Multi-objective Optimization of Feedback Parameters of Wheel Terrain Interaction of an Autonomous Vehicle

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Abstract—Optimal force distribution analysis is an integral part of research field of autonomous vehicle research in rough terrain. A set of quasi-static force analysis method of force distribution is proposed based on three-dimensional force of the vehicle on the rough ground surface. The algorithm tries to avoid excessive slip. A simulation study in MATLAB software is carried out in a typical three-dimensional terrain environment with regard to the power consumption of motors and forces of robot with related constraints. Simulation is employed on the typical three-dimensional terrain model. Pareto optimal solution sets was analyzed as a major concern. Furthermore, different Pareto fronts were obtained with different percentage of noise induced into the terrain with typical characteristics.

Index Terms—autonomous vehicle, multi-objective optimization, pareto optimal solution, robustness, wheel-terrain interaction

I. INTRODUCTION

Integrated mechatronics system results in much greater flexibility, easy redesign and programming, and the ability to carry out automated data collection and reporting [1]. In order to achieve good force distribution during locomotion or navigation, a trade-off design for a reconfigurable mobile robot based on multi-objective optimization with respect to terramechanics is proposed by Xu He et al. [2]. Perhaps all real-world problems are, in fact, multi-objective optimization problems for which there is no unique or single solution for it, but a set of solutions for which holds that there are no superior solutions considering all the objectives at the same time [3]. This concept of multi-objective optimization has already been considered for different aspects of wheeled mobile robots and other vehicles. Multi-objective optimizations for equipment configurations of earthmoving machines are explained [4-6]. A rough-terrain control (RTC) methodology is presented that exploits the actuator redundancy found in multi-wheeled mobile robot systems to improve ground traction and reduce energy consumption [5].

A compact and light weighted asymmetrical prototype is obtained with better trafficability and other prototypes can produce diversified configurations to meet specific requirements by using MOO for mobile robot with 5th wheel is proposed [7]. In this work, a new concept based on orthogonal forces is taken into consideration in three dimensional environments. The basic idea is to achieve a balance between the power consumption from motor used to steer and force experienced. This concept assumes that contact forces and power consumption are related and must ensure for minimum tip-over tendency. Due to noise impacts from different sources are also a problem which is addressed by assuming different level of noise and analyzing solution diversity in the base solutions. Non-dominated solution is chosen by genetic algorithm such that the trade-off solution will give better result in objective space. Numerical test problems involving constraints and some constrained engineering design problems which are often used in the evolutionary multi-objective optimization (EMO) literature are discussed [8]. Mah Ali et. All proposed control algorithm is derived from both the kinematic and dynamic modelling of a non-holonomic wheeled mobile robot that is driven by a differential drive system [9].

II. METHODOLOGY

The methodology comprises the sensing and compilation of data output from motor used, transfer of dynamic functions and physical data of vehicle used. Merely in first stage a prototype is developed in CATIA and exported to the dynamic environment to simulate. For the simulation, physical parameters are fitted as a constraint environment followed by mathematic modelling and parametric modelling. The real time data received is being further processed to get pareto optimal solutions in different noise levels. Kinematic analysis and mathematic modelling formulated and considered for variables for optimization in search of input output parameters of the vehicle. In this regard, wheel-terrain interaction basics for shear strength of terrain are taken basic inputs with variables and output of motor power, drawbar pull and normal load in particular terrain profile are taken. But, at

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the premises of these scenario smart decision-making system included with Multi-objective Evolutionary Algorithm (MOEA) applied pareto optimal solutions and further decision will be taken by the vehicle by self accordingly. Noise levels are determined to get robust solutions or trade-off solutions but at the extreme condition of tip-over typically physical reconfiguration system advised to take into consideration. Fig. 1 shows the basic structure and flow of this work.

This study is carried out based on a mobile robot represented with the Pro-Engineer model and real robot as shown in Fig. 2, which has four powered wheels and a fifth wheel for slip estimation. For this study the fifth wheel not considered for the interaction with selected terrain. Reconfigurable chassis facilitates mass distribution of robot such as adjustment of center of gravity, position variation of GPS antenna, camera mast and other scientific instruments embedded on it.

### A. Wheel Terrain Interaction

Coulomb's equation for maximum shear strength $\tau_{\text{max}}$ can be calculated which can withstand by ground with internal friction angle $\phi$ with soil cohesion $c$ and the corresponding wheel terrain model is shown in Fig. 3 [10].

$$\tau_{\text{max}} = (c + \sigma_{\text{max}} \tan \phi)$$

The known quantities the vertical load of the vehicle $W$, torque developed $T$, angular speed $\omega$ of the wheel, and wheel linear speed $V$ and sinkage $z$ measured with on-board sensors with special arrangements. The force balance equations for vertical load of the vehicle $W$, draw bar pull $DP$ and torque $T$ with wheel radius $r$ and width of the wheel $b$ are given by:

$$W = rb \left( \int_{\theta_1}^{\theta_2} \sigma(\theta) \cos \theta . d\theta + \int_{\theta_1}^{\theta_2} \tau(\theta) \sin \theta . d\theta \right)$$

$$DP = rb \left( \int_{\theta_1}^{\theta_2} \tau(\theta) \cos \theta . d\theta - \int_{\theta_1}^{\theta_2} \sigma(\theta) \sin \theta . d\theta \right)$$

$$T = r^2 b \int_{\theta_1}^{\theta_2} \tau(\theta) . d\theta$$

The shear stress is defined as;

$$\tau(\theta) = (c + \sigma(\theta) \tan \phi \left( 1 - e^{-\theta/(1-1\sin \theta_1 - \sin \theta_2)} \right)$$

![Figure 1. Basic flow of research work.](image1)

**III. KINEMATIC MODELLING**

A general approach to the kinematics modeling and analyses of autonomous all-terrain vehicles traversing uneven terrain is described in this paper. Based on a four-wheel autonomous vehicle, the model is derived for full 6 DOF motion, enabling movements in the, and directions, as well as rotations of pitch, roll and yaw. Differential kinematics is derived for the individual wheel motions in contract with the terrain. Then, the resulting equation of a single wheel motion is derived from composite equation for the vehicle motion.

One usually attempts to model a system that relied on a human operator, the difficulty in creating true autonomy lies in the ability to transform the thinking an action of the human operator into an effective set of behaviors and rules by which the autonomous system will abide. Such vehicles are mobile instrumentation platforms that have an integral navigation and control system, which work without any human participation. The autonomous vehicle as shown in Fig. 2, typically considered to work in an unknown environment with accessories equipped to perform intended use. They carry various types of sensors, cameras, and spot lights. Apart from this, robot also carries sensors, which helps in various functions such as detection drivers, tracking, and collecting different terrain parameters.
For wide range of sinkage coefficients in different terrain experience, the equations are simplified for computational purpose as follows:

\[
W = \frac{rb}{\theta_m(\theta_i - \theta_m)} \left\{ \frac{\sigma_m(\theta_i - \theta_m)}{\theta_m \theta_i - \theta_m} \cos \theta_i - \theta_i + \theta_m \right\} \\
\]

\[
DP = \frac{rb}{\theta_m(\theta_i - \theta_m)} \left\{ \frac{\sigma_m(\theta_i - \theta_m)}{\theta_m \theta_i - \theta_m} \cos \theta_i - \theta_i + \theta_m \right\} \\
\]

\[
T = \frac{1}{2} r^2 b (\tau_m \theta_i + c \theta_m) \\
\]

where, \( r, b, k \) and \( i \) are wheel radius, wheel width, shear deformation modulus and wheel slip ratio respectively. Where, wheel slip ratio \( i \) is taken as, \( i = 1 - \frac{\theta_j}{\theta_m} \). \( \theta_j \) is the entrance angle of wheel moving on terrain, \( \theta_e \) is the exit angle of wheel moving on soil and \( \theta_m \) is the angular position of maximum stress of soil acting on wheel.

Furthermore, Power model is considered for DC motor using pulse width modulation (PWM) amplifiers with respect to time scale of our interest can be written as [11];

\[
P_i(t) = V_{s,i}(t)I_{s,i}(t) = \frac{B_{s,i}}{K_{E,i}} \xi_i^2(t) + \frac{K_{F,i}}{K_{E,i}} \dot{q}(t) \xi_i(t) \\
\]

where, \( I_{s,i}(t) \) and \( V_{s,i}(t) \) are the equivalent DC current and voltage of \( i \)th rotor, \( R_{s,i} \) is the stator resistance, \( K_{E,i} \) is the transmission gear ratio and \( K_{E,i} \) is the equivalent torque constant. Also \( \xi_i(t) \) is the torque. The energy required to perform the operation can finally be computed by;

\[
E_{op} = \int_{0}^{T_f} \sum_{i=1}^{n} P_i(t)dt \\
\]

Optimization of the total energy consumption for operation time \( t \in [0, T_f] \), where \( T_f \) is the operation execution time. Simplified Power consumption with gears ratio \( n \) related to the motor torque \( \tau_i \) applied by the \( i \)th motor can be written as [12];

\[
P_d = \frac{Rr^2}{L_a K_m \eta_d} \sum_{i=1}^{4} T_i^2 \\
\]

where, \( r \) is the radius of the wheel; \( R \) is the motor resistance, \( L_a \) is the motor gear reduction ration, \( K_m \) is the torque constant of the motor, \( \eta_d \) is the efficiency of the motor and \( T_i \) is the tractive force of the \( i \)th wheel. In order to obtain the minimum energy consumption control methods, then total power consumption of the motor \( P_d \) should be minimized. In practical applications, especially in unknown environment, the soil types and characteristics are unknown, so wheel–terrain force coefficient \( \mu_0 \) is impossible to accurately predict. However, in order to carry out multi-objective optimization of traction, the concept of support force index is introduced such that better and more convenient way to express the optimization objective function can be achieved as mean equivalent traction coefficient:

\[
\Delta \mu_i = \frac{\sum_{j=1}^{n} \| \mu_{ij} \|}{n} \\
\]

The Eq. 13 is used to simulate the multi-objective optimization between traction and power consumption. The results obtained are explained in section V. These equations were considered as objective functions for MOO that optimizes for maximum traction or minimum power consumption depending on the terrain unevenness and necessity. The concept is to maximize traction when robot on highly uneven rough terrain while in relatively flat and easy terrain it would minimize power consumption and discussed further more in details.

IV. MULTI-OBJECTIVE OPTIMIZATION WITH NOISE

The fundamental principle in robustness is to minimize variation in performance caused by variations in different variables those can be considered as noise, thus providing insensitivity to different variables uncertainty. Noise stems from several sources, including sensor measurement errors, incomplete simulations of computational models, stochastic simulations and other environmental factors. Here noise parameters are considered from nature and characteristics of the terrain.

\[
\min_{x \in \mathbb{R}^n} F(x) = \{ f_1(x) + \delta_1, f_2(x) + \delta_2, ..., f_M(x) + \delta_M \} \\
\]

where, \( \delta_i \) is a scalar noise parameter added to the original objective function of \( f_i \) and \( F \) is the resultant objective vector. Each evaluation of the same solution results in different objective values can be defined mathematically for noisy Multi-objective Optimization (MOO) as in
equation 15. Fitness function on the basis can be written with including of noises as;

\[
\begin{align*}
P_i' &= P(T_i) \\
T_i' &= T_i(c + \Delta c, k + \Delta k, \phi + \Delta \phi) \\
N_i' &= N_i(c + \Delta c, k + \Delta k, \phi + \Delta \phi)
\end{align*}
\]  (15)

where, \( \Delta c, \Delta k \) and \( \Delta \phi \) represents respective terramechanics noise intensities in terms of soil cohesion, shear deformation modulus and internal friction angle. As a major concern for the effect of operational parameters of the autonomous mobile robot due to changes in terrain parameters, which influences greatly in remote operations, other noise sources like noise from measuring encoders, frictional variations etc. are not considered for the analysis. Different Pareto solutions in the form of Pareto front and corresponding values are obtained in the next section, which clearly shows the noise impacts due to related parametric variations. The parameters which are drawn through wheel terrain interaction are taken as input parameters as feedback for decision making and variation in noise level of those parameters recorded for optimization of vertical load of the vehicle \( W \), draw bar pull \( DP \) and torque \( T \).

Robustness can also be determined by sensitivity analysis of the data. The result in the Pareto front obtained in MATLAB can be analyzed for robust solutions by its changing tendency or gradient of the curve. In other word nature of curve for a small range or range of interest of the designer can be chosen for the analysis. For example, let us consider an optimal Pareto front obtained between power and normal load as shown in Fig. 4. The value of slope ratio that is slope function at one region say \( K_{S1} \) is compared to the value of another slope function say \( K_{S2} \). Lesser the slope function shows lesser the deviation in comparison and can be taken as more robust solution than other.

\[
\frac{\Delta y_2}{\Delta x_2} < \frac{\Delta y_1}{\Delta x_1}
\]

Figure 4. Sensitivity analysis.

Here, \( i.e. K_{S2} < K_{S1} \)

The solutions at the range interest of second region are more robust than first region.

More often than not, real-world problems are instantiations of the third type of multi-objective problems and this is the class of multi-objective problems that we are interested in. One serious implication is that a set of solutions representing the tradeoffs between the different objectives is now sought rather than a unique optimal solution.

The framework for MOEA is shown in Fig. 5 as given below which deliberately gives a framework for obtaining Pareto optimal front with different functions as follows.

\[
\begin{align*}
P &\leftarrow \text{Population initialization} \\
A &\leftarrow \text{Create external population or archive} \\
\text{While (Stopping criteria not satisfied)} & \quad \begin{align*}
P &\leftarrow \text{Eval}(P, A) \\
P &\leftarrow \text{Diversity}(P, A) \\
A &\leftarrow \text{Update}(P, A) \\
S &\leftarrow \text{Selection}(P, A) \\
P &\leftarrow \text{Variation}(S)
\end{align*}
\end{align*}
\]

Figure 5. Framework for MOEA

With reference to sensitivity analysis the final combined objective function with equation 6, 7, 8 and 15 has the overall constraint of noise intensities as;

\[
-0.1 \leq (\Delta c \text{ or } \Delta k \text{ or } \Delta \phi) \leq +0.1 \quad \forall \ i, i = \{1. \ . \ . \ . \ n\}
\]  (16)

Optimization of wheel-terrain interaction must consider physical constraints of the vehicle such that vehicle wheels remain in contact with terrain. In terms of wheel-terrain normal forces \( N \) must remain positive as a constraint;

\[
N_i > 0 \quad \forall \ i, i = \{1. \ . \ . \ . \ n\}
\]  (17)

Another constraint is that the torque produced must remain within the saturation limits that is;

\[
T_{i\ min} \leq (T_i, r) \leq T_{i\ max} \quad \forall \ i, i = \{1. \ . \ . \ . \ n\}
\]  (18)

The tractive force exerted on the terrain at the surface of contact must not exceed maximum force that the terrain can bear which can be simply approximated as Coulomb friction or force coefficient model as;

\[
T_i \leq \mu N_i \quad \forall \ i, i = \{1. \ . \ . \ . \ n\}
\]  (19)

Particularly in the equation 6-8, the constraint of angular positions has been limited for the purpose of this study as given below;

\[
15^0 \leq \theta_1 \leq 35^0 \\
15^0 \leq \theta_2 \leq 35^0 \\
0^0 \leq \theta_m \leq 15^0
\]  (20)

Parameter Settings of the simulation study considered for some of these examples are as stated in Table I which gives basic parameter setting for optimization environment.
### TABLE I. PARAMETER SETTING

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromosome</td>
<td>Binary coding; 15 bits per decision variable</td>
</tr>
<tr>
<td>Population</td>
<td>Population size 100; Archive (or secondary population 100)</td>
</tr>
<tr>
<td>Selection</td>
<td>Binary tournament selection</td>
</tr>
<tr>
<td>Crossover operator</td>
<td>Uniform crossover</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>0.8</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>( \frac{1}{\text{chromosomelength}} ) for ZDT1, ZDT4 and ZDT6; ( \frac{1}{\text{bitnumberper var table}} ) for FON and KUR</td>
</tr>
<tr>
<td>Ranking scheme</td>
<td>Pareto ranking</td>
</tr>
<tr>
<td>Diversity operator</td>
<td>Niche count with radius 0.01 in the normalized objective space.</td>
</tr>
<tr>
<td>Evaluation number</td>
<td>50000</td>
</tr>
</tbody>
</table>

### V. SIMULATION AND RESULTS

In this work, force distribution analysis carried out with the quasi-static force analysis of the autonomous vehicle. Multi-objective optimization (MOO) between various objectives is applied to get optimal solutions and corresponding Pareto fronts. Genetic algorithm from MATLAB® is used to get solutions by using 'gamultiobj' and 'fminimax' tools.

Note that all simulations shown below were done with an Intel® Core™ i7 CPU, M500@2.40 GHz processor, 4.00 GB installed memory(RAM) approximately with same processor load using MATLAB R2020a.

#### A. Simulation on Force Distribution

For simplicity the terrain model is considered as double Sine function as given in equation (13) and corresponding to Fig. 6 and 7.

\[ y = 10 \times \sin(x \times \pi/50) \]  

However, in real situation sinusoidal terrain may not be necessarily similar to be experienced.

#### B. Results

### TABLE II. SAMPLE PARETO SOLUTIONS FOR POWER(P), DRAWBAR PULL (DP) AND VERTICAL LOAD (W) WITHOUT AND WITH NOISE

<table>
<thead>
<tr>
<th>Trial 1(0% noise)</th>
<th>Trial 2(2% noise)</th>
<th>Trial 3(5% Noise)</th>
<th>Trial 4(10% Noise)</th>
<th>Trial 5(-10% Noise)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P</strong></td>
<td><strong>DP</strong></td>
<td><strong>W</strong></td>
<td><strong>P</strong></td>
<td><strong>DP</strong></td>
</tr>
<tr>
<td>5.04</td>
<td>15.15</td>
<td>6.40</td>
<td>5.14</td>
<td>15.45</td>
</tr>
<tr>
<td>10.90</td>
<td>10.30</td>
<td>10.81</td>
<td>11.12</td>
<td>10.50</td>
</tr>
<tr>
<td>4.79</td>
<td>15.54</td>
<td>7.08</td>
<td>4.88</td>
<td>15.85</td>
</tr>
<tr>
<td>5.40</td>
<td>14.63</td>
<td>10.01</td>
<td>5.51</td>
<td>14.92</td>
</tr>
<tr>
<td>8.72</td>
<td>11.51</td>
<td>8.59</td>
<td>8.89</td>
<td>11.74</td>
</tr>
<tr>
<td>5.22</td>
<td>14.88</td>
<td>10.48</td>
<td>5.32</td>
<td>15.18</td>
</tr>
<tr>
<td>****</td>
<td>****</td>
<td>****</td>
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<td>****</td>
</tr>
</tbody>
</table>

Tools in MATLAB degenerates into a random search under increasing levels of noise with greater variation leads to low robust solutions. The sample pareto solutions are tabulated as under. Trial one has been performed without noise that is on flat terrain, while trial 2 has been performed with 2% noise. Again, noise level increased to 5% in trial 3. Trial 4 and trial 5 are extreme noise levels of ±10% acted similar to sinusoidal wave as rough terrain and data extracted as an optimal solution set for further decision-making process. There may be number of pareto optimal solutions depending upon the real time situation to impart the better solution at the time. Furthermore, only sample solutions are tabulated here.

With these sample run for optimal solution with respect to power used and normal load in corresponding drawbar pull shows better trade-off solutions in the situations of different noise levels.

Fig. 8 shows the plot of power with respect to drawbar pull in the space of vertical load resulting trade off solution with 0% noise level while Fig. 9 with 2% noise level, Fig. 10 with 5%, Fig. 11 with 10% and Fig. 12 with -10% noise level. While Fig. 13 demonstrates Empirical Cumulative Distribution Function (CDF) of power at different noise levels in the solution region.

Figure 8. Surface plots of Power, DP and W for 0% noise level

Figure 9. Surface plots of Power, DP and W for 2% noise level

Figure 10. Surface plots of Power, DP and W for 5% noise level

Figure 11. Surface plots of Power, DP and W for 10% noise level

Figure 12. Surface plots of Power, DP and W for -10% noise level

Figure 13. Empirical Cumulative Distribution Function (CDF) of power at different noise levels
solutions along the obtained Pareto fronts. The non-dominated Pareto optimal solution is used to design motion control system of the mobile robot followed by experimental verification of real prototype autonomous vehicle.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Mahesh Kumar Isher conducted the research and wrote the paper. Ramchandra Sapkota and Sanjeev Maharjan provided scientific advisory of this paper; all authors had approved the final version.

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REFERENCES

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