

# Deposition Toolpath Pattern Comparison: Contour-Parallel and Hilbert Curve Application

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**Abstract**—For the construction of a piece using additive manufacturing technology, there are several methods of toolpath planning, which affect the quality and quantity of material deposited. In this document, we take an approach to commonly used patterns, and propose two path planning techniques based on contour-parallel and Hilbert curves, applying a smoothing process on them. To evaluate these methods, the areas of overfilling, underfilling and external filling are quantified; as well as the length of the deposited material, the travel moves and the estimated time in the printing process. A comparison is presented with the trajectories offered by a commercial software. The planning and quantization method has been implemented in MATLAB and evaluated on diverse geometries to be filled.

**Index Terms**—additive manufacturing, continuous fill pattern, contour-parallel, Hilbert curve, toolpath planning

## I. INTRODUCTION

In additive manufacturing applications, a recently researched topic is path planning, which describes print behavior and its quality. For this planning, some factors are considered, such as surface finish, piece density, the final mechanical properties presented by the print part, as well as the type of curves that the extruder will describe [1,2]. Some of the requirements of printing process are high degree of freedom of movement, high resolution and precision, high-speed movements and a compact size that allows printing [3,4]. These trajectories have been generated mainly from two approaches: trajectories based on cartesian movements or parametric movements [5].

The deposition toolpaths based on cartesian movements which were analyzed in this work were: space fill curves, a variant of these so-called zigzag curves and Hilbert curves; the latter belong to the group of continuous fractal curves. On the other hand, the toolpaths based on parametric motions investigated were spiral-based curves, as well as the iso-contours or contour-parallel which cover the area with geometries similar to the contours [6]. This section presents a compilation of these techniques, as well as some variants that were implemented on them.

Space-filling curves are the first approximation to a curve that forms a printing space, since its basis is the creation of equidistant parallel lines on the contour to be filled [5]. The main parameters for designing the curves are the desired final density, the distance between the lines, which is subject to the printing diameter of the extruder, and the printing angle, which changes the mechanical behavior of the piece.

The following analyzed patterns are the curves based on a zigzag, being quite similar to the previous one, however, these are continuous, and lines are generated that join the parallel lines [7]. Similarly, a design parameter on the curve is the angle with respect to the geometry, allowing to change the print direction [8,9]. It should be noted that these are the trajectories used commercially, because to their ease of design and great ability to fill various geometries, however, there are sharp turns that may not be desired. Due to this problem, smoothing methods have been proposed in these sections to minimize these changes [10,11].

The last curves based on cartesian movements analyzed is the Hilbert curve, which is a continuous fractal space fill curve described by the mathematician David Hilbert [12], and is presented in Fig. 1a. This curve is developed by means of a recursive algorithm that allows to generate such trajectory, having capacity for three-dimensional implementation. In addition, this curve can be inscribed on different geometries and as design parameter we present the print angle, as can be seen in Fig. 1b, in which Hilbert curve inscribed in a circle with a 45-degree angle of impression are presented [6].

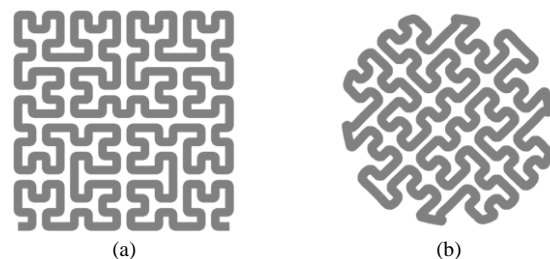


Figure 1. Hilbert curve implemented (a) Square region (b) Circular region.

Within paths based on parametric movements, there are contour-parallel tool paths, which depend directly on the geometry to be filled [12]. These contours reduce their dimension with respect to the geometrical center of

Manuscript received January 8, 2022; revised May 30, 2022.

This work was supported in part by the Research Vice-Chancellor of Universidad Militar Nueva Granada – 2021. under the project IMP-ING-3122.

the region to be analyzed, presenting variations due to their geometry [13]. As shown in Fig. 2, in order to complete some desired geometries, it is necessary to generate regions where the problem is solved locally. Furthermore, as discussed in the curves based on cartesian movements, these contours likely present the same problem of sharp turns [14,15]. Owing to the above mention, smoothing techniques with circles and spiral like fishtail have been applied in order to improve both the impression to be developed and the correct performance of the machine [11].

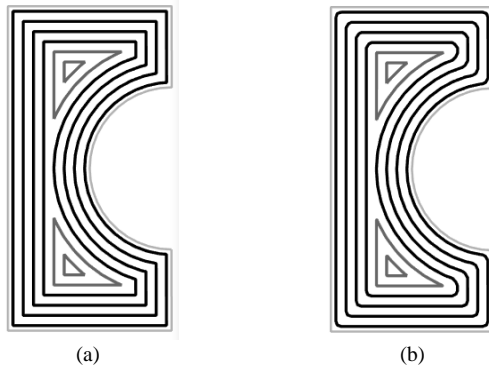


Figure 2. Filling pattern by contour-parallel tool paths (a) Normal (b) Smooth with circles.

Another problem that must be analyzed in the trajectories for additive manufacturing is the presence of islands within the geometry to be printed, which may occur due to the nature of the part. Thus, some methods have been developed to improve these profile changes [16]. As shown in Fig. 3, some techniques have been researched and explored to solve this problem, such as the use of a single contour for the path, or a continuous transition from an internal contour to external contours, modifications that have been focused on the extrusion process of CNC machines, but they are applicable in the process of deposition of material [17].

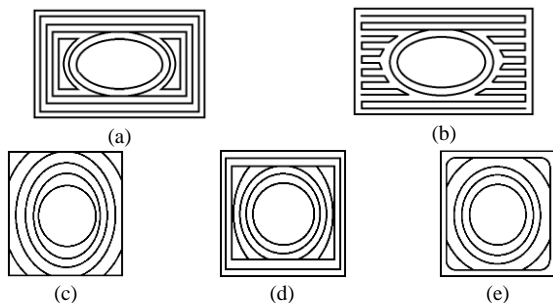


Figure 3. Cutting methods for machining a curved surface (a) Parallel contouring (b) Parallel cutting (c) Shape cutting (d) Contour and shape cutting (e) Profile transition [16].

Based on the contour-parallel tool paths, a series of spiral filling patterns are defined, which perform a continuous movement on the contours. Some of the advantages of the parametric path are the continuous deposition through a hollow entity compared to the zigzag-based filling techniques [10] and the smooth transition from one material printing region to another

[18]. Fig. 4a shows the basic behavior of a spiral over a region, however, new plans based on this pattern have been proposed, such as Fermat's spirals, which have the particularity of presenting a return over the center of the piece. It is worth mentioning that on these Fermat spirals modifications have been developed in which they become a cycle, allowing them to end at the starting point, as shown in Fig. 4c, allowing for better connectivity between layers in the printing process [12].

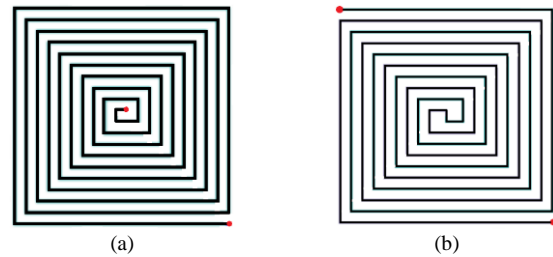


Figure 4. Spiral filling pattern (a) Normal (b) Fermat [12].

## II. METHODS

### A. Modifications

To improve the quality of the part to be printed, the following modifications are proposed to the print path patterns based on the contour-parallel and the Hilbert curves. These methods and modifications are described below.

#### 1) Connected contour-parallel tool paths.

Based on the contour-parallel, and considering the problem that these are not continuous paths, two methodologies are proposed for the connection between these closed curves, based mainly on the cutting of each contour, its contiguous contour and its union by means of rectilinear lines, as shown in Fig. 5a. For these methodologies, the connection starts from the internal contour and continuing towards the external one. As a first option, the following steps are proposed in the generation: initially, the internal contour is selected, a random cut is made with a distance corresponding to the thickness of the nozzle, after this, the line tangent to the contour is calculated in the selected points; with these lines, the cuts of these are searched in the contiguous contour, and their connection is made by a line. Once this process has been carried out on the first contour, these steps are performed cyclically on all contours.

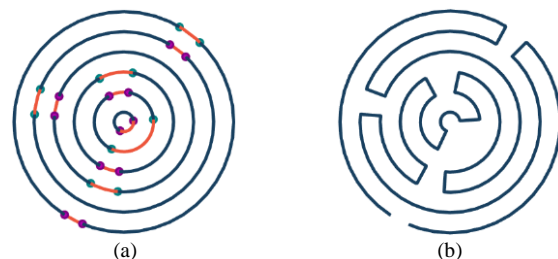


Figure 5. First process of connecting contour-parallel tool paths (a) Contour separation and gradient cutting (b) Generated path.

As a second contour connection option, and based on the results of the first connection process, a modification to it is proposed. Therefore, when the tangent lines were generated at the random points, the average angle between them is calculated, and this sets the direction of the lines that connect the contiguous contour, as shown in Fig. 6a. This modification allows to maintain the equidistance between the lines and between the points of arrival, minimizing the areas where no material is deposited.

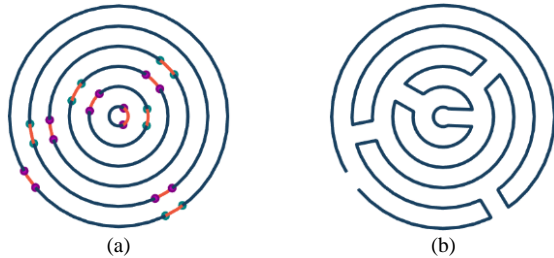


Figure 6. Second connection process of contour-parallel tool path (a) Contour separation and cut by average gradient (b) Generated path.

### 2) Hilbert curve optimization.

Based on Hilbert curves, and on Quadrees Decomposition (QD), which is a technique that divides an image into homogeneous 2D regions [19], it is proposed to approach this method to divide a 2D region to be filled, as shown in Fig. 7a for a circular contour. With such sections, the search for the Hamiltonian path connecting the centers of the QD regions [20] is proposed, where the connections of the network are given by its vicinity, allowing diagonal connections. When this path is solved, as shown in Fig. 7b, where all sections are connected, the Hilbert curve is reconstructed locally, taking into account the direction of the previous point and the next point, in order to connect all sections. This reconstruction process can be visualized in Fig. 7c.

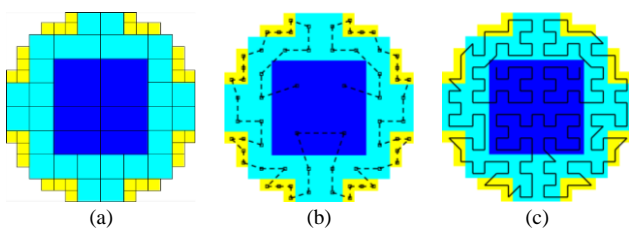


Figure 7. Generation of modified Hilbert curves (a) Region QD (b) Hamiltonian path in sections (c) Trajectory reconstruction.

### 3) Smoothing.

The Cubic Hermite interpolation is used in this approach, which is a third-degree spline that decreases the presence of sharp turns present in the conventional cubic spline. For its implementation, the corners that represent sharp turns in the trajectory are changed for two points equidistant to the turn; after that, 'pchip' MATLAB function is used, which performs such interpolation. The result of the smoothing over two cases of rotation is shown in Fig. 8.

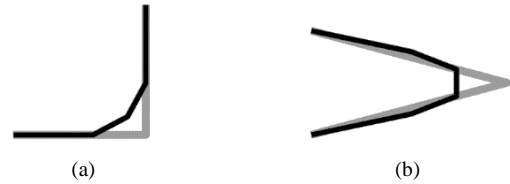


Figure 8. Smoothing in turns (a) Right angle (b) 30 °angle.

## B. Toolpath Patterns

To compare the trajectory planning techniques that are commercially used in the printing process, curves generated by the Slic3r software on parts with different geometries are extracted. This process is carried out obtaining the positions of the final effectors on the G code generated by the software, for its later debugging on a single layer. These trajectories are characterized by their complete filling configuration, being listed and exposed in Fig. 9.

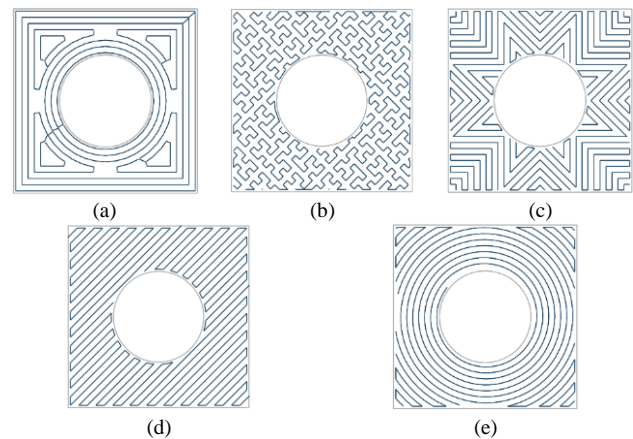


Figure 9. Filling patterns generated by Slic3r (a) Concentric (b) Hilbert curve (c) Octogram spiral (d) Rectilinear (e) Archimedean chords.

Considering the modifications specified in the previous section, seven trajectory planning techniques are proposed, in order to improve print quality and quantify its performance. These trajectories are shown in Fig. 10, and are listed below:

- Contour-parallel paths connected by gradient (Contour-A).
- Contour-parallel paths connected by average gradient (Contour-B).
- Hilbert curve modified by algorithm (Modified Hilbert).
- Rectilinear and smoothing (Soft rectilinear).
- Contour-parallel paths connected by gradient and smoothing (Soft contour-A).
- Contour-parallel paths connected by average gradient and smoothing (Soft contour-B).
- Hilbert curve modified by algorithm and smoothing (Soft modified Hilbert).

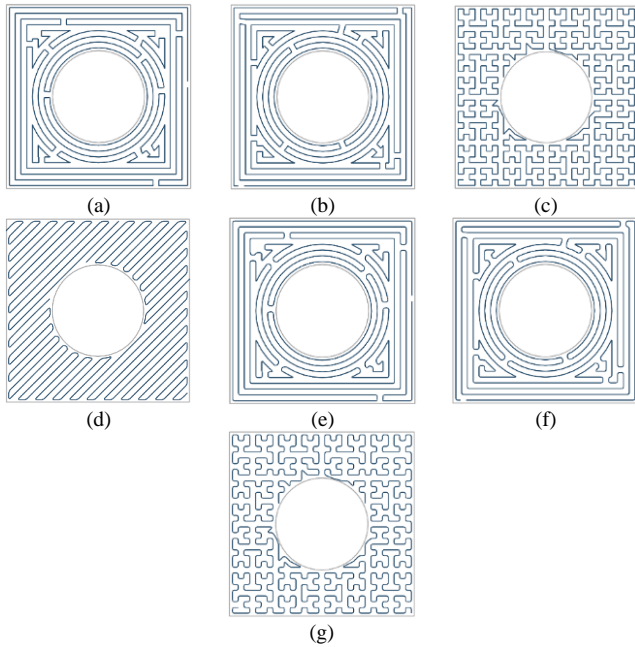


Figure 10. Filling patterns generated in MATLAB (a) Contour-A (b) Contour-B (c) Modified Hilbert (d) Soft rectilinear (e) Soft contour-A (f) Soft contour-B (g) Soft modified Hilbert.

C. Metrics

To evaluate the performance of the trajectory planning methods implemented commercially by the Slic3r software and the methods that were proposed in the previous section, the following metrics calculated on these are presented for their comparison.

1) Filling

The variables related to the filling process and its surface quality are quantified, which are overfilling, underfilling and external filling. The overfilling represents the area of the regions in which material is deposited more than once, shown in green in Fig. 11; whereas the underfilling represents the area of the regions within the contour in which no material was deposited, shown in red in Fig. 11. Finally, the external filling is the area in which material was deposited outside the contour, affecting the surface quality of the piece, shown in yellow. It should be noted that the aim is to minimize the area of these variables, a value of zero being an impeccable impression.

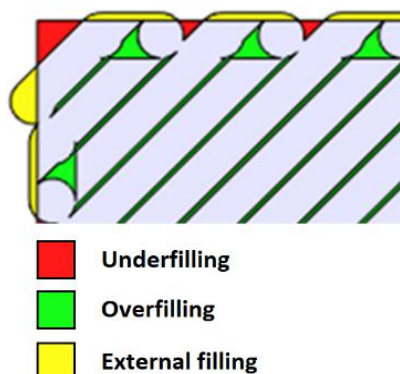


Figure 11. Filling metrics on rectangular section.

2) Length of deposited material and travel moves

These metrics are related to the length of displacement in the trajectory, and are mentioned in (1). As first variable, the length of the deposited material is quantified, which corresponds to the sections in the total trajectory where the extruder is enabled. As second variable, the length of the travel moves is also quantified, which are movements without extrusion between one point and another, needed to complete the printing.

$$L_{material} + L_{travel\ moves} = L_{trajectory} \quad (1)$$

3) Equations

The last metric is the deposition time, which can be estimated by means of speed ramp functions in the continuous movements, as it is observed in Fig. 12, being on each of the trajectory lines, due to the discretization of the patterns and the sequential programming of the printing machines in G-code [21,22]. To do this, the time in each linear movement is calculated by means of (2), and the sum of all the times consumed in the path is carried out.

$$t_n = \begin{cases} \frac{d_n}{v} + t_s & \text{if } \frac{d_n}{v} \geq t_s \quad (\text{ramp}) \\ \sqrt{\frac{4d_n}{a}} & \text{if } \frac{d_n}{v} < t_s \quad (\text{triangle}) \end{cases} \quad (2)$$

Where 'ts' is expressed as the rise time it takes to reach, the final effector, the maximum linear speed, which is given by 'ts = v/a', where 'v' is the maximum linear speed, set at 3000 mm/min, and 'a' is the acceleration of the effector, set at 1000 mm/s<sup>2</sup>. Finally, 'd' is expressed as the distance of the linear movement and is calculated in each line, to approximate the time in that section.

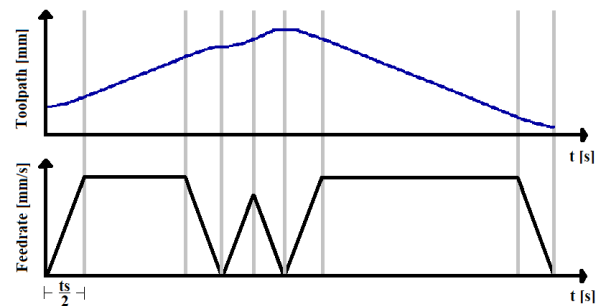


Figure 12. Toolpath and feedrate functions.

D. Case Study

Since each of the trajectories has a different behavior and value of the metrics, a study of 6 cases of contour geometries have been proposed, which are exposed in Fig. 13, where circular and square contours are considered, as well as their behavior in front of islands with these geometries. It is worth mentioning that the geometries without islands have dimensions of 6.4 x 6.4 mm, while the geometries with islands have dimensions of 12.8 x 12.8 mm. The thickness of the extruder nozzle to simulate the printing process is 0.4 mm.

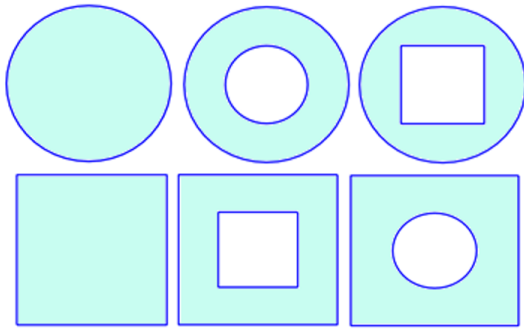


Figure 13. Layer geometries analyzed.

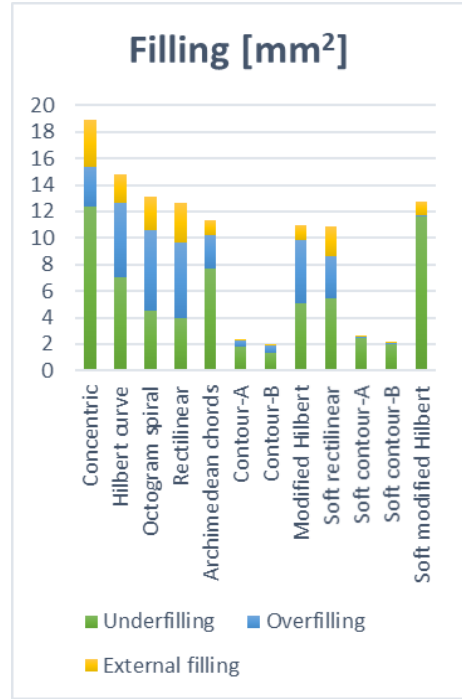
When each metric is calculated in the mentioned layers, an average of these are calculated using the weighted average, expressed in (3), where ‘x’ represents the metric to be averaged, ‘i’ the index of the layer, and ‘A’ the area of the layer. This average is made over the set of layers for each of the calculated paths.

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i} \quad (3)$$

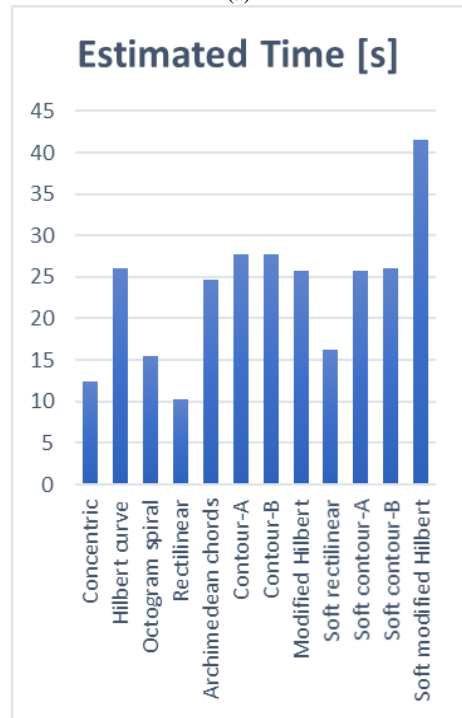
### III. RESULTS AND DISCUSSIONS

Bar graphs in Fig. 14 show the filling behavior and estimated print time of deposition patterns. The rectilinear trajectory had a 24% lower performance in the filling metrics with respect to the average, while the contour planning technique B is the one that had the best performance, with 398% above the average filling. However, the contouring techniques in general, with their smoothing process had the best performance in filling metrics. Is possible to remark that, the contour-B technique had the best performance in the underfilling metric, with 294% above average, while when evaluating the overfilling, the smoothing trajectories had a performance of 20% above average. Finally, when analyzing external filling, contour-A path planning method had 101% above average performance, being the best performing technique.

Regarding the estimated printing time metric, the rectilinear technique had a speedup of 2.26 with respect to the average, being the method that consumes less time in printing, due to the continuity in its movements. The next one that consumes less time was concentric with a speedup of 1.86 due to the low discretization that the software uses on the contours. The time they used was directly related to the number of points with which the path was approached, accelerating and decelerating the system to continue the path correctly. The methods proposed by contour A and B, with their smoothing modifications, had a speedup average of 0.87, making their time in printing increase notably with respect to the conventional process of rectilinear trajectory taken by the software as default.



(a)



(b)

Figure 14. Metric results (a) Filling (b) Estimated time.

Finally, Fig. 15 shows the results of the metrics related to the movements made by the trajectories. Regarding the deposited material metric of the trajectories, the modified Hilbert trajectory planning had the best performance, with 9.5% less material than the average. The methods that deposited more material in the analyzed layers were the Hilbert and rectilinear curves generated by Slic3r, with a 5% increase of material with respect to the average. It is stated that the smoothing process saved material in 1.5% with respect to the methods that were not smoothed.

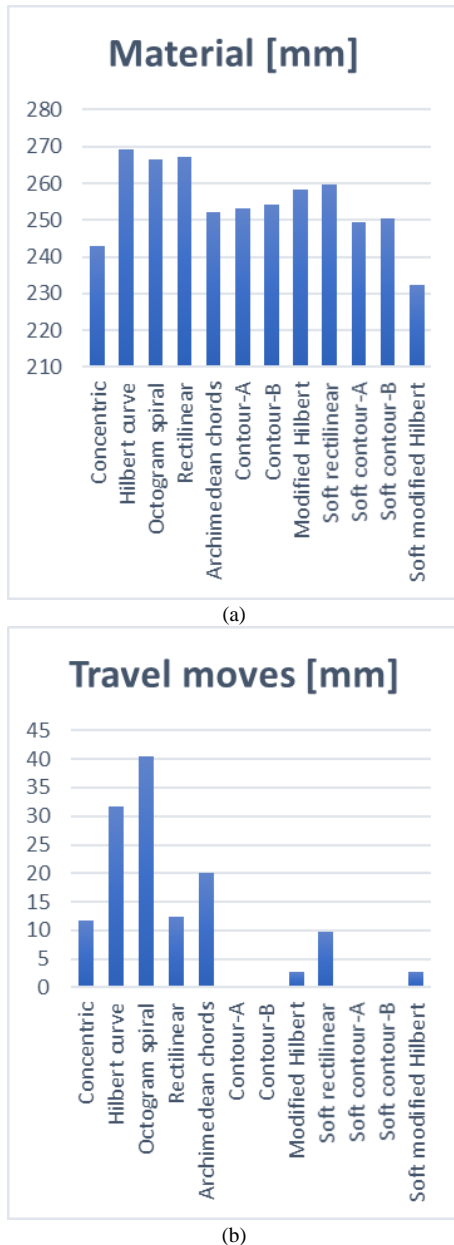


Figure 15. Metric results (a) Deposited material (b) Travel moves.

The travel moves metric expresses unnecessary paths in the printing process, and it can be seen in Fig. 15b that, modified paths based on contour-parallel do not present any movement of this type, being the desired in any path. On the other hand, it is exposed that the trajectories generated by the software Slic3r are the ones that present a great amount of movements of this type, being their Hilbert curves the ones that stand out the most. The modified Hilbert curves have some travel moves, nevertheless, it is indicated that they are only presented in one geometry of the six analyzed.

#### IV. CONCLUSION

In the present analysis of patterns used in additive manufacturing, a good performance of commercially implemented patterns was verified, emphasizing the good behavior of concentric and rectilinear patterns, which are

based on contour-parallel and zigzag respectively. However, the above modifications greatly improved the surface and filling quality during the printing process, compromising the time. The behavior of the trajectory planning techniques, as well as the quantified metrics, depend to a great extent on the geometry of the layer being analyzed. For this reason, it is recommended to study the methods specifically on the geometry to be printed, to guarantee the best printing among the exposed patterns.

The soft contour-B pattern is recommended toolpath planning when is seeking to improve the surface quality, fill the piece and have no travel moves, however this method compromises a considerable increase in printing time with respect to the rectilinear pattern used conventionally. The Hilbert's method modified by QD optimization was proposed. This pattern together with a smoothing process presents better results in the material decrease, at the same time that has low presence of travel moves, however, due to its great amount of turns in the nature of the pattern, it increases notably the time in the printing process, as well as in the filling metrics.

The importance to analyze several performance metrics in the printing process is emphasized, and to consider the relevance given to the application of the piece to be manufactured, because in some processes it may be in the interest of minimizing time more than the surface or structural quality, whereas in others a better filling quality predominates. As a future work, it is exposed the possibility of redefining the orientation in the patterns based on cartesian movements, providing scope to an improvement in the metrics, likewise, the analysis of metrics that relate the effort and consumption that the printer develops, and the quantification of metrics on manufactured pieces.

#### CONFLICT OF INTEREST

The authors declare no conflict of interest.

#### AUTHOR CONTRIBUTIONS

Mauricio Mauledoux and Oscar F. Aviles conducted the research; Diego A. Nunez and Juan C. Guacheta Alba analyzed the data; Juan C. Guacheta Alba wrote the paper; All authors revised the manuscript and they had approved the final version.

#### ACKNOWLEDGMENT

This paper is a research product from the IMP-ING-3122 high-impact project funded by the Research Vice-Chancellor of Universidad Militar Nueva Granada – 2021. Producto derivado del proyecto IMP-ING-3122 financiado por la Vicerrector á de Investigaciones de la Universidad Militar Nueva Granada – Vigencia 2021.

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