Fractional Order Integral Sliding Mode Tracking Control of a Third-Order Double-Acting Electrohydraulic Actuator Model

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Abstract—This article presents a fractional order integral sliding mode control method for tracking control of sinusoidal, multi-sinusoidal, and chaotic trajectories in an Electro-Hydraulic Actuator (EHA) system. To acquire the proposed controller, the linear model of the EHA system is determined utilizing the parametric identification method, and the model's parameters are estimated utilizing the MATLAB System Identification toolbox. Incorporating a fractional order integral sliding surface into the conventional sliding mode control algorithm yields a fractional order integral sliding mode controller. Using Lyapunov's theory of stability, the closed-loop system with the proposed controller is evaluated for its stability. Through simulation, the control output of the tracking control system was acquired in order to assess the performance of the trajectory tracking control. The results demonstrate that the proposed controller outperforms the conventional sliding mode controller in trajectory tracking control.

Index Terms—fractional order integral sliding mode tracking control, electrohydraulic actuator, particle swarm optimization

I. INTRODUCTION

Electro-Hydraulic Actuator (EHA) systems are particularly practical and trustworthy owing to their small size in comparison to their power, their tremendous force-producing capabilities, and their rapid response times [1]–[3]. These properties make them particularly well-suited for use in construction equipment, which is sometimes referred to as mobile hydraulic machines [4]–[7]. The EHA system is recognized to be a nonlinear system with significant uncertainty due to the servo valve's high-frequency behaviour and external disturbances [8]–[11]. As a result, controller design for the EHA system can be difficult because of its uncertainty, nonlinearity, and disturbances.

Researchers and scientists are always on the lookout for more effective techniques to govern nonlinear complex systems. For almost seven decades, linear proportional integral and derivative controllers (PID) have been employed to regulate large plants [8], [12], but they have consistently failed to provide satisfying results for nonlinear, time-varying, uncertain, and complex systems.

Numerous classical nonlinear controllers are available, including gain scheduling [13], model reference adaptive control (MRAC) [14], self-tuning regulator (STR) [15], [16], and sliding mode controller (SMC) [17]–[23]. The sliding mode controller is one of these control systems that has gained considerable popularity among academics and
scientists due to its robust nature. SMCs are a subclass of variable structure controllers whose core design is based on Lyapunov stability theory, ensuring the overall system's bounded input and bounded output stability [24]. It is a high-gain controller in which the output of the system rapidly reaches the sliding surface and attempts to retain its place on it.

Despite decades of study about SMC, important technical issues such as the consequences of un-modelled dynamics, parameter uncertainty, chattering, and adaptive behaviour have attracted researchers and scientists. To address these issues, SMC included quick oscillations in behaviour have attracted researchers and scientists. To address these issues, SMC included quick oscillations in the controller output, which may cause damage to the system's final control element [25]. To solve these issues, many technological systems have been devised and are included in the conventional SMC. Conventional variable order control methods and intelligent procedures are now being used together in a new trend [26].

Therefore, this paper explains the effort involved in developing a fractional order integral sliding mode controller (FOISMC) by modifying the sliding surface in the conventional SMC algorithm for a third-order double-controller (FOISMC) by modifying the sliding surface in developing a fractional order integral sliding mode system's final control element [25]. To solve these issues, many technological systems have been devised and are included in the conventional SMC. Conventional variable order control methods and intelligent procedures are now being used together in a new trend [26].

III. METHODOLOGY

A. System Identification and Experimental Setup of EHA System

System identification involves modeling of the EHA system with prior knowledge by physical-based modeling and utilizing of input and output data which obtained through open-loop real-time experiment. In 2000, Knohl and Unbehauen explained in [30] regarding physical-based modeling for the EHA system presented in Fig. 1 by ignoring nonlinearities and producing a model structure based on a continuous transfer function as shown in (5) with three parameters: $a_1$, $a_2$, and $a_3$, where $x_p$ and $u$, are system output and input, respectively [8][28].

$$\frac{X_p(s)}{U(s)} = \frac{a_1}{s(s^2 + a_2s + a_3)}$$

$$\dot{x}_p = a_1u - a_2\dot{x}_p - a_3x_p$$

In the scientific and/or engineering community, fractional calculus has a three-century history but is largely unknown. The topic has been related to mathematicians for three centuries, but it has only recently been associated with engineering, physics, and economics. As reported in [27], the basic operator in fractional calculus is (1), where $a$, $t$, and $r$ represent the operation's limit order and common real number, respectively. The generic fractional difference integral has three definitions: Grunwald-Letnikov (2), Riemann-Liouville (3), and Caputo (4) for $n - 1 < r < n$.

$$\begin{align*}
D^r_t f(t) &= \lim_{h \to 0} h^{-r} \sum_{j=0}^{t-a/h} (-1)^j \binom{r}{j} f(t - jh) \\
D^r_t f(t) &= \frac{1}{\Gamma(n-r)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{-n+r} f(\tau) d\tau \\
D^r_t f(t) &= \frac{1}{\Gamma(r-n)} \int_a^t (t-\tau)^{-n+r} f(\tau) d\tau \\
\end{align*}$$

II. THEOREM AND DEFINITION OF FRACTIONAL CALCULUS

In the scientific and/or engineering community, fractional calculus has a three-century history but is largely unknown. The topic has been related to mathematicians for three centuries, but it has only recently been associated with engineering, physics, and economics. As reported in [27], the basic operator in fractional calculus is (1), where $a$, $t$, and $r$ represent the operation's limit order and common real number, respectively. The generic fractional difference integral has three definitions: Grunwald-Letnikov (2), Riemann-Liouville (3), and Caputo (4) for $n - 1 < r < n$.

$$D^r_t = \begin{cases} 
\frac{d^r}{dt^r} & \text{for } r > 0 \\
1 & \text{for } r = 0 \\
\int_a^t (t-\tau)^{-r} d\tau & \text{for } r < 0
\end{cases}$$

Additionally, a linear model is adequate for tracking control with advanced control strategies to achieve high performance of an EHA system in trajectory tracking applications, and the linear model of an EHA system is frequently valid over a relatively broad range [29]. In addition, the parameters of the model in (5) were estimated using the MATLAB System Identification toolbox.

The experimental workbench for the EHA system, depicted in Fig. 2, was used to collect time domain input-
output data. The EHA system workbench consists of a real-time control system based on MATLAB/Simulink and PCIe-6321, a moving cylinder, and a hydraulic power supply with an 8 MPa supply pressure and a 16.6 L/min flow capacity. The hydraulic cylinder has a bore of 40 mm and a rod of 25 mm. It is governed by a proportional directional valve that has a maximum flow rate of 80 L/min and a maximum supply pressure of 31.5 MPa. Real-time cylinder displacement is measured using a wire displacement sensor.

B. Controller Design

In the early 1960s, SMC was created as a variable-structure control. The most crucial step in SMC design is the construction of the sliding surface mathematical equation. The illustration of the error signal (e) is shown in Fig. 4, and the equation (6) identifies e and its third-order derivative, where r and xp are the reference input and piston displacement signals, respectively.

\[
e = r - x_p, \quad \ddot{e} = \ddot{r} - \ddot{x}_p
\]  

(6)

\[
s = \left(\lambda + \frac{d}{dt}\right)^{n-1} e + k_i D^{-\alpha} e
\]

\[
= \lambda^2 e + 2\lambda \ddot{e} + \ddot{r} + a_1 u + a_2 \ddot{x}_p + a_3 x_p + k_i D^{-\alpha} \dot{e}
\]

(7)

\[
\dot{s} = -\epsilon \text{sign}(s), \epsilon > 0,
\]

(9)

\[
u = \frac{1}{a_1}(\lambda^2 \ddot{e} + 2\lambda \ddot{e} + \ddot{r} + a_2 \ddot{x}_p + a_3 x_p + k_i D^{-\alpha} \dot{e} + \epsilon \text{sign}(s))
\]

(10)

C. Stability Analysis

The primary objective of the controller design is to guarantee that the feedback control system operates continuously in a steady manner. According to the Lyapunov stability theorem, when condition \(s \dot{s} < 0\) is
met, the whole system becomes stable and approaches the sliding surface. The Lyapunov function (11) is used in this paper with the constraint that \( V(0) = 0 \) and \( V(t) > 0 \) for \( s \neq 0 \). To ensure that the trajectory transitions smoothly from the reaching to sliding phase and maintains stability, it is required to adhere to the reaching condition marked as in (12). With reference to (13) and (14), it is evident that the criterion \( s\dot{s} < 0 \) has been met in this case.

\[
V = \frac{1}{2} s^2 \quad (11)
\]

\[
\dot{V} = s \dot{s} < 0, \quad s \neq 0 \quad (12)
\]

When \( s > 0 \), \( \text{sign}(s) = 1, \epsilon > 0 \),

\[
\dot{V} = -s(\epsilon \text{sign}(s)) = -s(\epsilon) < 0 \quad (13)
\]

When \( s < 0 \), \( \text{sign}(s) = -1, \epsilon > 0 \),

\[
\dot{V} = -s(\epsilon \text{sign}(s)) = s(\epsilon) < 0 \quad (14)
\]

D. Particle Swarm Optimization

Fig. 5 depicts how the PSO algorithm is used in practice. To get the ideal values for the design gain parameters, the PSO was constructed by imitating the swarm social behaviour of birds flocking and fish schooling. Particles, or swarms of people, explore the whole population in a high-dimensional search region for the best global solution at a particular position and velocity. Every particle in the swarm has a fantastic response and follows a simple principle by replicating its previous success. Furthermore, the particle's placement in the neighbourhood is impacted by the particle's personal best position, with the global best position being the best solution among the personal best positions.

![Figure 5. Common process of the optimization algorithm.](image)

In this work, the particle population size, maximum number of iterations, and social and cognitive coefficients are all set to 20, 50, 2, and 2, respectively by using the heuristic approach. In addition, the fitness function is written in (15), where \( m \) is the total number samples.

\[
J = \sqrt{\frac{1}{m} \sum (e)^2} \quad (15)
\]

Furthermore, the proposed controllers' effectiveness for tracking control is defined by the MSE index in (16), where a lower value of the MSE index indicates a better capability of the controller for effective tracking control in the EHA system.

\[
MSE = \frac{1}{m} \sum (e)^2 \quad (16)
\]

IV. SIMULATION RESULTS AND DISCUSSION

This section shows the evaluation of tracking control performance for the FOISMC and conventional SMC controllers using MATLAB/Simulink (R2021b) with 1 ms of sample time and optimized controllers’ variables using PSO algorithm. To construct the necessary trajectory, sinusoidal, multi-sinusoidal, and chaotic signals are employed. The values of \( a_1, a_2, \) and \( a_3 \) are 144400, 372.3, and 7855, respectively, which are developed by the MATLAB System Identification toolbox procedure. Furthermore, the variable values of the proposed and conventional SMC controllers, which were developed by the PSO process, are listed in Table I.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>395.42</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>6.49</td>
</tr>
<tr>
<td>( k_i )</td>
<td>-</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-</td>
</tr>
</tbody>
</table>

The trajectory profiles are illustrated in Fig. 6, which were utilized to evaluate the performance of FOISMC and conventional SMC controllers, respectively. Taking into consideration the tracking performance with associated error signals shown in Figs. 7 and 8, it can be clearly seen that these two controllers can track the three signal trajectories. However, the FOISMC’s tracking performance outperformed the conventional SMC.

![Figure 6. Trajectories signal.](image)
Furthermore, Table II summarizes the results of a detailed investigation relating to the mean square error (MSE) based on sinusoidal, multi-sinusoidal, and chaotic trajectory tracking control using conventional SMC and FOISMC controllers. The MSE values obtained from tracking sinusoidal, multi-sinusoidal, and chaotic trajectories using conventional SMC controller are 0.3557 mm$^2$, 13.8973 mm$^2$, and 0.2562 mm$^2$, respectively. Meanwhile, the MSE values acquired from the same trajectories tracking using FOISMC controller significantly decrease to around 98%, 98.1%, and 65% of the conventional SMC’s MSE value, respectively. Therefore, it is revealed that the FOISMC is the preferable controller as compared to the conventional SMC controller for the trajectory tracking control in the EHA system.

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>Conventional SMC</th>
<th>FOISMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sinusoidal</td>
<td>0.3557</td>
<td>0.0071</td>
</tr>
<tr>
<td>Multi-sinusoidal</td>
<td>13.8973</td>
<td>0.0750</td>
</tr>
<tr>
<td>Chaotic</td>
<td>0.2562</td>
<td>0.0888</td>
</tr>
</tbody>
</table>

V. CONCLUSION

This study focuses on a fractional order integral sliding mode control method for tracking control of sinusoidal, multi-sinusoidal, and chaotic trajectories in an EHA system. To obtain the proposed controller, the linear model of the EHA system is determined using the parametric identification technique, and the model’s parameters are estimated using the MATLAB System Identification toolbox. A fractional order integral sliding mode controller is then presented by integrating a fractional order integral sliding surface into the conventional sliding mode control algorithm. The stability of the closed-loop system with the proposed controller is evaluated using Lyapunov’s theory of stability. Through simulation work, the tracking control system's control output was gathered in order to evaluate the trajectory tracking control's performance. Results reveal that the proposed controller for trajectory tracking control outperforms the conventional sliding mode controller. In addition, future research should assess the proposed controller's performance on a real-time hardware platform and compare it to other sliding mode control methods such as fractional order proportional-integral-derivative sliding mode and fuzzy proportional-integral-derivative controllers.

CONFLICT OF INTEREST

Authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

M. F. Ghani and R. Ghazali conducted the research, analyzed the data, and wrote the paper. H. I. Jaafar, C. C. Soon, Y. Md. Sam, and Zulfatman review the manuscript. All authors had approved the final version manuscript.
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