Adaptive Plan Using Sigmoid Function for Nonlinear Topology Optimization

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Abstract² Topology optimization is a process that distributes material into the necessary position of a design area under the action of external force. The main purpose of this process is to decrease the mass of structure whilst still ensuring its strength. In this field, proportional topology optimization (PTO) is a popular non-sensitivity technique. This method updates material density through the relationship between the maximum stress at each iteration and allowable stress of the material. Additionally, the target of the material amount is added or removed by a certain ratio of the total number of elements. This renders the optimization to reach convergence a time-consuming process. This paper assumes that the ratio of moving material at each iteration has a significant effect on the convergence of the optimization process. Thus, this paper proposes adaptive moving material using the Sigmoid function for the proportional technique. A cantilever with nonlinear characteristic material is used to verify the effectiveness of this approach.

Index Terms² topology optimization, adaptive plan, volume, cantilever, nonlinear

I. Introduction

Topology optimization methods allow designers to determine the best structural layout under many requirements of load and constraint. In general, structural optimization is divided into three classes: size, shape, and topology optimization. The purpose of size optimization is to obtain the best sections, while the aim of shape optimization is to find the best node positions of the predefined nodes of the structure. Both are based on the predefined design layout. Topology optimization redistributes the material of a given design space until the optimal layout is achieved through many iterations. Presently, the result of topology optimization is often used for the preliminary concept of design process and thus plays an important role in constructing the structure.

First, in the literature on topology optimization with OLQHDU PDWHULDO methods are described in [1]-[4]. In recent years, upgraded methods of topology have been developed as follows. Guo et al. applied moving morphable components to conduct topology optimization, this solution was sufficient to substantially enhance the computational cost associated with topology optimization [5]. Furthermore, Zhang et al. developed an approach to preserve the smoothness of optimal design by using B spline curves for the boundaries of moving morphable components or moving morphable voids [6]. In [7], Daicong Da et al. presented an evolutionary topology optimization method with smooth boundary representation for continuum structures. Most recently, Hao Deng and Albert C. To applied deep learning to describe the density distribution of a material. This method assures the smoothness of the boundary and overcomes the checkerboard problem in the optimal layout [8]. Yun-Fei Fu et al. developed a smooth-edged material distribution strategy using the Heaviside smooth function for optimizing topology [9]. Xiaodong Huang introduced a floating projection topology optimization technique using a material penalization model to obtain a smooth layout [10].

For linear topology optimization, the deformation of material is small under the action of external force, and it is applied in many practical design problems. However, in many situations, nonlinear analysis must also be considered, including energy absorption structures or crashworthiness design. For instance, Lei Li et al. presented the density-based framework to enhance the ability of energy absorption of an optimal structure [11]. X Huang et al. introduced two sensitivity numbers to adjust the principal design parameters by applying an adjoint method [12]. Recently Suphanut Kongwat and Hiroshi Hasegawa applied the novel weight filtering method to prevent stress of the element from fluctuation problems under cyclic load [13]. Additionally, a class of nature-inspired evolutionary algorithms has also been adopted for topology optimization [14]-[17].

In topology optimization, the proportional method is a non-sensitivity approach, where the density of the element is updated by using the ratio between the stress of this element and the stress summation of all the elements in the design space at the current generation. The target of the total density is added or removed depending on the relationship between the maximum stress and allowable stress. This moving material is set from 0.001 to 0.002 of the total number of elements. Fixing the moving material

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during the optimization process results in a high cost of computation; it takes a long time to reach convergence. Thus, this paper proposes an adaptive volume factor for adding or removing material. This factor is generated by the Sigmoid function. The moving material is set to be high at the beginning and gradually reduce through each iteration. The reduction in moving material at a later iteration enhances the ability of exploitation and improves the convergence speed. The result is validated on a benchmark model: cantilever with nonlinear material.

To this end, the rest of this paper is organized into six sections. Section 2 depicts the cantilever model with a nonlinear material. Section 3 describes the methodology for topology optimization. Section 4 explains the density filtering technique. A proportional optimization algorithm is described in Section 5. Section 6 addresses the numerical example and presents a discussion. Finally, Section 7 includes some brief conclusions.

II. Optimization Problem

A. Bilinear Elastoplastic Material

The design domain is assigned to the bilinear elastoplastic material properties, which are input as 285 MPa of yield stress; allowable stress of 600 MPa; RXQJ¶V modulus of 207 GPa; and tangent modulus of 13,921 MPa, IRU 3RLVVRQ¶V UDWLR DQ -9 IRU RXQJ¶V PRGXOXV assigned to void regions E_{min}. The properties of bilinear elastoplastic material are depicted in Fig. 1.

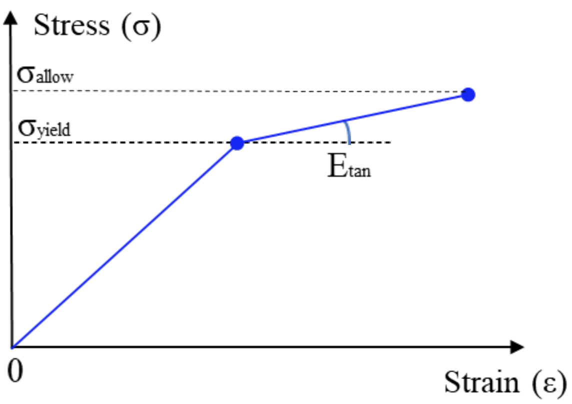

Figure 1. Properties of bilinear elastoplastic material.

B. Optimization Procedure

This research adopted compliance as an objective function, where the target is to minimize the compliance, while the maximum stress value is less than the allowable stress. The optimization problem is described as follows:

$$Min\ C = F^T.\ U \tag{1}$$

subject to

$$\sigma_{max} \leq \sigma_{allow} \tag{2}$$

where U is the displacement vector, F is the external force vector, σ_{max} is the maximum stress, and σ_{allow} is the stress limit.

III. Solid Isotropic Material With Penalization Method

The solid isotropic material with penalization method (SIMP) is the most popular method for topology optimization [18]. The density distribution of material within a design domain is assigned a binary value:

- x_e = 1 where the material is required (black).
- x_e = 0 where the material is removed (white).

To avoid the on±off nature of the problem, it causes a matrix of stiffness singular; the density of the element varies continuously from x_{min} to 1; x_{min} is the minimum allowable relative density value for empty elements that are greater than zero. Since the material relative density can vary continuously, the Young¶V modulus of the material at each element can also vary continuously and is computed by the power law:

$$E_e = E_{min} + x_e^p.(E_o - E_{min}) \tag{3}$$

where:
E_{min} is the elastic modulus of void element, and E_o is the elastic modulus of solid element. The penalty value (p) for the modified SIMP approach is set to 3.

IV. Density Filtering

The PTO method incorporates a density filtering. In the work of Bruns [19], a simple cone density filtering is introduced as follows:

$$\zeta_i = \frac{\sum_{j=1}^{n} w_{ij} d_j}{\sum_{j=1}^{n} w_{ij}} \tag{4}$$

$$w_{ij} = max\ (0, r_{min} - r_{ij}) \tag{5}$$

where ζ_i is the filtered density of element i; d_i is the non-filtered density of element i; w_{ij} is the filtering weight of elements i and j; r_{min} is the prescribed filtering radius; r_{ij} is the distance between elements i and j.

V. Proportional Topology Optimization Algorithm

In the PTO for the SIMP method, the update function is used to renew the density of the design variable through each iteration. The proportional technique is a non-sensitive method which builds an equation based on a ratio of stress and uses this parameter to update the density value as in Eq. 6.

$$\rho_i^{new} = \rho_i^{prev} + xRem.\ (\frac{\sigma_i^q}{\sum_{i=1}^{N} \sigma_i^q}).\ \zeta_i \tag{6}$$

where ρ_i^{new} is the new elemental density for the next iteration, and ρ_i^{prev} is the elemental density from the previous iteration; for the first proportional loop, we assume that the density value of the previous iteration is zero, σ_i is the elemental stress, and $xRem$ is the remaining material amount. xRem is defined as follows:

- At the first loop, *xRem* is set to *xTar*, where *xTar* is the target material amount. In previous research, this value has been defined by removing or adding 0.001 to 0.002 of the number of elements depending on the relationship between the maximum stress and the allowable stress [13], [20]. Thus, the optimization takes too long to reach convergence. This paper proposes an adaptive strategy to replace the fixed volume ratio of the total target density in the traditional topology optimization method. This ratio is generated by the Sigmoid function as in Eq. 7.

$$r_{adap}=\frac{1}{1+e^{-10/iter}} \tag{7}$$

where r_{adap} is the adaptive volume ratio, and iter is the current iteration. The gait of the r_{adap} chart depends on the current iteration, as shown in Fig. 2. In the beginning of the iteration, this value is set to be high to obtain a high speed and must be small for proper exploitation. As a result, we believe that it will steadily provide an optimal layout and reduce the calculation cost. The target material amount is calculated by Eq. 8.

$$xTar= \sum_{i=1}^{n} x_i \pm 0.01*r_{adap}*(\text{total number of elements}) \tag{8}$$

where

$$\begin{cases} + ;\, if\ \sigma_{max} \leq \sigma_{allow} \\ -\ if\ \sigma_{max} \geq \sigma_{allow} \end{cases} \tag{9}$$

- The new density amount is revised through each loop by comparing it to the target material amount as in Eq. 10.

$$xRem_{new} = xTar - \sum_{i=1}^{n} \rho_i^{new} \tag{10}$$

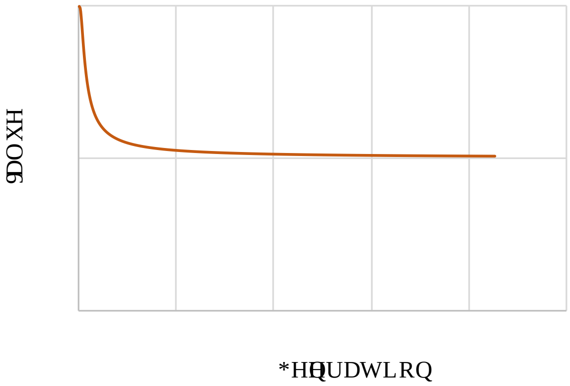

Figure 2. Adaptive parameter at each iteration.

If $xRem_{new}$ is less than 0.001, the updating process is terminated. Otherwise, $xRem_{new}$ is set to *xRem*, and the updating process is repeated.

The pseudocode of the algorithm is described below:

1. Building a cantilever model, assign the first density element to all elements.
2. While termination condition is not reached:

   Implementing FE analysis.

   Compliance calculation.

   Checking stop criteria; break if satisfied.

   Running proportional optimization algorithm.

   Generating adaptive volume factor at current iteration by Eq. 7.

   Calculating TA.

   Setting RA = TA.

   While remaining material amount (RA) is not small enough:

      Applying filtering weight and calculating new density value by Eq. 6.

      Calculating new amount of material (CA).

      Updating RA = TA - CA.

      The process is repeated.

Where TA is the target material amount; RA is the remaining material amount; CA is the current material amount.

VI. Numerical Examples

A. *Comparison of Two Methods*

1) *Test case 1:*

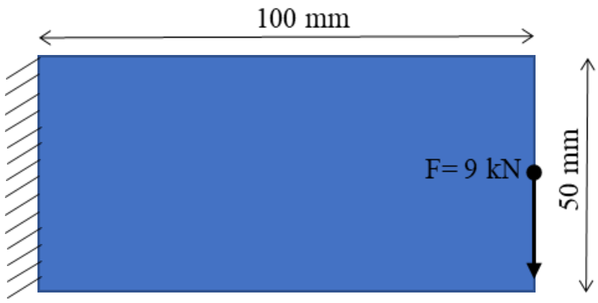

Figure 3. Cantilever model in test case 1.

In this case, a numerical example, as described in Fig. 3, is considered. The design area is applied by a force of 9 kN in the downward direction at the middle corner, and it is discretized to 100x50 elements.

The comparison of two proportional methods for topology optimization problem in Test case 1 is shown in Fig. 4. For the old approach, the optimal layout is obtained when the compliance is approximately 137 and the maximum stress is 556 MPa. As can be seen in Fig. 4, the previous approach reached convergence at 133 generations, while the proposed method achieved this at 42 generations and a compliance of 132, and maintained 64.34% of the initial material amount. Thus, the convergence speed is improved by 68.42%, and the distribution of the material during the optimization process is illustrated in Fig. 5.

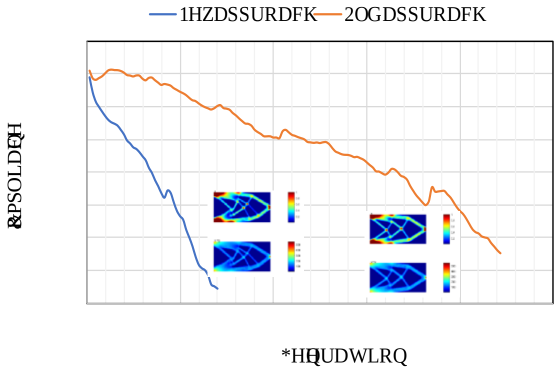

Figure 4. Speed comparison of two methods in test case 1.

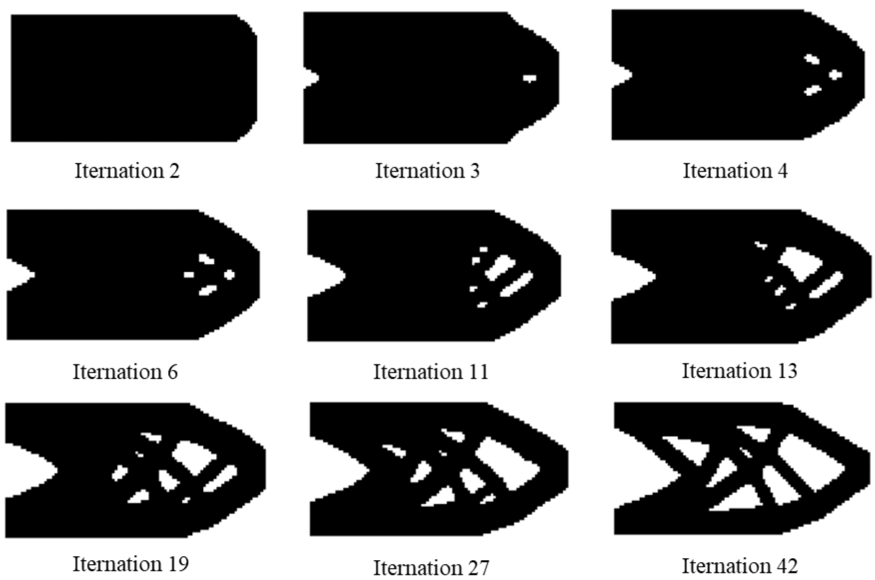

Figure 5. Material distribution during the optimization process of test case 1.

2) *Test case 2:*

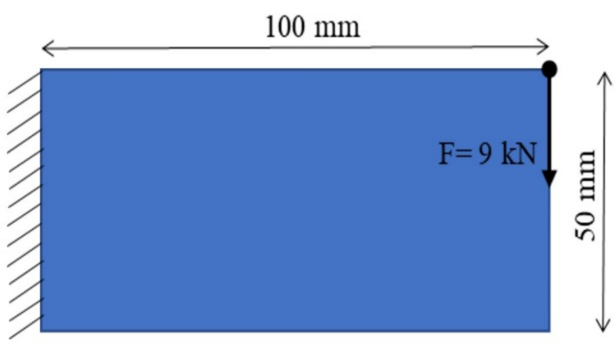

Figure 6. Cantilever model in test case 2.

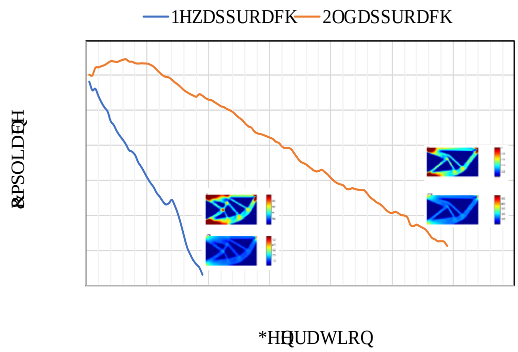

Figure 7. Speed comparison of two methods in test case 2.

In this case, a numerical example described in Fig. 6 is considered. The design area is applied by a force of 9 kN in the downward direction at the top corner, and it is discretized to 100x50 elements.

The comparison of two proportional methods for topology optimization is shown in Fig. 7. For the old approach, the optimal layout is obtained when the compliance is approximately 116 and the maximum stress is 563 MPa. As can be seen in Fig. 7, the previous approach reached convergence at 118 generations, while the proposed method achieved this at 38 generations and a compliance of 112, and maintained 63.58% of the initial material amount. Thus, the convergence speed is improved by 67.8%, and the distribution of the material during the optimization process is illustrated in Fig. 8.

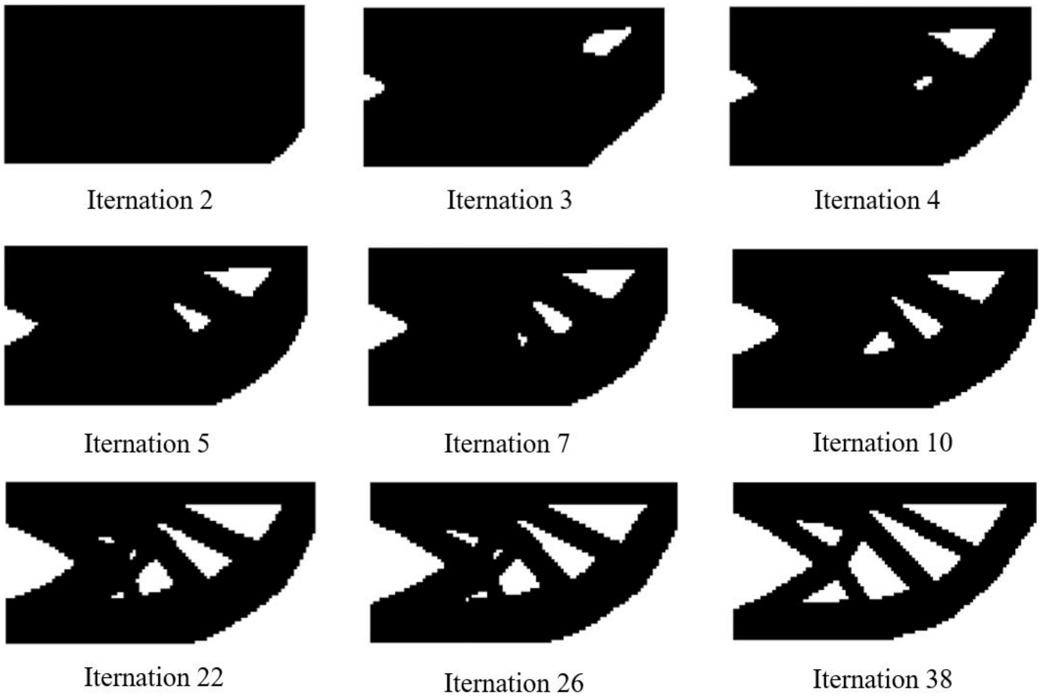

Figure 8. Material distribution during the optimization process of test case 2.

B. *Investigation for the Adaptive Volume Factor*

In this section, the numerical example of a cantilever beam in Test case 1 is used to investigate the effect of the adaptive volume factor for the proportional topology optimization as in Eq. 11.

$$r_{adap} = \frac{1}{1+e^{-\frac{\beta}{iter}}} \tag{11}$$

where β is from 5 to 50. The gait of r_{adap} charts depends on the current iteration, as shown in Fig. 9. The results of numerical examples are shown in Fig. 10. It can be seen that, in general, when β is increased from 5 to 50, the convergence speed also increases by 23.81%. The optimization process has the highest computation cost and reaches convergence at 52 generations when $\beta = 5$. From $\beta = 30$ to 50, the convergence speed is similar, and the optimal layout is achieved at 42 generations.

The results of all numerical examples are presented in Table I, in which all the optimal layouts are distinct, and it is uncomplicated to sketch out the real structure for the purpose of manufacture.

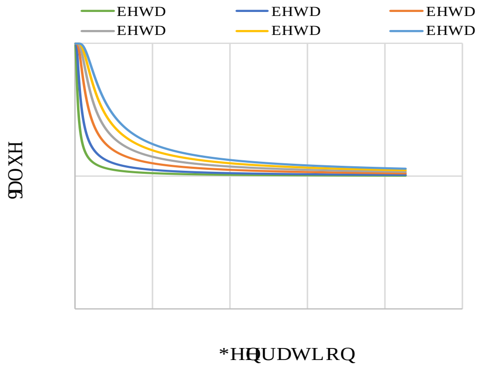

Figure 9. Gait of r_{adap} charts.

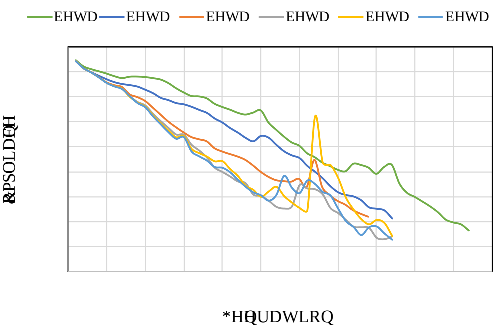

Figure 10. Results with six adaptive strategies.

TABLE I. RESULT OF NUMERICAL EXAMPLE.

β	Optimal layout	β	Optimal layout
$\beta = 5$		$\beta = 10$	
$\beta = 20$		$\beta = 30$	
$\beta = 40$		$\beta = 50$	

VII. CONCLUSIONS

This research proposes an adaptive approach to the proportional method. The optimization procedure is implemented based on MATLAB, and the numerical analysis is performed by LS-DYNA. The target of element density amount is adapted to enhance the convergence speed of the optimization process. The target material amount and the elemental stress are used to update the new element density. The full stress design criterion and the filtering technique are applied in this paper. Then, the optimal layout is obtained when the difference between the maximum stress and allowable stress is small enough or the compliance is constant. A cantilever model was used to validate the effectiveness of the proposed method.

CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

AUTHOR CONTRIBUTIONS

Van-Tinh Nguyen designed the methodology and simulated the cantilever problem in LS-DYNA. Ngoc-Tam Bui evaluated and commented on the methodology. Ngoc-Linh Tao and Thanh-Trung Nguyen reviewed and edited the manuscript. All authors contributed to the preparation of the manuscript; all authors approved the final version of the manuscript.

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