Bound Gait Reference Generation of a Quadruped Robot via Contact Force Planning

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Abstract—**This paper presents a reference generation technique for bounding quadruped locomotion. Synthesis of the reference is carried out by forming contact forces. Contact forces are planned with the aid of linear and angular momentum conservation and impulse laws. For the purpose of obtaining a stable and continuous bound gait, linear and angular momentum changes are set to be equal to zero over a full gait cycle. This condition allows the robot body to keep its initial dynamics at the end of the cycle, thus, producing a stable bound gait throughout the cycles. Periodicity in vertical linear velocity and angular velocity is obtained. These settings also result in almost constant horizontal body linear velocity. Suitable contact forces are produced with an optimization algorithm. Sequential Quadratic Programming (SQP) is utilized as an optimization solver. Linear and angular momentum laws and a non-slipping condition are applied as constraints of optimization. A full-dynamics simulation environment is employed to test the proposed reference generation algorithm. Results verify the validity of the proposed reference generation method for quadruped bounding.**

Index Terms—**quadruped robot, legged robot locomotion, reference generation, force planning, simulation, optimization**

I. Introduction

The legged robot field developed significantly in the last few decades [1-8]. Legged robots, especially quadrupeds, have considerable advantages on rugged terrain over wheeled and tracked robotic land platforms. It is foreseen that, quadruped robots will be assigned to operations such as mine-sweeping, exploration, logistic distribution, search and rescue operations, disaster areas, space, and military applications.

Still, using quadruped robots in the aforementioned tasks is not a straightforward process. These robots not only have high degrees of freedom (DOF), but also complex and non-linear dynamics. In order to prevent unstable motions, a reference generation method must either be based on well-defined stability criteria, or it should be adjusted by gained experience, by a learning algorithm or an optimization process.

Legged robots are floating-base robots and they do not have any fixed attachment points in the world. Since their body center of mass (COM) is not directly actuated, they are classified as under-actuated robots. Nevertheless COM can be controlled indirectly by proper joint trajectories and contact forces [6, 9-11]. Therefore, reference generation plays a vital role in the control of floating-base robots [12-14].

There are several types of quadrupedal gaits such as crawl, trot, bound, etc. [15, 16]. Nature provides us knowledge about how animals change their gait type according to their locomotion speed in order to keep their balance meanwhile utilizing their energy efficiently [17]. Hence, it is important to succeed in a variety of quadrupedal gaits including static motion [18, 19] as well as dynamic motion [6, 8, 11, 20].

The bound is a gait type in which front and rear legs act in pairs (front pair and rear pair) [16]. High speed locomotion by quadrupedal animals exhibits this gait. In this highly dynamic gait, the body stabilization problem is pronounced. When front or rear legs take off, a limited support area is formed on the ground. This area is far from the ground projection of the COM. This distance produces high angular acceleration around the COM and affects the robot's balance adversely. During bound gait, it is also likely that hard ground impacts are observed because of high angular acceleration and short swing times. Under these circumstances, the creation of a robust reference generation method for the bound gait is challenging.

A. Related Work

A gait reference generation technique, when designed suitably, can achieve a number of desired features for the bound. First, it can effectively increase the robot's stability and energy efficiency. Second, capacity of terrain adaption can be improved, preventing robot from falling. In recent years, the majority of research on the reference generation techniques of bounding quadruped robot locomotion have focused on three main areas, namely, model-based, bio-inspired, and learning approaches.

1) Model based approaches

Model-based approaches are very popular among reference generation methods. Vukobratovic introduced the Zero Moment Point (ZMP), which has become a widely used stability criterion for legged robot reference generation [21]. Orsolino et al. proposed a ZMP-based reference generation technique for improving quadruped robot lateral stability during the front/hind stance phases using the periodic limit cycle approach to acquire a stable bounding gait. [22] describes the procedures they followed

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to conduct real-world outdoor experiments of HyQ bounding at various speeds and omni-directional movements. Winkler et al. presented a strategy for optimizing reference trajectories. They integrated rapid ZMP criterions-based approaches into the optimization process handling body motion, footholds, and center of pressure simultaneously [23]. Although the ZMP method is frequently used in the literature, it requires pre-defined foothold positions. Its control performance and accuracy are questionable, and its environmental adaptability needs improvement.

Raibert proposed a straightforward and convenient three-point reference generation method based on the Spring-Loaded Inverted Pendulum (SLIP) model [24] and created bound gait by varying the forward speed, bounce height, and foot touchdown time of the robot [16]. Poulakakis et al. further generalized the SLIP model to a planar three-link structure with two compliant legs and one rigid rod. The bound gait was tested on a quadruped robot "Scout-II" that has one DOF located on each leg [25, 26]. Wang et al. contributed to the field by investigating the variation of the system energy during bounding [27]. Although these applications offered various advantages, the main drawback of the works mentioned above remains the adoption of simplified one-DOF spring-damper type leg structures. This limitation was removed in Gong et al. by performing bounding locomotion with a 20-DOF quadruped robot in 2018 [28]. However, in their model, several control parameters are not optimal.

Park et al. developed a bounding gait motion generating technique and applied it at their quadruped robot "MIT Cheetah." A simple impulse planning technique is provided for designing vertical and horizontal force profiles that result in zero net impulse on the system throughout one complete gait cycle. These force profiles are produced by employing third order Bezi ér polynomials [6, 11]. Although the ease of computing the integral of B ézier polynomials makes it a suitable candidate in planning the vertical forces balancing the gravity, sticking to a particular function class imposes restrictions on generated forces.

2) Bio-inspired approaches

Humans and animals are both extremely stable and also adaptable to their surroundings. Their rhythmic motions are regulated by their spinal cord's Central Pattern Generator (CPG), which is a self-oscillating neuronal network. Inspired from this, Kimura et al. utilized a CPG based reference generation algorithm. They coupled the spring mechanism with a neural CPG oscillator network. The running gait changed from pronk to bound with the help of the mutual entrainment between hips of the robot [29]. Furthermore, they validated their method by implementing bound gait on their quadruped robot “Tekken” [30]. Righetti et al. described a method for building CPGs with coupled oscillators that separately regulate the ascending and descending phases of the oscillations. They utilized a feedforward bound trajectory generator and two separate quadruped robots (Aibo and Ghostdog) to perform bound gait [31]. Li et al. utilized CPG to generate rhythmic signals along with a CPG modulation system to synchronize the limb phases. Their simulated quadruped robot performed a seamless and quick gait change from trot to bound gait [32]. Despite high adaptability, the mathematical model of CPG has significant limitations. It is not straightforward to tune parameters of CPG equations.

3) Learning approaches

There is a growing interest in utilizing learning algorithms to create suitable bound references for quadruped robots. Singla et al. developed a learning method for quadrupedal bound locomotion. They utilized reference trajectories learnt via deep reinforcement learning (D-RL), based on kinematic motion primitives. Experiments with their quadruped robot "Stoch," these kinematic motion primitives were successful in creating a bound gait [33]. Li et al. suggested using D-RL to build a neural network controller that can execute a bound gait. They were able to effectively adapt the proposed approach to the physical "Jueying Mini" robot. Proposed effective reward function based on contact points protected the robot from experiencing dramatic oscillations during bounding [34]. These learning algorithms are promising, but they are still complex and require high computing resources.

B. Our Contribution

In this study, we present a reference generation method for bounding quadruped locomotion. In the previous subsection, drawbacks of the approaches in the literature are briefly mentioned. Taking these points into account, we can state the contributions of this paper as follows:

- We use a biologically consistent articular joint leg to get a more accurate model unlike previous SLIP based techniques that used one-DOF spring-damping legs.
- We combine SLIP variables (forward speed, bounce height, and foot touchdown) with momentum and impulse laws to obtain efficient dynamic motion.
- We execute force planning via convex optimization without a specific function class assumption. This way, environmental adaptability of the reference generation technique is enhanced. Furthermore, the proposed method can synthesize references without predefined linear body positions.
- We achieve smaller vertical changes at COM utilizing new force planning method with convex optimization. Less body oscillations provide energy efficiency and easily trackable references.

The paper is structured as follows: Section 2 explains the proposed reference generation method. In Section 3, the kinematic arrangement and generalized dynamic equations of the quadruped robot used in the simulations are described. Section 4 presents the simulation results. The paper is concluded in Section 5.

II. Reference Generation

Detailed full-body dynamic equations of a quadruped robot are well studied in the literature [35]. However, utilizing a simplified robot model has the advantage of providing researchers the basic principles and insight into reference generation. Simplified versions of a robot model are commonly applied in the literature [6, 10, 11, 36]. The bound gait has symmetric leg motions with respect to the sagittal plane. The pitch motion of the body is central for the balance of a robot, and it takes place on this plane. The symmetricity of the motion with respect to the sagittal plane permits the modeling of the motion of a quadruped as a planar two-legged robot for the bound gait. The generated references would be identical for both the right and left sides in three-dimensional motion.

A. Reference Generation Model

Two models are employed in this paper. One of them is a detailed dynamics model used for simulation purposes, as a test bed. The second is a simplified model exploited for reference generation purposes. This second model is described in this section. The first and second model will be referred to as the detailed and simplified models in the text to follow. Furthermore, the simplified model has two versions which are called “simplified model” and “equivalent simplified model” in this work. Similar to previous work [6, 11, 37], in our simplified model, we assume mass-free legs since they are considerably lighter compared with the body mass (less than 10%) in our detailed model. For reference generation purposes the total mass (bodyand legs) of our robot is bulked on the body. The COM is located in the middle of the body. The simplified model is illustrated in Fig. 1. Under the assumptions above, the mathematical model of the body is presented as

$$m_b \ddot{x}_b = F_{fx} + F_{rx} , \tag{1}$$

$$m_b \ddot{y}_b = F_{fy} + F_{ry} - m_b g , \tag{2}$$

$$I_b \ddot{\theta}_b = P_{bfx} F_{fy} - P_{bfy} F_{fx} + P_{brx} F_{ry} - P_{bry} F_{rx} , \tag{3}$$

where F_{fx} and F_{rx} are the horizontal contact forces and F_{fy} and F_{ry} are the vertical contact forces acting on the front and rear feet, respectively. P_{bfx} and P_{brx} denote the horizontal distances while P_{bfy} and P_{bry} denote the vertical distances between COM and front, and rear feet. x_b is the horizontal linear position, $\mathrm{y_b}$ is the vertical linear position and θ_b is the angular position of the body. m_b represents the mass of the body, I_b is the inertia of the body around COM and g stands for the gravitational acceleration. Link connections are assumed to be rigid in the derivation of the moments around COM.

The stability of a quadruped robot is mostly related to its body motion. The body is exposed to high linear and angular accelerations during the bound gait. Hence, the motion of the body may be subject to instability. Taking the reaction forces into account is an important step in maintaining stability. By virtue of the massless leg model, an equivalent simplified model can be generated. In our equivalent model, contact forces are moved to the hips of the robot. The lacking moment reactions, occurring due to the foot positions with respect to the hip positions, are also added. The equivalent simplified model helps us focus on body stability. The angular motion equation for the body in the new model is given below:

$$I_b \ddot{\theta}_b = \left(\frac{L_b}{2}\cos\theta_b\right) F_{fy} - \left(\frac{L_b}{2}\sin\theta_b\right) F_{fx} - \left(\frac{L_b}{2}\cos\theta_b\right) F_{ry} + \left(\frac{L_b}{2}\sin\theta_b\right) F_{rx} + M_{fh} + M_{rh} , \tag{4}$$

$$M_{fh} = P_{hfx} F_{fy} + P_{hfy} F_{fx} , \tag{5}$$

$$M_{rh} = P_{hrx} F_{ry} + P_{hry} F_{rx} . \tag{6}$$

Here M_{fh} and M_{rh} are the moments at the hips due to the front and rear contact forces, respectively. P_{hfx} and P_{hrx} represent the horizontal, P_{hfy} and P_{hry} the vertical distance, from the hips to the front and rear feet. The body length is denoted by L_b. The linear motion equations in (1) and (2) remain unaffected.

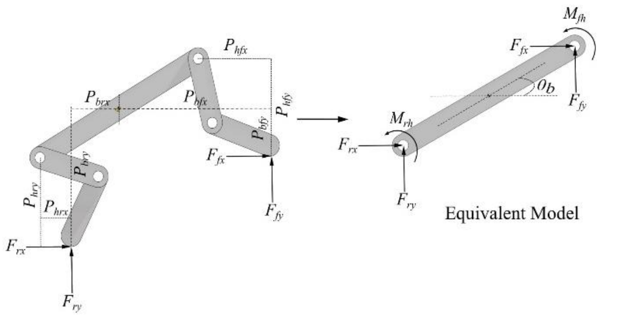

Figure 1. Simplified model and equivalent simplified model.

B. Contact Force Planning

The limit cycle behavior is one of the criteria reported in the literature to achieve gait stability in legged robots [38]. In order to maintain the limit cycle behavior in a bound gait, periodic motion in every individual gait cycle (the sum of stance and swing times of a single leg) is aimed at. If perfect periodicity is achieved, motion will be stable throughout the cycles. To accomplish this task, linear and angular momentum changes in every individual gait cycle have to be zero. If we apply this concept to the simplified model, we obtain periodicity at the linear and angular motions of the body in the sagittal plane. Consequently, the bound gait with a limit cycle behavior can be produced. If (1) and (2) are integrated over a gait cycle duration T the equations become

$$m_b\left(\dot{x}_b(T) - \dot{x}_b(0)\right) = \int_0^T \left(F_{fx} + F_{rx}\right) dt \tag{7}$$

$$m_b\left(\dot{y}_b(T) - \dot{y}_b(0)\right) = \int_0^T \left(F_{fy} + F_{ry}\right) dt - \int_0^T m_b g dt \tag{8}$$

Left hand sides of these equations represent the horizontal and vertical linear momentum changes through the whole gait cycle, respectively. Right hand sides of the equations are the net impulses in horizontal and vertical directions. The net impulses created by the contact forces are set to zero through a gait cycle for periodicity. Then, the linear momentum at the start of a gait cycle and at the end of the gait cycle become identical. The contact forces are planned by employing zero linear momentum change requirement:

$$\int_0^T \left(F_{fx} + F_{rx}\right) dt = 0\,, \tag{9}$$

$$\int_0^T \left(F_{fy} + F_{ry}\right) dt - \int_0^T m_b g dt = 0\,. \tag{10}$$

The vertical forces are chosen positive to balance the net impulse created by gravity. The integrals of front and rear horizontal forces over a gait cycle (i.e., impulses) are chosen in the same magnitude and at opposite directions to cancel each other. The periodicity of angular body motion is achieved by a predefined angular position reference. Due to its periodicity, basic trigonometric cos function is chosen as the reference trajectory of the angular body position:

$$\theta_{ref} = A cos(\omega_b t + \phi)\,, \tag{11}$$

$$\ddot{\theta}_{ref} = -A\omega_b^2 cos(\omega_b t + \phi)\,. \tag{12}$$

Here θ_{ref} is the reference of angular body position, A is the amplitude of body oscillations, ω_b is the frequency of body oscillations and ϕ is phase difference. $\ddot{\theta}_{ref}$ is the angular acceleration that relates the contact forces and body angular position. Given these quantities, our main goal is to design contact forces which provide the desired angular body position with the zero linear momentum change constraint. Suitable contact force references are determined by formulating and solving an optimization problem.

In addition to the angular body reference, a predefined foot position reference is designed to calculate the moment terms in (4) and solve the inverse kinematic problem. The reference trajectory of the foot is constructed using polynomials by considering the stride length and maximum vertical swing foot clearance. In order to reduce impact forces, swing foot is set to arrive to the floor with low velocity (zero, if possible). Finally, the reference joint positions are found by solving the inverse kinematic problem between the robot hip and foot positions.

C. *Optimization Process*

As mentioned in the previous section, the production of appropriate contact forces is indispensable for the reference generation method. Previous work in [6,11] proposed a solution which utilized third-order Bézier polynomials for planning contact forces. The ease of integration of a Bézier curve makes it suitable to plan the vertical forces that balance gravity. This planning method involves only a particular function class which imposes restrictions on planned forces. However, in the contact force planning approach proposed in this paper, no specific function class is assumed a priori. Such design freedom necessitates the derivation of a general optimization scheme. Discretization is applied on all quantities in one gait cycle time for the optimization process. Denoting all contact force values in one gait cycle by the vector $\boldsymbol{F} = \left[\overline{F}_{fx}\ \overline{F}_{fy}\ \overline{F}_{rx}\ \overline{F}_{ry}\right] \in \mathbb{R}^{(i_t)\times(4)}$, where f and r stand for front and rear, respectively, we get,

$$\overline{F}_{fx} = \left[f_{fx_1}\ f_{fx_2}\ \cdots\ f_{fx_{i_t}}\right]^T,$$
$$\overline{F}_{fy} = \left[f_{fy_1}\ f_{fy_2}\ \cdots\ f_{fy_{i_t}}\right]^T,$$
$$\overline{F}_{rx} = \left[f_{rx_1}\ f_{rx_2}\ \cdots\ f_{rx_{i_t}}\right]^T,$$
$$\overline{F}_{ry} = \left[f_{ry_1}\ f_{ry_2}\ \cdots\ f_{ry_{i_t}}\right]^T. \tag{13}$$

Here, subscript i_t denotes the terminal iteration number in one gait cycle time (i.e., the ratio of one gait cycle time to the sampling time). Our goal is to obtain a solution which minimizes the difference between the predefined angular body acceleration $\ddot{\theta}_{ref}$ and the actual angular body acceleration $\ddot{\theta}_b$, produced by the contact forces, at each time step. The following objective function is employed:

$$\min_{\boldsymbol{F}} \sum_{i=1}^{i_t} |\ddot{\theta}_{ref_i} - \ddot{\theta}_{b_i}(\boldsymbol{F})|^2\,. \tag{14}$$

In (14), i denotes an iteration index. The angular body position is formed to suit being utilized in an objective function by inserting (5), (6) and (13) into (4):

$$\begin{aligned}\ddot{\theta}_{b_i}(\boldsymbol{F}) = &\left(\tfrac{L_b}{2I_b}\cos(\theta_{b_i}) + P_{hfx_i}\right) f_{fy_i} - \\ &\left(\tfrac{L_b}{2I_b}\sin(\theta_{b_i}) + P_{hfy_i}\right) f_{fx_i} - \\ &\left(\tfrac{L_b}{2I_b}\cos(\theta_{b_i}) - P_{hrx_i}\right) f_{ry_i} + \\ &\left(\tfrac{L_b}{2I_b}\sin(\theta_{b_i}) - P_{hry_i}\right) f_{rx_i}\,.\end{aligned} \tag{15}$$

Solutions minimizing this objective function are expected to yield proper contact forces. The next step is to declare the constraints for this problem. Any functional class is not preassigned for solutions. However, we know that the output responses should be bounded. Solutions should not have highly fluctuating behavior and they should possess a smoothness property for continuity in time. This constraint over the solution can be imposed by limiting the values in the neighboring time indices by $|f_{i+1} - f_i| \leq \delta_1$. Since this constraint should be in effect for each pair of neighboring time indices, we obtain $4i_t$ constraints on the problem. Because we know that the vertical impulse is equal to the net impulse created by the gravity, the sum of the values must be bounded:

$$\sum_i^{i_t}\left(f_{fy_i}+f_{ry_i}\right)=\sum_i^{i_t} m_b g. \tag{16}$$

We also know that the net impulses created by the horizontal forces have the same total magnitudes at inverse directions,

$$\sum_i^{i_t} f_{fx_i} = -\sum_i^{i_t} f_{rx_i}. \tag{17}$$

As a last step, the relation between the parts of the solution vector is stated. In order to move without slipping, each corresponding value of f_x must be smaller than the product of the ground friction coefficient μ and the vertical contact force f_y. This yields a multiplier constraint of the form $f_{x_i} \le \mu f_{y_i}$. The constraint must hold for each time step (μ is a positive constant). This provides $2i_t$ extra constraints. Finally, the full problem can be stated as

$$\begin{aligned} \min_{\boldsymbol{F}} \quad & \sum_{i=1}^{i_t} |\ddot{\theta}_{ref_i} - \ddot{\theta}_{b_i}(\boldsymbol{F})|^2 , \\ s.t. \quad & \sum_i^{i_t}\left(f_{fy_i}+f_{ry_i}\right) = \sum_i^{i_t} m_b g , \\ & \sum_i^{i_t} f_{fx_i} = -\sum_i^{i_t} f_{rx_i} , \\ & |f_{i+1} - f_i| \le \delta_1 , \\ & f_{x_i} \le \mu f_{y_i} . \end{aligned} \tag{18}$$

This objective function contains only first order values of particular f_i elements and is linear. Similarly, due to the convexity of the absolute value function, the summation constraint is a convex constraint. Thus, our derived optimization problem turns out to be a convex problem with a unique global minimum and can be solved in closed form [39]. To solve the aforementioned problem, we used SQP solver of fmincon function in MATLAB 2018b.

D. Stability Analysis

A limit cycle is a periodic orbit that is either asymptotically stable (marginally stable) or unstable. When we think about limit cycle dynamics with the addition of ground impacts to legged locomotion, it is necessary to adopt an extra stability analysis. The Poincaré map is a widely used technique for defining a limit cycle's stability [40]. In our research, we will analyze the dynamics of the n dimensional system, $x = f(x)$ and create a surface of section, S, that has $n-1$ dimensions. Given these conditions, the Poincaré map is defined as a mapping from S to itself,

$$P: \{(x,t)|t=0\} \to \{(x,t)|t=T\}, \quad x^* = P(x^*) \tag{19}$$

where x^* is the fixed point of the Poincaré map. For any x^*, if $P(x^*)$ exists then the stability analysis of a limit cycle becomes the stability analysis of a fixed point on a discrete map. As a result, local limit cycle stability can be deduced via an eigenvalue analysis. It is possible to discretize (19) by linearizing it around the fixed point x^* that represents the periodic orbit outcome in a discrete linear system,

$$x(k+1) - x^* = \left.\frac{\partial P}{\partial x}\right|_{x=x^*} (x(k) - x^*) \tag{20}$$

The characteristic multipliers are the eigenvalues (λ_i) of the derivative matrix of the Poincaré map. We know that if the eigenvalues of this matrix are smaller than one, the resulting periodic orbit is stable [41].

III. Quadruped Robot Model

A. Kinematic Arrangement

The detailed quadruped model employed in simulations consists of 18 DOF with 3 DOF on each leg. The remaining 6 DOF are the position and orientation of the floating base. Every DOF on legs are rotational. Each leg has an adduction/abduction (a/a) joint on the hip, flexion/extension (f/e) joints on the hip and the knee. There is an illustration of the detailed quadruped model in Fig. 2.

B. Generalized Robot Dynamics

The generalized position coordinates, velocities and accelerations of the quadruped robot are denoted by x, $\dot{x}$ and $\ddot{x}$, respectively. These vectors include both the floating-body information with respect to an inertial frame, and information of joint variables.

$$x = \left[x_{bd}^T \;\; q_j^T\right]^T,\; \dot{x} = \left[v_{bd}^T \;\; \dot{q}_j^T\right]^T,\; \ddot{x} = \left[a_{bd}^T \;\; \ddot{q}_j^T\right]^T, \tag{21}$$

where $x_{bd} \in SE(3)$ is the position and orientation of the robot body with respect to the inertial frame. $q_j \in \mathbb{R}^{12}$ is the joint angular position vector of the quadruped robot with 12 joints. The motion equations of a quadruped robot in contact with the environment is expressed as:

$$M(x)\ddot{x} + C(x,\dot{x}) + G(x) + J_C(x)^T F_C = S^T \tau , \tag{22}$$

where $M(x) \in \mathbb{R}^{(6+12)\times(6+12)}$ is the inertia matrix of a quadruped robot, $C(x,\dot{x}) \in \mathbb{R}^{(6+12)}$ is the Coriolis and centripetal forces, $G(x) \in \mathbb{R}^{(6+12)}$ is the gravitational force, $J_C(x) \in \mathbb{R}^{(6\times4)\times(6+12)}$ is the contact Jacobian of a quadruped robot with respect to the world frame, $F_C \in \mathbb{R}^{(6\times4)}$ is the vector of 24 linearly independent contact forces and torques which are applied by the robot to the environment, $S \in R^{(12)\times(6+12)}$ is the selection matrix of the actuated joints, $\tau \in R^{12}$ is the vector of actuated joint torques. Since there is no actuation on the body of quadruped robot, the S matrix is $[0_{12\times6} \;\; I_{12\times12}]$.

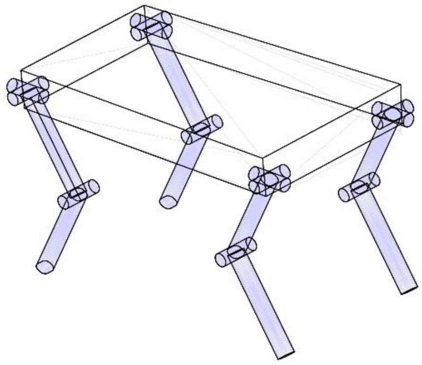

Figure 2. Kinematic arrangement of the detailed quadruped robot.

IV. Simulation Results

The goal of this section is to present the applicability of the proposed reference generation method. Hence, it is assumed that generated references are perfectly tracked. The validity of the reference generation method for quadruped bounding gait via contact force planning is shown on a simulated quadruped robot model. This simulation environment is constructed in MATLAB & Simulink. The overall scheme of the simulation environment is shown in Fig. 3.

For simplicity, the body and legs of the robot are modelled as rectangular blocks. Simulation and locomotion parameters are given in Table I. The predefined foot positions are shown in Fig. 4.

Contact force planning is carried out based on the predefined angular body position and foot references. As mentioned in Section III, the duration of a single cycle is defined as the summation of stance and swing periods of the leg (either front or rear). Experimental findings indicate that swing time remains relatively constant in the range of $0.22 - 0.3$ s in the bound gait although locomotion speed changes [42]. Following these findings, we select swing time to be 0.22 s in our simulations to reach higher speeds. Stance time is also determined as the same value for the sake of symmetry. Hence, one cycle duration is 0.44 s. The results of force planning are shown in Fig. 5. It is observed that all the horizontal force values are inside in the friction cone.

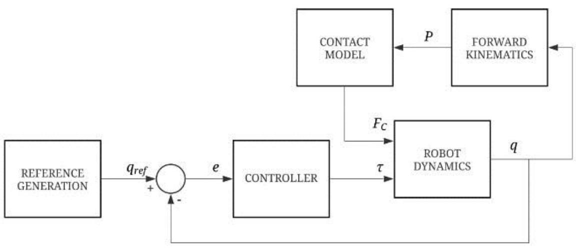

Figure 3. Simulation environment block diagram.

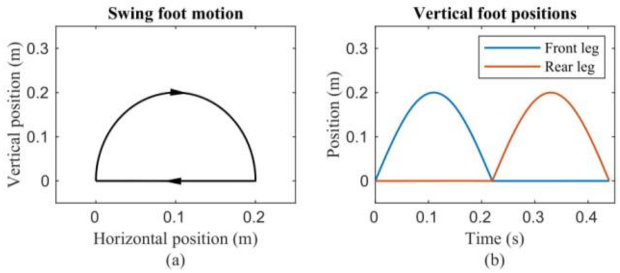

Figure 4. Foot positions in one cycle, (a) foot motion in space, (b)vertical foot position in time.

TABLE I. Simulation and Locomotion Parameters

Simulation Parameters		
Definition	Symbol	Value (unit)
Body length, height, and width	$L_b - H - W_b$	$1 - 0.15 - 0.6\ (m)$
Shank length, height, and width	$L_s - H_s - W_s$	$0.4 - 0.06 - 0.1\ (m)$
Thigh length, height, and width	$L_t - H_t - W_t$	$0.4 - 0.06 - 0.1\ (m)$
Body and sum of leg masses	$m_b - m_l$	$36 - 4\ (kg)$
Gravitational acceleration	g	$9.81\ (kgm/s^2)$
Sampling time	t_s	$0.5\ (ms)$
Friction coefficient	μ	0.3
Locomotion Parameters		
Definition	Symbol	Value (unit)
Gait cycle	t_g	$0.44\ (s)$
Swing time	t_{sw}	$0.22\ (s)$
Stance time	t_{st}	$0.22\ (s)$
Step height	h_{st}	$0.2\ (m)$
Step length	l_{st}	$0.2\ (m)$

After force planning, the next step is to test the generated references. We performed 5 s simulations for this purpose. Robot body variables are shown in Fig. 6 and Fig. 7. Although multiple simulations are successfully run with various angular body references, only one sinusoidal reference with 22 ° amplitude is presented here.

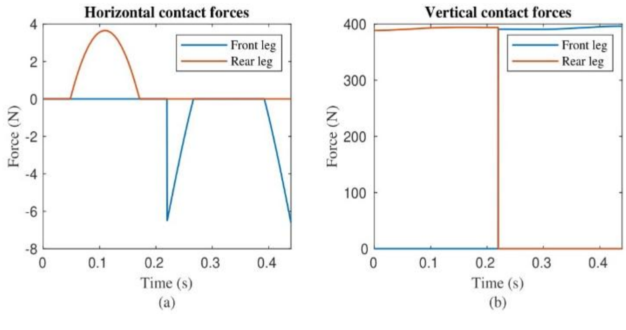

Figure 5. Forces generated by optimization, (a) horizontal force, (b) vertical force.

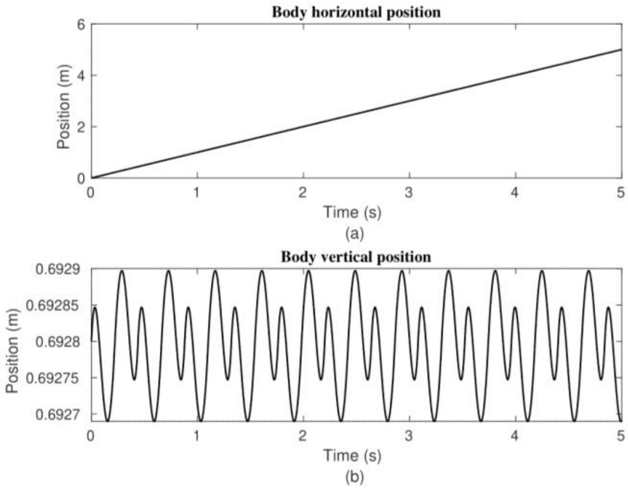

Figure 6. Body linear positions, (a) horizontal position, (b) vertical position.

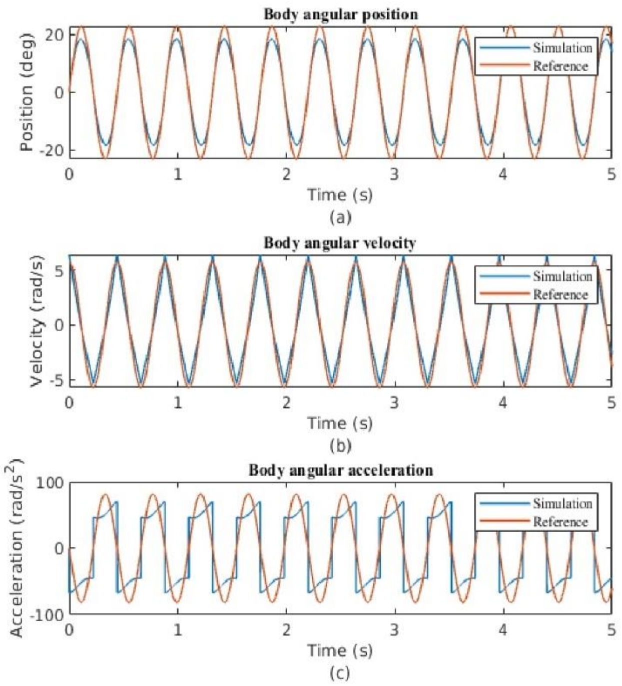

Figure 7. Body angular variables in a sagittal plane, (a) position, (b) velocity, (c) acceleration.

To present the stability of the technique, Poincaré analysis is applied. Poincaré stability criterion indicates that if the return map converges to a fixed point, a hybrid system with impact effects is stable [43]. A fixed point has been computed numerically through simulation. On par with our expectations, the simulation results reveal that the eigenvalues of the derivative matrix of the Poincaré map are less than one. Furthermore, the periodic orbit of the body pitch motion has stable limit cycle behavior. In Fig. 8, we show this periodic orbit for body pitch motion.

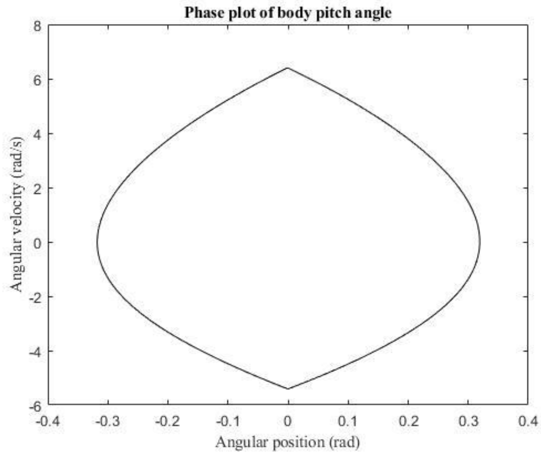

Figure 8. Phase plot of body pitch angle.

In addition to the reference generation method presented above, we also simulated a vertical impulse planning method with third order Bézier polynomials proposed in [6,11], under the same conditions. Results show that our presented method has less oscillations in vertical body position compared to Bézier polynomial modeling. The results are shown in Fig. 9.

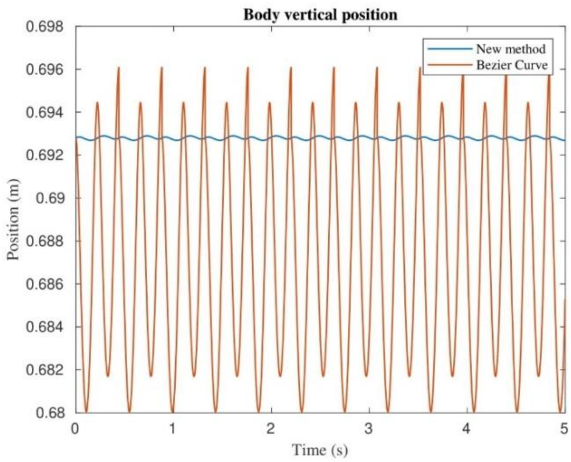

Figure 9. Comparison of the proposed method and Bézier polynomials.

V. Conclusion

We have successfully demonstrated stable quadruped bound gait in the simulation environment. Simulation results show that periodicity is achieved for the vertical linear and pitch motions of the robot body at consecutive cycles. With the proposed reference generation method, the robot retained its initial horizontal velocity of approximately $1\ m/s$ at consecutive cycles. There are small oscillations in the horizontal velocity due to the changes in horizontal friction forces. With the aid of the reference generation method presented in this paper, we found that there are relatively small friction forces (with the magnitude of approximately $6\ N$ at most.) in the horizontal direction. Therefore, the oscillations of the horizontal velocity are insignificant. It is not easy to track the acceleration reference perfectly due to not having contact forces during the swing phase. The transition between swing and stance phases produces sudden switches between positive and negative values of the angular acceleration of body. Nevertheless, it is observed that these effects do not deteriorate the periodicity of the motion and can be easily coped with setting an appropriate offset to obtain the desired angular motion.

The proposed method successfully produces the reference of body vertical position. Despite high amplitude of the body angular position (almost 20 degrees), there is less than millimetric changes at the body vertical position during motion. It improves the vertical body stability significantly over the method employing Bézier polynomials. With a less amount of oscillation in the vertical body movement, the potential energy radiation of the body is smaller. Therefore, reference generation method is more energy efficient. It creates trajectories which are more suitable to control. Another advantage of the proposed method is that it can synthesize references without predefined linear body positions. It generates bound gait references only with predefined angular body reference and foot reference trajectories.

The directions to follow in our future work are as follows. We are planning to construct a control architecture to track the planned position and force references. Also, we will design a body yaw controller to control the motion direction of the robot and finally, we aim to explore the proposed reference generation method on a real quadruped robot.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

The contributions of authors to this work are as follows: OKA developed the reference generation method, constructed simulation environment, executed simulations, interpreted the results, and wrote the paper. KE provided academic support, supervised the research, and gave valuable feedback, laboratory equipment and workstation facilities.

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