

International Journal of Mechanical Engineering and Robotics Research

ISSN 2278 – 0149 www.ijmerr.com Vol. 1, No. 2, July 2012 © 2012 IJMERR. All Rights Reserved

Research Paper

ON THE STEADY-STATE PERFORMANCE OF HYDRODYNAMIC FLEXIBLE JOURNAL BEARINGS OF FINITE WIDTH LUBRICATED BY FERRO FLUIDS WITH MICRO-POLAR EFFECT

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The present work deals with the micro-polar ferro fluid lubrication theory to the problem of the static characteristics of finite flexible journal bearings. Based on a magnetic field distribution with a gradient in both circumferential and axial directions, a modified Reynolds equation has been derived by incorporating the characteristics of micro-polar fluid and using the magnetic forces on magnetic particles as the body-external forces. The numerical solution of the discretized Reynolds equation by the finite difference technique with an appropriate iterative procedure has resulted in the film pressure distribution. With the help of the film pressure, load capacity, side leakage flow and frictional parameter are obtained for various parameters. The results indicate that the influences of micro-polar and magnetic effects on the bearing performance characteristics are signicantly apparent.

Keywords: Flexible journal bearings, Hydrodynamic lubrication, Static characteristics, Magnetic fluids, Micro-polar effect

INTRODUCTION

Ferro fluids are suspensions of single domain magnetic particles, stabilized by surfactants in the standard lubricating fluids. When a magnetic field is applied to the ferro fluid, each particle experiences a force that depends on the magnetization of the magnetic material of the particles and on the strength and position of the applied field (Rosensweig, 1985). As a controllable fluid in terms of its position, orientation and movement, the ferro fluids have different applications in a variety of engineering devices and systems such as in lubrication and sealing of lubricated bearings. Therefore, this invokes interest in carrying out more research for the improvement of selfacting characteristics of journal bearings lubricated with ferro fluid.

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S K Guha, 2012

Several studies (Tipei, 1982 and 1983; Sorge, 1987; Cheng et al., 1987; and Zhang, 1991) have been reported in the field of ferro fluid - lubricated journal bearings. All the above investigations assume that the ferro fluid behaves as a Newtonian fluid. The research work by Osman (1999) is the first one that considered the non-Newtonian behavior of the ferro fluid, using a parabolic distribution of magnetic field model in the axial direction. Osman et al. (2001) analyzed the static and dynamic characteristics of finite journal bearings lubricated with ferro fluid, considering the perturbation technique. Later on, Nada and Osman (2007) obtained the static characteristics of ferro fluid - lubricated journal bearings with coupled stress effects.

As the ferro magnetic lubricant contains the magnetic particles together with the chemical additives for better lubricating effectiveness, constituting special substructure, the concept of Newtonian fluid approximation is not a suitable approach. A number of microcontinuum theories has been developed for describing the peculiar behavior of fluids containing substructure, which can translate, rotate or even deform independently (Ariman and Turk, 1973 and 1974). The theory of micropolar fluids and the field equations were first developed by Eringen (1966) from the basic theories of micro-continua. Based on this work, extensive studies (Prakash and Sinha, 1975a, 1975b and 1976; Zaheeruddin and Isa, 1978; Tipei, 1979; Singh and Singh, 1982; Huang et al., 1988; Khonsari and Brewe, 1989; and Das et al., 2002) on the performances of journal bearings lubricated with micro-polar fluid have been reported in the literature. All these works have not included the elastic deformation of bearing surface.

It has long been realized that the assumption of perfectly rigid bearing shell in the analysis may be unrealistic especially for bearings operating with small film thickness, i.e., at higher values of eccentricity ratios. Because, the higher eccentricity ratios result in higher film pressure distribution in the bearing clearance, which cause the deformation of the bearing surfaces and consequently the changes in the configuration of film profile. Eventually, the performance characteristics of lubricated journal bearing are changed as compared with the case of a rigid bearing shell. Conway and Lee (1977) carried out the analysis of short flexible journal bearings. The investigation reveals that the average film pressure deteriorates with increase in flexible parameter. In several studies (Donoghue et al., 1967; Brighton et al., 1968; Conway and Lee, 1975; and Majumdar et al., 1988), the effect of elastic deformation of bearing surface has been considered in the analysis of performance of finite hydrodynamic journal bearings. Later on, a few investigations have dealt with the elastohydrodynamic effect (Oh and Huebner, 1973; and Jain et al., 1981) in the field of lubricated journal bearings.

However, so far there is no investigation available that addresses the effect of elastic deformation of bearing surface on the steadystate characteristics of finite hydrodynamic journal bearing under the micro-polar lubrication with ferro fluid. The objective of this paper is to reveal the micro-polar effect on the steady-state performance of flexible journal bearings lubricated with magnetic fluids. In the present analysis, a more generalized Reynolds equation is derived taking into account of the non-Newtonian behavior of the ferro fluid. More realistic magnetic field models are used. Influence of the micro-polar and magnetic parameters on the static characteristics of magnetized bearings is investigated.

MODIFIED REYNOLDS EQUATION

The induced magnetic force per unit volume of a ferro fluid under the effect of magnetic field is given by Cowley and Rosensweig (1967) and Zelazo and Melcher (1969).

$$\boldsymbol{F}_{m} = (\nabla \times \boldsymbol{h}_{m}) \times \boldsymbol{B} + \mu_{0} \boldsymbol{M}_{g} (\nabla \boldsymbol{h}_{m}) \qquad \dots (1)$$

Where, *B* is the magnetic field density vector and $(\nabla \times h_m)$ represents the induced free current. Since the electrical properties of the ferro fluid are similar to those of the base fluid, they are non-conductive and no free currents are induced. The first term, then, can be cancelled and the equation reduces to

$$F_m = \mu_0 M_g(\nabla h_m) \qquad \dots (2)$$

Considering isothermal conditions and linear behavior of the ferro fluid, the magnetization M_a of the magnetic material is:

$$M_g = X_m h_m$$

The induced magnetic force can be defined as:

$$F_m = \mu_0 X_m h_m (\nabla h_m) \qquad \dots (3)$$

This force will be considered as a body external force in the governing equations as derived as follows:

The governing equations for the steadystate flow of the micro-polar fluid, written in the tensor notation as mentioned in the reference (Khonsari and Brewe, 1989) can be simplified considerably for the incompressible fluid $(\rho = \text{constant})$ in the absence of inertial forces and in the presence of the magnetic forces as the external body forces. Moreover, the thermodynamic pressure can be replaced by the film pressures (Khonsari and Brewe, 1989). Having made the afore-mentioned assumptions, the so called governing equations for the micro-polar lubrication reduces to as follows:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \qquad \dots (4)$$

$$\frac{\left(2\,\mu+\chi\right)}{2}\cdot\frac{\partial^2 V_x}{\partial y^2} + \chi \frac{\partial V_3}{\partial y} - \frac{\partial p}{\partial x} + F_{mx} = 0$$
...(5)

$$\frac{\partial p}{\partial y} = 0 \qquad \dots (6)$$

$$\frac{(2 \mu + \chi)}{2} \cdot \frac{\partial^2 V_z}{\partial y^2} - \chi \frac{\partial V_1}{\partial y} - \frac{\partial p}{\partial z} + F_{mz} = 0$$
...(7)

$$\gamma \frac{\partial^2 v_1}{\partial y^2} + \chi \frac{\partial V_z}{\partial y} - 2\chi v_1 = 0 \qquad ...(8)$$

$$\gamma \frac{\partial^2 \mathbf{v}_2}{\partial \mathbf{y}^2} - 2\chi \mathbf{v}_2 = \mathbf{0} \qquad \dots (9)$$

$$\gamma \frac{\partial^2 v_3}{\partial y^2} - \chi \frac{\partial V_x}{\partial y} - 2\chi v_3 = 0 \qquad \dots (10)$$

Now, by solving Equations (7) and (8) with the following boundary conditions

at
$$y = 0$$
, $V_z = 0$, $v_1 = 0$...(11)

at
$$y = h$$
, $V_z = 0$, $v_1 = 0$...(12)

 V_1 and v_1 are obtained.

at y = 0, $V_x = 0$, $v_3 = 0$...(13)

at y = h, $V_x = U$, $v_3 = 0$...(14)

 V_{y} and v_{3} are obtained.

So, in presence of magnetic forces as the external body forces, the velocity components, V_x and V_z of the micro-polar fluid are derived as:

$$Vx = \frac{y(y-h)}{2\mu} \left(\frac{\partial p}{\partial x} - F_{mx} \right) + \frac{h}{2\mu} \cdot \frac{N^2}{m}$$

$$\left\{ \sinh my - \frac{(\cosh my - 1)(\cosh mh - 1)}{\sinh mh} \right\}$$

$$\left(\frac{\partial p}{\partial x} - F_{mx} \right) - \frac{U}{2 \left\{ \frac{2N^2}{m} - \frac{h(\cosh mh + 1)}{\sinh mh} \right\}}$$

$$\left[\frac{2y(\cosh mh + 1)}{\sin hmh} + \frac{2N^2}{m} (\sinh my) \right]$$

$$\left\{ \frac{(\cosh my - 1)}{\sin hmy} - \frac{(\cosh mh + 1)}{\sinh mh} \right\} \right] \qquad \dots (15)$$

$$V_z = \frac{y(y-h)}{2\mu} \left(\frac{\partial p}{\partial z} - F_{mz} \right) + \frac{h}{2\mu} \cdot \frac{N^2}{m}$$

$$\left\{ \sinh my - \frac{(\cosh my - 1)(\cosh my + 1)}{\sinh mh} \right\}$$

$$\left(\frac{\partial p}{\partial z} - F_{mz} \right) \qquad \dots (16)$$

where, $m = 2N \sqrt{\frac{\mu}{\gamma}}$

Substituting V_x and V_z in the continuity Equation (1) and integrating across the film thickness using the following boundary conditions

 $V_y(x, 0, z) = V_y(x, h, z) = 0$ (non-squeezing case) the modified Reynolds equation is obtained as:

$$\frac{\partial}{\partial x} \left[g(\Lambda, N, h) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[g(\Lambda, N, h) \frac{\partial p}{\partial z} \right]$$
$$= 6 \mu U \frac{\partial h}{\partial x} + \frac{\partial}{\partial x} \left[F_{mx} g(\Lambda, N, h) \right]$$
$$+ \frac{\partial}{\partial z} \left[F_{mz} g(\Lambda, N, h) \right] \qquad \dots (17)$$

where,

$$g(\Lambda, N, h) = h^3 + 12h\Lambda^2 - 6h^2\Lambda N \coth\left(\frac{Nh}{2\Lambda}\right)$$

$$\mathsf{N} = \left(\frac{\chi}{2\mu + \chi}\right)^{\frac{1}{2}}, \ \Lambda = \left(\frac{\gamma}{4\mu}\right)^{\frac{1}{2}}$$

 μ is the viscosity of the base fluid as in the case of the Newtonian fluids and χ and γ are two additional viscosity coefficients of the micro-polar ferro fluids. These viscosity parameters are grouped in the form of two parameters viz. N^2 and Λ . *N* is a dimensionless parameter called the coupling number as it signifies the coupling of the linear and angular momentum equations. Λ is called the characteristics length as it characterizes the interaction between the micro-polar fluid and the film gap.

 F_{mx} and F_{mz} are the magnetic force components which are defined as per Equation (3) as follows.

$$F_{mx} = \mu_0 X_m h_m \frac{\partial h_m}{\partial x},$$

$$F_{mz} = \mu_0 X_m h_m \frac{\partial h_m}{\partial z} \qquad \dots (18)$$

With the following substitutions

$$\theta = \frac{x}{R}, \ \overline{z} = \frac{2z}{L}, \ \overline{h} = \frac{h}{C}, \ \overline{p} = \frac{pC^2}{\mu \Omega R^2}$$
$$\ell_m = \frac{C}{\Lambda}, \ U = \Omega R, \ \overline{h}_m = \frac{h_m}{h_{m0}}$$

Equation (17) reduces to its nondimensional form as:

$$\frac{\partial}{\partial \theta} \left[\overline{g} \left(\ell_m, N, \overline{h} \right) \frac{\partial \overline{p}}{\partial \theta} \right] + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \overline{z}} \left[\overline{g} \left(\ell_m, N, \overline{h} \right) \frac{\partial \overline{p}}{\partial \overline{z}} \right]$$
$$= 6 \frac{\partial \overline{h}}{\partial \theta} + 4 \left(\frac{L}{D} \right)^2 \alpha \frac{\partial}{\partial \theta} \left[\overline{g} \left(\ell_m, N, \overline{h} \right) \overline{h}_m \frac{\partial \overline{h}_m}{\partial \theta} \right]$$
$$+ 4 \alpha \frac{\partial}{\partial \overline{z}} \left[\overline{g} \left(\ell_m, N, \overline{h} \right) \overline{h}_m \frac{\partial \overline{h}_m}{\partial \overline{z}} \right] \qquad \dots (19)$$

where,

$$\overline{g}\left(\ell_{m}, N, \overline{h}\right) = \overline{h}^{3} + \frac{12\overline{h}}{\ell_{m}^{2}} - \frac{6N\overline{h}^{2}}{\ell_{m}} \coth\left(\frac{N\ell_{m}\overline{h}}{2}\right),$$
$$\alpha = \frac{(h_{m0})^{2}\mu_{0}X_{m}C^{2}}{\mu\Omega L^{2}}$$

MAGNETIC FIELD MODEL

The applied magnetic field is that produced by displaced finite current carrying wire. It has a field gradient in both axial and circumferential directions of the bearing. In the nondimensional form, it is defined by Nada and Osman (2007).

$$\overline{h}_{m}(\theta, \overline{z}) = \beta \left[\sin \left\{ \tan^{-1} \left(\lambda + \lambda \overline{z} \right) \beta \right\} \right]$$
$$\overline{h}_{m}(\theta, \overline{z}) = \beta + \sin \left\{ \tan^{-1} \left(\lambda - \lambda \overline{z} \right) \beta \right\} \quad \dots (20)$$

where,

$$\beta = \left\{ 1 + K^2 - 2K \cos(\psi - \theta) \right\}^{-0.5}$$
$$\overline{h}_{m0} = \frac{h_m}{h_{m0}}, \ h_{m0} = \frac{I}{4\pi R}, \ \lambda = \frac{L}{2R} = \frac{L}{D}$$
$$K = \frac{R_w}{R} = \text{Distance Ratio Parameter}$$

FILM THICKNESS OF FLEXIBLE BEARING

In case of a flexible bearing shell, the steadystate film thickness, *h* is given by

$$\overline{h} = \frac{h}{C} = 1 + \varepsilon_0 \cos \theta + \overline{\delta} \left(\theta, \overline{z} \right)$$

Where, $\overline{\delta}(\theta, \overline{z})$ is the non-dimensional deformation of bearing surface. Following a method similar to that as mentioned in Donoghue *et al.* (1967), $\overline{\delta}(\theta, \overline{z})$ is obtained as follows.

$$\overline{\delta}\left(\theta, \overline{z}\right) = 2\left(1+\sigma\right) F\left[\overline{p}_{0,0} d_{0,0} + \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \overline{p}_{k,n} d_{k,n} \cos\left(n\theta + \beta'_{k,n}\right) \cos\left(k\pi \overline{z}\right)\right]$$
$$(k, n) \neq (0, 0) \qquad \dots (21)$$

where as per the Majumdar *et al.* (1988), the followings are defined.

$$\overline{p}_{0,0} = \frac{1}{\pi} \int_{0}^{2\pi} \int_{0}^{1} \overline{p} \cdot d\theta \cdot d\overline{z}$$

$$\overline{\rho}_{k,n} = \frac{2}{\pi} \left[A_c^2 + A_s^2 \right]^{\frac{1}{2}}, \ \beta' = \tan^{-1} \left[-\frac{A_s}{A_c} \right]$$
$$A_c = \int_0^{2\pi} \int_0^1 \overline{\rho} \cos k\pi \overline{z} \cdot \cos n\theta \cdot d\theta \cdot d\overline{z}$$
$$A_s = \int_0^{2\pi} \int_0^1 \overline{\rho} \cos k\pi \overline{z} \cdot \sin n\theta \cdot d\theta \cdot d\overline{z}$$
$$F = \text{Flexibility parameter} = \frac{\mu \Omega R^3}{EC^3},$$
$$G = \frac{E}{2(1+\sigma)}$$

$$d_{k,n}$$
 = Distortion Coefficient = $\frac{Gu_r}{Rp}$,

 u_r = Radial displacement of bearing liner

NUMERICAL PROCEDURE

Equation (19) is discretized into finite difference form and solved by the Gauss-Seidel iterative technique using the overrelaxation factor, satisfying the appropriate boundary conditions. The boundary conditions are as follows:

$$\overline{p}(\theta \pm 1) = 0, \frac{\partial \overline{p}(\theta, 0)}{\partial \overline{z}} = 0$$
$$\overline{p}(\theta_2, \overline{z}) = \frac{\partial \overline{p}(\theta_2, \overline{z})}{\partial \theta} = 0$$

Where, θ_2 is the angular location of the film rupture.

Prior to starting the iteration process, the steady-state film pressure is initially determined considering the bearing liner as rigid one. With the help of this pressure expressed in double Fourier series, \overline{p}_{kn} and

 $\beta'_{k,n}$ are obtained by numerical integration method. From the known values $\overline{p}_{k,n}$, $\beta'_{k,n}$ of and the distortion coefficient, $d_{k,n}$, the nondimensional deformation, $\overline{\delta}(\theta, \overline{z})$ is computed. With this new deformation, $\overline{\delta}(\theta, \overline{z})$, Equation (19) is solved again to get new film pressure distribution. This process is repeated through iteration technique till convergence is achieved. With the help of this film pressure after convergence, the steady-state performance characteristics of flexible bearing are obtained.

BEARING PERFORMANCE CHARACTERISTICS

Load Capacity

The non-dimensional load components which are shown in Figure 1 are as follows:

$$\overline{W}_{r} = -\int_{0}^{1}\int_{0}^{\theta_{2}} \overline{p} \cos \theta \cdot d\theta \cdot d\overline{z},$$

$$\overline{W}_{\phi} = \int_{0}^{1}\int_{0}^{\theta_{2}} \overline{p} \sin \theta \cdot d\theta \cdot d\overline{z} \qquad ...(22)$$

The dimensionless load carrying capacity is given by

$$\overline{W} = \sqrt{\overline{W}_r^2 + \overline{W}_{\phi}^2} \qquad \dots (23)$$

where $\overline{W} = \frac{WC}{\mu \Omega R^2 L}$,

Attitude angle,
$$\phi = \tan^{-1} \left(-\frac{\overline{W}_{\phi}}{\overline{W}_{r}} \right)$$
 ...(24)

End Leakage Flow

The end leakage flow can be obtained by integrating the axial velocity component, V_z as given by Equation (16) across the end section of bearing. The dimensionless end leakage flow is calculated by



$$\overline{Q} = -\frac{1}{6} \int_{0}^{2\pi} \overline{g} \left(\ell_{m}, N, \overline{h} \right)$$

$$\left\{ \frac{1}{2\lambda^{2}} \left. \frac{\partial \overline{p}}{\partial \overline{z}} \right|_{\overline{z}=1} - 2\alpha \overline{h}_{m} \left. \frac{\partial \overline{h}_{m}}{\partial \overline{z}} \right|_{\overline{z}=1} \right\} \cdot d\theta \quad \dots (25)$$

where $\overline{Q} = \frac{2Q}{C\Omega LR}$, $\lambda = \frac{L}{D}$

Frictional Parameter

The shear stress at the journal surface is given by:

$$\tau_{s} = \mu \left. \frac{\partial V_{x}}{\partial y} \right|_{y=h} \qquad \dots (26)$$

Based on Equation (15), the above expression for τ_s reduces to:

$$\tau_{s} = \frac{h}{2} \left(\frac{\partial p}{\partial x} - F_{mx} \right) + \frac{\mu U}{2} \left\{ \frac{1}{\frac{h}{2} - \frac{N \Lambda (\cosh \varphi - 1)}{\sinh \varphi}} \right\}$$
...(27)

where $\varphi = \frac{Nh}{\Lambda}$

The frictional force at the journal surface is:

$$F_{s} = 2 \int_{0}^{\frac{L}{2}} \int_{0}^{2\pi} \tau_{s} \cdot \mathbf{R} \cdot d\theta \cdot dz \qquad \dots (28)$$

After substituting the expression for τ_s from Equation (27) into Equation (28), the nondimensional frictional force becomes as follows:

$$\overline{F}_{s} = \int_{0}^{1} \int_{0}^{2\pi} \left[\frac{\overline{h}}{2} \cdot \frac{\partial \overline{p}}{\partial \theta} - 2K_{c}\alpha \,\lambda^{2}\overline{h}\overline{h}_{m} \frac{\partial \overline{h}_{m}}{\partial \theta} \right] \\ + \frac{K_{c}}{\left\{ \overline{h} - \frac{2N}{\ell_{m}} \tan h\left(\frac{N \,\ell_{m} \,\overline{h}}{2} \right) \right\}} d\theta \cdot d\overline{z} \dots (29)$$
where $\overline{F}_{s} = \frac{F_{s}C}{\mu \,\Omega \,R^{2}L}, \ \lambda = \frac{L}{D}$

As per the Ariman and Turk (1974), Kc is a control number. Inside the full film region, Kc is equal to 1.0 and outside this region (partial film region), $K_c = \overline{h}_{min}/\overline{h}$ which is less than 1.0. \overline{h}_{min} is the non-dimensional minimum film thickness.

Frictional parameter, f(R/C) is defined as:

$$f\left(\frac{R}{C}\right) = \frac{\overline{F}_s}{\overline{W}}$$

RESULTS AND DISCUSSION

Results for the steady-state characteristics are exhibited in Figures 2-10 for flexible journal bearings lubricated with micro-polar ferrofluids at various values of non-dimensional characteristics length, I_m , coupling number, N, flexibility parameter, F, eccentricity ratio, ε_0 and magnetic force coefficient, α . The numerical computations pertain to a finite journal bearing of L/D = 1.0 with H/R = 0.3, $\sigma = 0.4$, K = 1.2and $\psi = \pi/2$ for parametric study. The conditions for magnetic and non-magnetic lubricants are characterized with $\alpha = 0.1$ and $\alpha = 0$ respectively.

Load Capacity

Figure 2 shows the dimensionless load capacity, \overline{W} as a function of bearing flexibility parameter, F at various values of eccentricity ratios for both magnetic and non-magnetic lubricant cases. A scrutiny of the figure reveals



that as eccentricity ratio increases, the dimensionless load increases for all values of flexibility parameter. Further, the family of curves shows the declining trend with increase in *F* for all values of \mathcal{E}_0 . It is also observed by comparing the non-magnetic curves with the magnetic ones that the non-magnetic case has an adverse effect on the load capacity.

Effect of I_m on the load capacity can be studied from Figure 3. Here, too, it is observed that the other factors remaining same, an increase in I_m deteriorates the load capacity and tends to Newtonian load value as I_m is increased more and more. For a value of I_m , load capacity decreases with increase in F but at lower values of F, the change in load parameter with F is significant.



Variation of the load capacity as a function of flexibility parameter, *F* is shown in Figure 4 for various values of N^2 , keeping the nondimensional characteristics length, $I_m = 20$ and eccentricity ratio, $\varepsilon_0 = 0.6$. It is found that at any *F* value, the load capacity improves with N^2 . This is because of the fact that an increase in N^2 means a strong coupling effect between linear and angular momentum. This eventually gives enhanced effective viscosity and hence non-dimensional load capacity improves. This has also been reported by Khonsari and Brewe (1989) while analyzing the performance of plane journal bearings lubricated with micropolar fluids. Moreover, the effect of the magnetic field coefficient is to significantly increase the load capacity for all values of N^2 at any value of *F*.

End Flow Rate

Variation of end flow rate (from clearance space) of the bearing with F is presented in Figure 5 at various eccentricity ratios. For both magnetic and non-magnetic lubrication

cases, the effect of eccentricity ratio is to increase the end flow rate at any value of flexibility parameter. It is further observed that end flow rate improves with increase in *F* for any value of eccentricity ratio and the increase in the end flow is more conspicuous at higher





values of *F*. Moreover, there is a considerable decrease in the side leakage for ferromagnetic lubrication. It is found that the magnetic lubrication has two opposing effects. The first effect is an increase in pressure and then an increase in the pressure gradient at the end section. This effect causes an increase in the side leakage. The second effect is the sealing effect by the magnetic force (F_{mz}) at the end section, which causes a decrease in the side leakage. The net effect is the resultant of the above two opposite effects. The results indicate that F_{m_z} has more predominant effect.

The effect of I_m on end flow rate is shown in Figure 6. For a particular value of F, the end flow rate decreases with increase in I_m . This decrease in end flow is more conspicuous at higher values of F. Here, too, it is observed that the effect of \dot{a} is to decrease the end flow rate significantly at any value of F.



Figure 7 shows the variation of the end flow rate with respect of *F* for various values of *N*². It is observed from the figure that the end flow increases with *F* at any value of *N*². In case of magnetic lubrication, the rate of increase in \overline{Q} is more predominant. Moreover, the effect of *N*² is to decrease the end flow rate at any value of *F* for both types of lubrication. The effect of magnetic force coefficient, α on \overline{Q} follows the similar trend when *N*² is taken as a parameter.

Frictional Parameter

Frictional parameter, f(R/C) is shown as a function of bearing flexibility parameter, *F* for various values of eccentricity ratio in Figure 8. It is observed that the frictional parameter increases with *F* at any value of eccentricity ratio, ε_0 for both magnetic and non-magnetic lubricating conditions. Moreover, the effect of eccentricity ratio is to decrease the frictional parameter at any value of *F*. It is clearly shown that the magnetic lubricant has a significant





decreasing effect on the frictional parameter at all values of *F*. This is due to higher load capacity obtained by the magnetic effect at all values of F, when ε_0 is taken as a parameter.

Variation of frictional parameter with F for various values of I_m is shown in Figure 9. The effect of I_m is to deteriorate the frictional parameter

at any value of F. As I_m tends to ∞ indicating the loss of individuality of the micro-structure in the micro-polar lubricant, the frictional parameter





approaches the value of that in the classical lubrication. At any value of I_m , the frictional parameter improves with increase in *F*. Here, also the magnetic lubrication condition results in the lower value of frictional parameter when I_m is taken as a parameter.

Figure 10 shows the variation of the frictional parameter as a function of *F* for a number of N^2 . It can be discerned from the figure that the effect of N^2 is to increase the frictional parameter for both types of lubrication at any value of *F*. Moreover, the frictional parameter tends to increase with *F* at any value of N^2 . Here, too, the magnetic lubrication has potential in lowering the frictional parameter, when N^2 is taken as a parameter.

CONCLUSION

The conclusions that may be drawn from the above analysis are summarized as follows:

- At any value of flexibility parameter for both types of lubricant, the effect of eccentricity ratio is to:
 - Improve load carrying capacity.
 - Improve end flow rate.
 - Decrease frictional parameter.
- At any value of flexibility parameter, an enhancement in the non-dimensional characteristics length of micro-polar ferrofluid decreases the load capacity, end flow rate and frictional parameter for both type of lubrication.
- At any value of flexibility parameter for both types of lubrication, the effect of coupling number is to:
 - Improve load capacity.
 - Decrease end flow rate.
 - Improve frictional parameter.

- The effect of flexibility parameter is to deteriorate load capacity when eccentricity ratio or micro-polar characteristics length or coupling number is taken as a parameter. Such effect is not observed in cases of end flow rate and frictional parameter.
- The magnetic micro-polar lubricant has potential for achieving higher load capacity, lower end flow rate and lower frictional parameter as compared to the nonmagnetic micro-polar lubricant.

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APPENDIX

Nomenclature				
В	Magnetic field intensity vector.			
С	Radial clearance.			
$d_{k,n}$	Distortion coefficient at harmonics, k and n.			
D	Bearing diameter.			
$\boldsymbol{e}_{_{0}},arepsilon_{_{0}}$	Steady-state eccentricity, $\varepsilon_0 = e_0/C$.			
E	Young's modulus of flexible lining material.			
f	Frictional coefficient.			
F	Flexibility parameter, $F = \mu \Omega R^3 / EC^3$.			
F _m	Induced magnetic force per unit volume of ferro-fluid.			
F _{mx}	Component of induced magnetic force per unit volume of ferro-fluid along x-direction.			
F_{mz}	Component of induced magnetic force per unit volume of ferro-fluid along z-direction.			
F_{s}, \overline{F}_{s}	Shear force on the journal surface, $\overline{F}_s = F_s C/\mu \Omega R^2 L$.			
G	Lame's constant, $G = E/2(1 + \sigma)$.			
h, \overline{h}	Film thickness of ferro-fluid, $\overline{h} = h/C$.			
h_m, \overline{h}_m	Magnetic field intensity, $\overline{h}_m = h_m / h_{m0}$.			

APPENDIX (CONT.)

Nomenclature				
<i>h</i> 0	Characteristics value of magnetic field intensity.			
Н	Thickness of flexible bearing liner.			
Ι	Strength of the current passing through the wire.			
к	Distance ratio parameter, $K = R_0/R$			
I _m	Non-dimensional characteristics length of micro-polar ferro-fluid, $I_m = C/\Lambda$.			
L	Bearing length.			
к	Axial harmonic.			
$M_{_g}$	Magnetization of the ferro-fluid.			
Ν	Circumferential harmonic.			
Ν	Coupling number, $N = \chi/(2\mu + \chi)$.			
р, <u></u>	Film pressure, $\overline{p} = pC^2/(\mu \Omega R^2)$.			
$p_{k,n} \ \overline{p}_{k,n}$	Film pressure expressed in double Fourier's series, $\overline{p}_{k,n} = p_{k,n} C^2 / (\mu \Omega R^2)$.			
Q, \overline{Q}	Side leakage flow of lubricant, $\overline{Q} = 2Q/(C\Omega LR)$			
R	Radius of journal.			
$R_{_0}$	Displaced distance from the wire position to the bearing centre.			
u _r	Radial displacement of flexible bearing liner.			
U	Velocity of journal surface, $U = \Omega R$.			
V_i	Velocity components of ferro-fluid along i – direction, $i = x$, y and z .			
$V_{1}^{}, V_{2}^{}, V_{3}^{}$	Micro-rotational velocity components about the axes.			
W, \overline{W}	Steady-state load capacity, $\overline{W} = WC/(\mu \Omega R^2 L)$.			
W_{r}, \overline{W}_{r}	Radial component of the steady-state load, $\overline{W}_r = W_r C / (\mu \Omega R^2 L)$.			
$W_{_{\phi}}, \ \overline{W}_{_{\phi}}$	Transverse component of the steady-state load, $\overline{W}_{\phi} = W_{\phi} C / (\mu \Omega R^2 L)$.			
X	Cartesian coordinate axis along the circumferential direction, $\theta = x/R$.			
X_m	Susceptibility of magnetic fluid.			
Y	Cartesian coordinate axis along the film thickness direction.			
Z, \overline{Z}	Cartesian coordinate axis along the bearing axis, $\overline{z} = 2z/L$.			
$\beta_{k,n}^{l}$	Attitude angle at harmonics <i>k</i> and <i>n</i> .			
Γ	Viscosity coefficient of the micro-polar ferro-fluid.			
δ	Deformation of the flexible bearing liner.			
μ	Newtonian viscosity coefficient.			
χ	Spin viscosity coefficient of the micro-polar ferro-fluid.			
λ	Slenderness ratio of journal bearing, $\lambda = L/D$.			

APPENDIX (CONT.)

Nomenclature	
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Λ	Characteristics length of the micro-polar ferro-fluid, $\Lambda = (\gamma/4\mu)^{\frac{1}{2}}$.
μ_{0}	Permeability of free space or air ($\mu_0 = 4 \pi \times 10^{-7} \text{ AT/m}$).
α	Magnetic force coefficient, $\alpha = \frac{h_{m0}^2 \mu_0 X_m C^2}{\mu \Omega L^2}$.
σ	Poisson's ratio of flexible bearing liner material.
ϕ	Attitude angle.
Ψ	Position angle of the displaced wire magnetic model, $\psi = \pi/2$.
Ω	Angular speed of rotation of journal.
θ	Angular coordinate, $\theta = x/R$.