23})\,q_2' \\ +(-m_3L_2L_{c3}S_3-m_3L_1Lc_3S_{23})\,q_3' \end{array} \\ \\ \begin{array}{l} (m_2L_1Lc_2S_2+m_3L_1L_{c3}S_{23} \\ +m_3L_1L_2S_2)\,q_1' \\ +(-m_3L_2Lc_3S_3)\,q_3' \end{array} & (-m_3L_2Lc_3S_3)\,q_3' & \begin{array}{l} (-m_3L_2Lc_3S_3)\,q_1' \\ +(-m_3L_2Lc_3S_3)\,q_2' \\ +(-m_3L_2Lc_3S3)\,q'_3 \end{array} \\ \\ \begin{array}{l} (m_3L_2Lc_3S_3+m_3L_1Lc_3S_{23})\,q_1' \\ +(m_3L_2Lc_3S_3)\,q_2' \end{array} & \begin{array}{l} (m_3L_2Lc_3S_3)\,q_1' \\ (m_3L_2Lc_3S_3)\,q_2' \end{array} & \text{Zero} \end{pmatrix}$$

Next, the potential energy of the three link manipulator in terms of (θ_1, θ_2 and θ_3) equals:

$$P_1 = m_1gL_{c1}(1 - \cos\theta_1) \quad \ldots(5)$$

$$P_2 = m_2g[L_1(1 - \cos\theta_1) + L_{c2}(1 - \cos(\theta_1 + \theta_2)] \quad \ldots(6)$$

$$P_3 = m_3g[L_1(1 - \cos\theta_1) + L_2(1 - \cos(\theta_1 + \theta_2)) + L_{c3}(1 - \cos(\theta_1 + \theta_2 + \theta_3))] \quad \ldots(7)$$

$$P = P_1 + P_2 + P_3$$

$$g1(q) = \partial p/\partial\theta_1 = (m_1L_{c1} + m_2L_1 + m_3L_1)g\sin\theta_1 + (m_2L_{c2} + m_3L_2)g\sin(\theta_1 + \theta_2) + m_3gL_{c3}\sin(\theta_1 + \theta_2 + \theta_2) \quad \ldots(8)$$

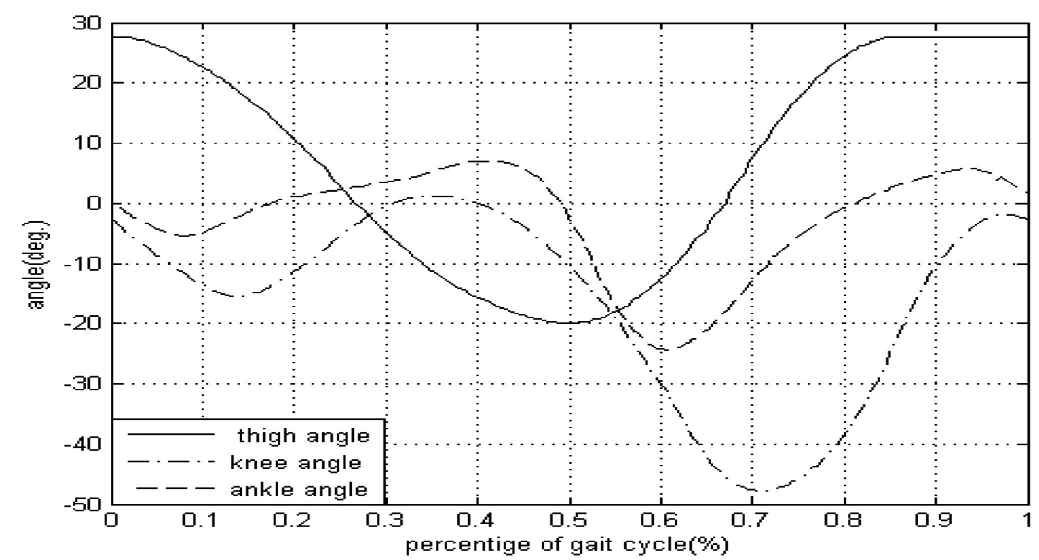
$$g_2(q) = \partial p/\partial\theta_2 = (m_2L_{c2} + m_3L_2)g\sin(\theta_1 + \theta_2) + m_3gL_{c3}\sin(\theta_1 + \theta_2 + \theta_3) \quad \ldots(9)$$

$$g_3(q) = \partial p/\partial\theta_3 = m_3gL_{c3}\sin(\theta_1 + \theta_2 + \theta_3) \quad \ldots(10)$$

The above three curves for the indicated three angles were tabulated by taking more than (240) readings through the complete gait cycle, and 100% gait cycle duration of complete gait cycle), then these readings are used to obtain a useful polynomial that represent this variation of the angle during the gait cycle with respect to the time. The obtained angles functions are the variation of these angles with respect to the time during the complete gait cycle.

The given angle curves and the estimated polynomial angle curves are viewed in Figure 3, the first and the second derivatives of these angles (i.e., the velocity and the acceleration) are obtained by differentiation of these functions once and twice respectively.

Figure 3: Variation of the Angles of Thigh, Knee and Ankle with the Percentage of Gait Cycle



The anthropometric parameters of the human segments which are function of the mass and the length of the body (James, 1993; Winter, 2005; Julie, 2008; Roozbeh, 2008; and Marko and Fabio, 2009) as shown in Figure 4 and Table 2 are used in the analysis.

The right hand of the equation of motion is:

$$\begin{pmatrix} \tau_{01} \\ \tau_{02} \\ \tau_{03} \end{pmatrix} = \begin{pmatrix} M_{theigh} - M_{knees} \\ M_{knee} - M_{ankles} \\ M_{ankle} \end{pmatrix} + J^T \begin{pmatrix} F_x \\ F_y \\ 0 \end{pmatrix} \quad ...(11)$$

The first new approach in our methodology is cleared in taking into consideration the ankle joint as a distinct joint and not fixed to the shank in addition to that, the centre of mass of the foot is calculated and the link is assumed to pass through this point. this mean that the third angle (i.e., ankle angle) consists of two angle

Figure 4: Anthropometric Data for the Human Lower Body

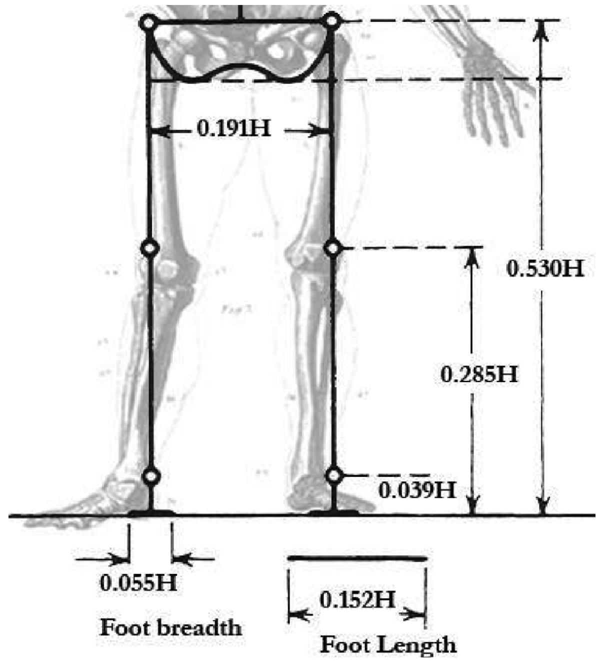


Table 2: The Anthropometric Parameters of the Human Segments

Measure	Thigh	Shank	Foot
Mass	m_1 = 0.1*m	m_2 = 0.0465*m	m_3 = 0.0145*m
Moment of Inertia	I_1 = m_1*(0.323*L_1)^2	I_2 = m_2*(0.302*L_2)^2	I_3 = m_3*(0.475*L_3)^2
Centre of Mass	L_{c1} = 0.433*L_1	L_{c2} = 0.433*L_2	L_{c3} = 0.5*L_3
	From hip	From knee	From ankle

(i.e., the variation in the ankle joint during the gait cycle and angle between the vertical line from the ankle joint to the ground and the line from the ankle joint through the centre of mass of the foot) the position and the length of the foot through the centre of mass point are functioned with respect to the length of the body, this permits to change the above data with different tested bodies.

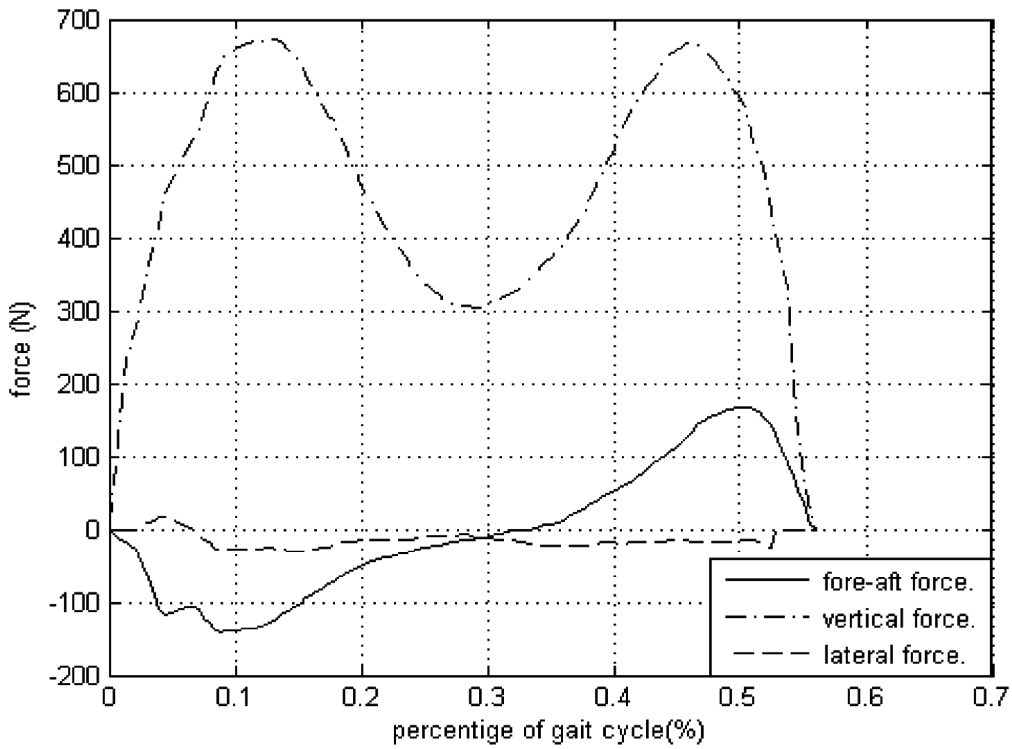
The variation of the ground reaction forces, i.e., the vertical forces, the horizontal forces (fore-aft forces) and the lateral forces are taken from reference (Michael, 2007).

In these curves we have two peaks, the first one after the heel strike of the foot with the ground and the second is before the toe off. The vertical forces represent the ground reaction due the weight of the body during gait cycle and the two peaks are positive, while for the fore-aft force and the lateral force the first peak is negative and the second peak is positive, the lateral force is not considered in our calculation. The curves for the ground reaction forces were tabulated by taking more than (240) readings through the complete gait cycle (100% gait cycle or duration of complete gait cycle), then these readings are used to obtain a useful polynomial that represent the variation of these forces with respect to the time during the complete gait cycle.

The used force curves are obtained from testing of a person weighted (55 kg) (Michael, 2007); the obtained tabulated data were functioned with respect to the weight of the body to be useful for different tested bodies. The given ground reaction forces and the estimated polynomial of these force are viewed in Figure 5. The given ground reaction forces are acting on a variable point of contact of lower face of the foot, but most researchers are assumed to be acted on single point almost the time of gait cycle and others are assumed only two or three points. The second new approach in this study is performed by assuming the point of contact of the applied ground reaction forces are varied continuously during the time from heel strike point to the toe off point were acted forces reach zero (i.e., swing phase). The foot is assumed to be as two triangles one is small and the other is bigger, as in Figure 2a, the point of contact assumed to be continuously varied, therefore we have at each time, the length of the foot from the ankle joint to the point of contact and also the angle between the line from the ankle joint to the point of contact and the vertical line from the ankle joint.

In calculating the hand of the ground reaction forces act on the ankle joint, we have two angles, first the angle of the ankle joint (i.e., θ_3) and the angle of the line of contact and the vertical foot line (θ_4), for the knee joint we have three angles (θ_2, θ_3, θ_4) and for the hip joint we have four angles (θ_1, θ_2, θ_3, θ_4).

Figure 5: Variation of the Ground Reaction Forces with the Percentage of Gait Cycle



The dimensions of the foot (the vertical and the horizontal dimension of the triangles) are functioned with respect to the length of the body. The acted torques on the ankle, knee and hip joints are obtained from substituting Equation (11) in Equation (1).The above equations are solved by using MATLAB program.

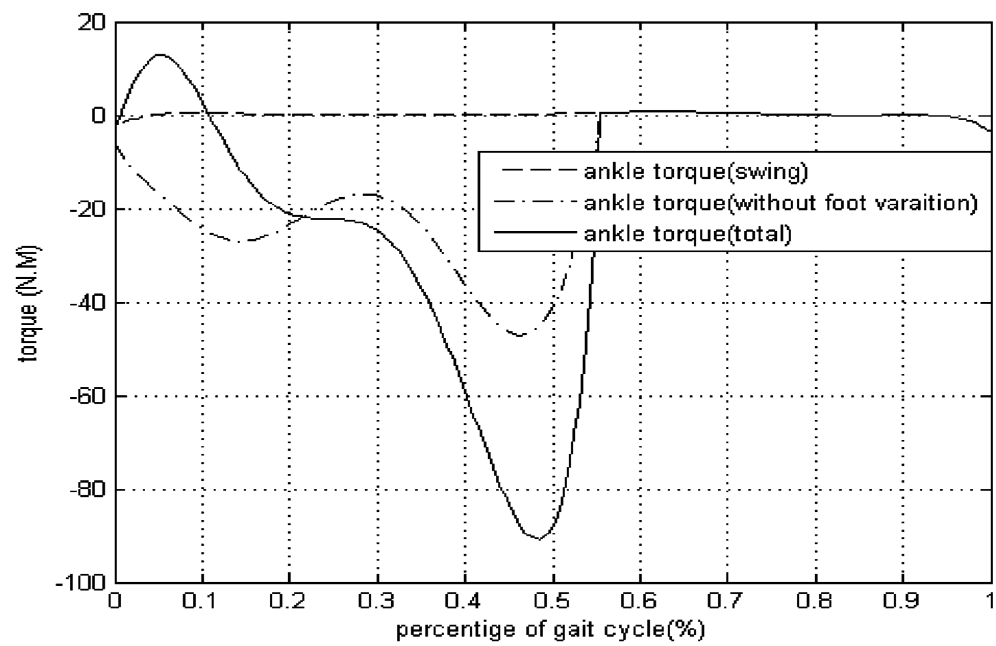
RESULTS AND DISCUSSION

Figure 3 represents the variation of the thigh, knee and ankle joints respectively (12) and their estimated curves obtained from polynomial fittings of each angle data, the obtained fittings curves show an acceptable variation of these angles with respect to the measured angles, these estimated polynomials for each of the three angles are used to obtain the variation of the velocity and acceleration of the three joints during complete gait cycle which important in the calculations of the equations of motions of the three joints.

Figure 5 represents the variation of the ground reaction forces which are the vertical ground reaction force, the horizontal (fore-aft) ground reaction force and the lateral ground reaction force, which are acted on the points of action on the foot during the gait cycle, the obtained fitting curves show an acceptable variation of these forces in compared to the measured forces, the estimated polynomials for each of the three forces are functioned with respect to the mass of the tested body (M), therefore the obtained polynomials are useable for different human bodies with different mass and length (L).

Figure 6 shows the calculated torque variation applied at the ankle joint during complete gait cycle for three cases, the first one shows the calculated torque at the ankle joint by estimation of the lower limb as a swing joint only, i.e., the action of the ground reaction forces and the variation of the points of contact of these forces on the foot and the foot shape

Figure 6: Variation of the Ankle Torque with the Percentage of Gait Cycle



are not considered, the figure shows that there is two small peaks at the heel strike and the toe-off times during gait cycle. The second one shows the variation of torque on the ankle joint but with the action of these forces are fixed in one point on the foot during the gait cycle, by representing the foot as a line from the ankle joint, the figure shows also two peaks one near the heel strike and the other near toe-off time during gait cycle but the peaks are both have same negative values because the action of these ground reaction forces are acted on fixed point and the variation of the point of action on the foot and the position of the centre of gravity of the foot are not considered.

The length from the ankle joint to the point of action of the ground reaction forces during complete gait cycle is calculated by considering the shape of the foot as two triangles and the point of action varied with time from heel strike point to the toe-off point (stance phase). The variation of the angle between two lines, the line from the ankle joint to the ground reaction force point of action and the vertical line from the ankle joint to the foot, this angle with the line of foot is important to calculate the hand of the vertical and horizontal ground reaction forces with respect to the ankle joint.

The obtained line of action and angle of action are functioned with respect to the length of the human body (L). The third one shows that two peaks also but with higher values due to the variation of the hand of action of the ground reaction forces on the foot which is varied continuously during the gait cycle, in addition to that, the first peak (near the heel strike) have positive value but lower than the second peak (near the toe-off) which have negative value. The obtained values for the two peaks and the higher value at the toe-off point are similar to big extent with that obtained from other researchers. The third one shows the higher effect of foot shape and the variation of point of action on the applied torque which corrects the values obtained from estimation of the foot as a lined link.

Figure 7 shows the variation of the applied torque on the knee joint during the complete gait cycle for three cases. The first one shows the calculated values of the knee torque by considering the swing case only, the figure shows two small peaks near heel strike and toe-off times during gait cycle ,the peak near the toe-off is higher. The second one shows the knee torque with the action of ground reaction forces acted on fixed point during the gait cycle, the figure shows one peak near toe-off time, this is because the variation of line and the angle of action are not considered. The third one shows the knee joint torque with the action of the ground reaction forces, the variation the line of action and angle of action during gait cycle, the figure shows two peaks with higher one near the heel strike point. The obtained curve for knee joint torque is also similar to big extent to that obtained from other researchers.

Figures 8 show the variation of the applied torque on the thigh joint during complete gait cycle for three cases. The first one shows the calculated values of the thigh torque by considering the lower limb as a swing joint only; the figure shows two small peaks near the heel strike and toe-off time. The second one shows the variation of thigh torque with the action of ground reaction forces acted on fixed point during gait cycle, the figure shows one peak

Figure 7: Variation of the Knee Torque with the Percentage of Gait Cycle

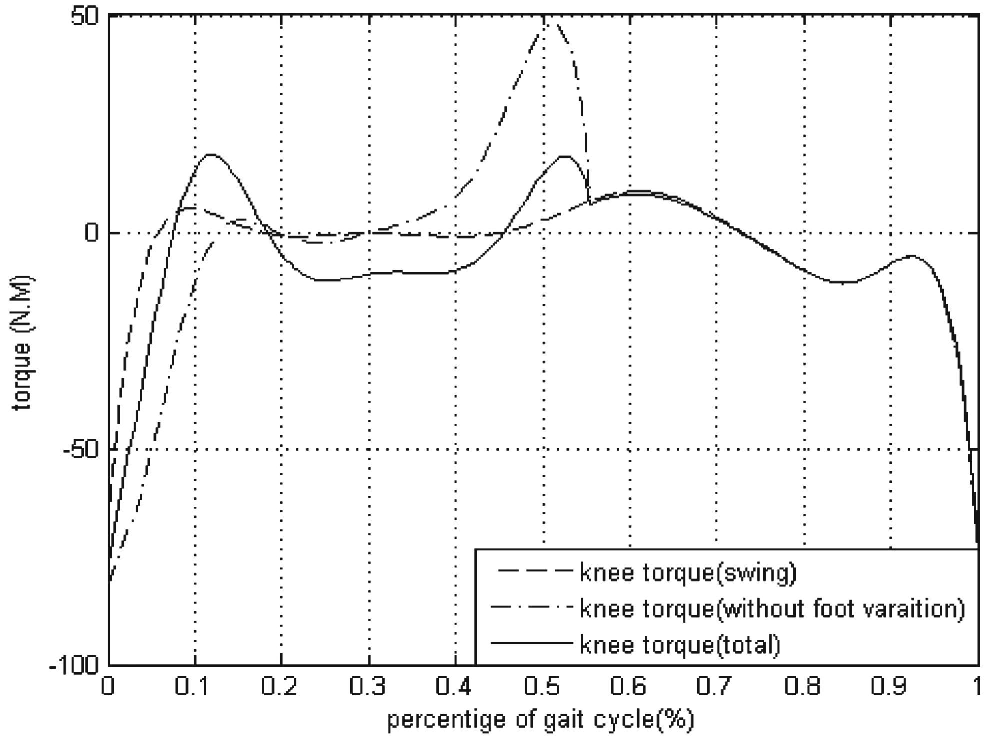
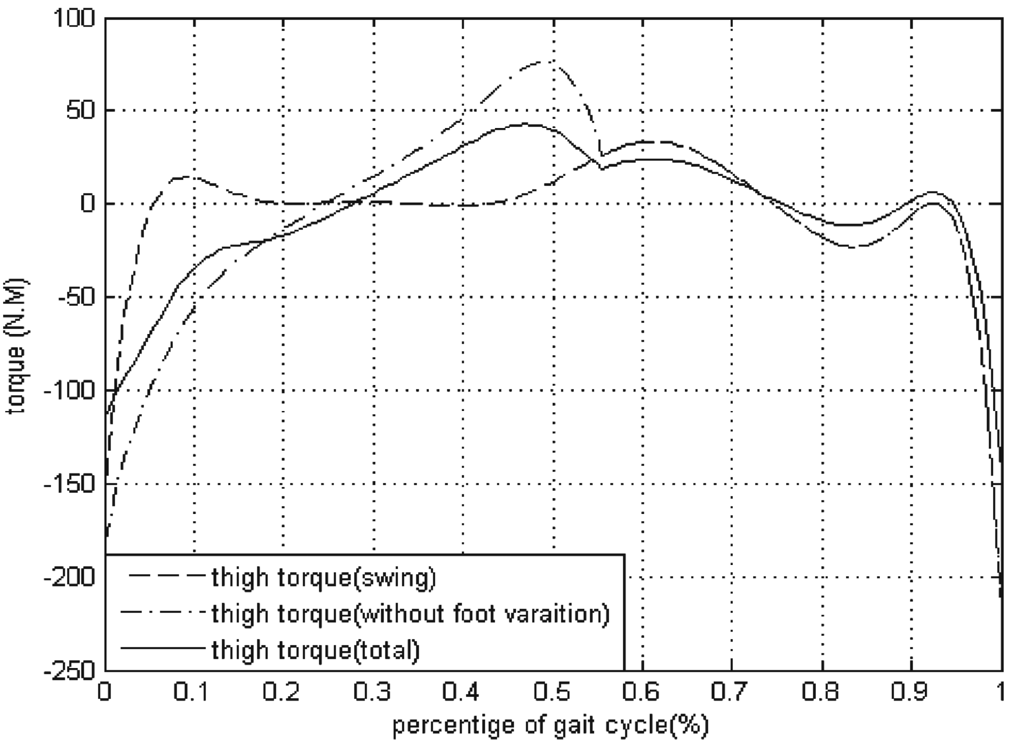


Figure 8: Variation of the Thigh Torque with the Percentage of Gait Cycle



near the toe-off time. The third one shows the variation of the thigh torque with the action of ground reaction forces and the variation of line of action and angle of action during the gait cycle, the figure shows one peak near the toe-off point but with smaller value than that of second figure due to the variation of line and the angle of the foot which made the maximum value to be smaller. The obtained values from the third figure are also similar to big extent with that of the other researchers.

The obtained torque on the thigh, knee, and ankle joints obtained from the method estimated in this paper are fitted with big extent with that obtained from the other researchers which they obtained the estimated torque from different ways, for example, from experimental tests on human lower limb joints during gait cycle (1). This paper shows that we can obtain the applied torque on the lower limb joints from simple experimental tests with the absent of specialized equipments for measuring the velocity, the acceleration, the torque, etc., at the lower limb joints, the needed experimental tests are only the treadmill for calculating the ground reaction forces and the variation of the thigh, knee, and ankle joints angles during gait cycle.

The constructed MATLAB program and the obtained curves are very important in the design of the lower limb components like socket, knee joint, ankle joint and foot, also the used components for amputee persons can be input data for our MATLAB program to obtain the loads at the lower limb joints for the used lower limb components.

REFERENCES

1. Andrew Chu *et al.* (2005), "On the Biomimetic Design of the Barkeley Lower Extremity Exoskeleton (BLEEX)", Proceeding of the IEEE, International Conference on Robotics and Automation, Barcelona, Spain.
2. Faramand F *et al.* (2006), "Kinematic and Dynamic Analysis of the Gait Cycle of Above-Knee Amputee", *Scientia Iranica*, Vol. 13, No. 3, pp. 261-271.
3. Han Yali and Wang Xingsong (2011), "The Biomechanical Study of Lower Limb During Human Walking", *Science China Technological Sciences*, Vol. 54, No. 4, pp. 983-991.
4. Hong-Liu, Zhao-Hong Xu *et al.*(2009), "Inverse Dynamic Compound for Intelligent Artificial Leg Based on PD-CMAC", University of Shanghai for Science and Technology Issue, Vol. 8.
5. James Lanphier Patton (1993), "Forward Dynamic Modeling of Human Locomotion", MSC Thesis, Department of Material Science and Mechanics, Michigan State University.
6. Jian Zhang *et al.* (2010), "Gait Analysis of Powered Bionic Lower Prosthesis", Proceedings of the IEEE, International Conference on Robotics and Biomechanics, Tianjin, China.
7. Julie Ann Thompson (2008), "Design, Construction and Validation of Cadaver Knee Motion Testing Device", Undergraduate Honors Thesis, Mechanical Engineering, Ohio State University.
8. Luis I Lugo-Villeda *et al.* (2009), "A Mechatronic Analysis and Synthesis of

Human Walking Gait", Proceedings of the IEEE International Conference on Mechatronics, Spain.

9. Mark W Spong *et al.* (1992), *Robot Modeling and Control*, 1st Edition, John Wiley & Sons Inc., New York.

10. Marko Ackermann and Fabio G Cozman (2009), "Automatic Knee Flexion in Lower Limb Orthosis", *J. of the Braz. Soc. of Mech. Sci. & Eng.*, Vol. 31, No.4, p. 305.

11. Michael W Whittle (2007), "Gait Analysis an Introduction", Butterworth Heinemann Elsevier Ltd.

12. Obinata G *et al.* (2004), "A New Method for Identifying Link Models of Lower Limbs", Proceedings of the IEEE EMBS San Francisco, USA.

13. Roozbeh Borjian (2008), "Design, Modeling and Control of an Active Prosthetic Knee", *Applied Science in Mechanical Engineering*, Ph.D. Thesis, Waterloo, Ontario, Canada.

14. Shabnam Pejhan *et al.* (2008), "Design Optimization of Above-Knee Prosthesis Based on the Kinematics of Gait".

15. Winter D A (2005), *Biomechanics and Motor Control of Human Movement*, 3rd Edition, John Wiley & Sons.

APPENDIX

Nomenclature

C_{123}	: Cosine of $(\theta_1 + \theta_2 + \theta_3)$
$C(q \cdot q')$	: Is the christoffel symbol matrix which matches with first derivative of the generalized coordinates (q_1', q_2', q_3') and $(q_i' = \theta_i')$, this matrix is also divided into two types. Terms involving a product of the type (q_i^2) are called centrifugal, while those involving a product of the type $(q_i' q_j')$ where $(i \neq j)$ are called coriolis terms
$D(q)$	: Is the inertia matrix which matches with the second derivative of the generalized coordinates (q_1'', q_2'', q_3'') and $(q_i'' = \theta_i'')$
F_x	: The horizontal ground reaction forces acted at the foot during gait cycle
F_y	: The vertical ground reaction forces acted at the foot during gait cycle
$g(q)$	: Is the gravitational potential energy which is function of the generalized coordinates (q_i) only (the mass multiplied by the gravitational acceleration and the height at its centre of mass).
I_1	: Moment of the inertia of the thigh
I_2	: Moment of the inertia of the shank
I_3	: Moment of the inertia of the foot
q_i	: The generalized coordinates which represents the joint angles $(\theta_1, \theta_2, \theta_3)$
q_i''	: The second derivative of the generalized coordinates (q_i)
L	: Body length
L_1	: Length of the thigh
L_{c1}	: Distance of the center of mass of the thigh from the hip
L_2	: Length of the shank
L_{c2}	: Distance of the center of mass of the shank from the knee
L_3	: Vertical length of the foot
L_{c3}	: Vertical distance from the ankle joint to the center of mass point of the foot
M	: Mass of the body
M_{ankle}	: Acted torque on the ankle joint
M_{ankles}	: The torque at ankle at the swing phase
M_{knee}	: Acted torque on the knee joint
M_{knees}	: The torque at knee at the swing phase
M_{thigh}	: Acted torque on the thigh joint
m_1	: Mass of thigh
m_2	: Mass of shank
m_3	: Mass of foot
S_{123}	: Sine of $(\theta_1 + \theta_2 + \theta_3)$
W	: Body weight
W_3	: Horizontal length of foot
W_{31}	: Horizontal length of foot from heel strike point to the center of mass point
W_{32}	: Horizontal length of foot from center of mass point to the toe off point
W_{c3}	: Distance from the ankle joint to the center of mass point
θ_1	: Angle between the thigh line and the vertical line
θ_2	: Angle between the shank line and the thigh line
θ_3	: Angle between the foot vertical line and the shank line
τ_{0i}	: Induced and applied torque