# Particle Shapes Effect on Segregation Phenomena in 2D Binary Granular Mixtures under Vertical Vibration

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Abstract—The objective of this study is to experimentally investigate the effect of particle shape on the segregation coefficient and segregation pattern of 2D binary granular mixtures under a vertical vibration. Polyoxymethylene (POM) cylinder with four different cross-sectional shapes, i.e. circle, triangle, square, and hexagon, was used as a particle to prepare the binary granular mixtures. Each mixture consisted of 500 particles for each two species of the shape. A vertical vibration was then applied to the mixtures by varying its level via the dimensionless accelerations ( $\Gamma$ ) from 1 to 5, while the vibrational amplitude was at 5 mm. The segregation phenomena were analyzed by the segregation coefficient and segregation pattern. The experimental results show that the segregation does not occur due to the effects of both particle shape and dimensionless acceleration. However, it was observed for all cases that a particle reorganization began when  $\Gamma > 2$ . In addition, convection rolls were found in some cases.

*Index Terms*— granular materials, segregation, particle shape effect, vertical vibration

# I. INTRODUCTION

Granular materials are widely used in many engineering process applications, such as rocks, sands, soils, plastic bead, grains, pharmaceutical particles. Granular materials in engineering term are conglomeration of solid particles with various sizes, shapes, and mechanical properties such as friction, porosity, and density, whose macroscopic mechanical behaviors are governed by the interaction forces between particles [1-4]. One of interesting phenomena of the granular media is a segregation. The segregation in granular mixtures can be defined as a process that a homogeneous bulk solid of component or species in mixed configuration transform to spatially non-uniform configuration by a relative movement within the material [5]. The segregation frequently occurs in many industrial processes during transportation and handling of granular media under vibrated or shaken systems, such as agriculture, material science, food processing, and pharmaceutical products [6, 7]. However, such the segregation is an undesirable phenomenon in the industries because it causes a significant issue in terms of low product yield and out of spec products. For this reason, the study on effect of particle properties on segregation is very interested topic since it can fulfill the knowledge and understanding about segregation phenomena. These are very useful to prevent such the segregation in many industrial processes. Most of previous simulation and experimental studies have been carries out mainly in a topic of particle size induced segregation under a vertically vibrated bed [8–10]. The well-known phenomena is *Brazil nut effect* (BNE), where the larger particles rise to the top of the mixture [8, 11–13]. Nevertheless, although there are many studies in the literature about segregation phenomena in the granular mixtures due to various particle properties, the effect of particle shape still unexplored.

In this study, the effect of particle shapes on segregation phenomena in a vertically vibrated bed of 2D granular binary mixtures is experimentally investigated by changing four different particle shapes and five different levels of a vertical vibration characterized by a dimensionless acceleration ( $\Gamma$ ). The paper is organized as follows. The characterization of samples and experimental setup are described in Section II. The segregation phenomena are then systematically analyzed in term of segregation coefficient and segregation pattern in Section III. Finally, the conclusions were presented in Section IV.

# II. EXPERIMENTAL SETUP

Polyoxymethylene (POM) cylindrical rod was used as a particle to prepare a granular sample consisting of 1000 particles. Two species of particle differing in terms of a cross-sectional shape, i.e. circle, triangle, square, and hexagon, were randomly placed inside a rectangular container. Each species has the same number of particles. It must be noted that a cross-sectional area for each shape and a length for the rod were about  $64 \text{ mm}^2$  and 60 mm, respectively [14]. In this study, six different binary granular mixtures were systematically tested, as shown in Fig. 1. Table I also provides information for each tested configuration. To the best knowledge of the authors, macroscopic behaviors of granular materials in 2D and 3D are the same. This is a reason why the present study performs only 2D analysis of the segregation, i.e. only the cylindrical face was considered [15].

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Figure 1. Initial configuration of each tested binary granular mixtures: a) CH, b) CS, c) CT, d) HS, e) HT, and f) ST

Under a gravity, the samples were then subjected to vertical vibrations in the manner of a sinusoidal wave provided by a scotch-yoke mechanism of a vertical testing machine, as shown in Fig. 2. Note that a vibrational frequency can be adjusted by an electric. invertor connecting to an electric motor. The level of the vibration was characterized by the dimensionless acceleration, which can be defined as

$$\Gamma = \frac{A\omega^2}{g} \tag{1}$$

where A is the vibrational amplitude,  $\omega$  is the angular frequency which can be expressed as a function of the vibrational frequency (f), i.e.  $2\pi f$ , and g is the gravitational force. Note that the dimensionless acceleration ( $\Gamma$ ) used in the present study was progressively varied from 1 to 5, while the vibration amplitude (A) was fixed at 5 mm. Each experiment was performed up to 20 minutes. During the experiments, the particle movement inside the granular samples was recorded along the cross-sectional plane by a high-frame rate video (240 fps).

TABLE I. TESTED CONFIGURATIONS

Species 1	Species 2	Case Name
Circle	Hexagon	СН
Circle	Square	CS
Circle	Triangle	CT
Hexagon	Square	HS
Hexagon	Triangle	HT
Square	Triangle	ST

#### **III. SEGRAGATION ANALYSIS**

Most of previous studies of segregation phenomena in granular materials under vibrations mainly focused on the effect of the particle size. Two significant mechanisms that induces the segregation were also introduced. These can be summarized as follows.

The first mechanism is a void filling, which is also known as a percolation. This mechanism is caused from the geometrical effect at the local scale. Under the vibration, particles can move either upward or downward inside the container. During the expansion phase, all the particles attempt to move upward into the top of the container. Available spaces between the particles are thus created. The particles can then simply fill into such spaces during the compaction phase, especially the smaller particles. Therefore, the larger particles are forced to migrate upward into the top of the container [8, 16, 17]. At the end, the BNE is generally observed. The void filling strongly influences when the ratio of the diameter size between the particles was increased [18]. However, it must be noted that this mechanism only dominates in the case of a small excitation [19].



Figure 2. Schematic view of vibrational testing machine

	<b>D</b>	P	P	1	C	
	$P_B$	$P_M$	$P_T$	and	Segr	egation pattern
If	$\geq 0.33$	≥ 0.33	≥ 0.33	-	PU	Pure uniform
Else if	$0.3 \le P_B \le 0.4$	$0.3 \le P_B \le 0.4$	$0.3 \le P_B \le 0.4$	-	U	Uniform
Else if	$0.55 \leq P_B < 0.98$	-	-	$ P_T - P_M $	В	Bottom
				< 0.14		
Else if	-	$0.55 \le P_M < 0.98$	-	$ P_T - P_M $	С	Central
				< 0.14		
Else if	-	-	$0.55 \le P_T < 0.98$	$ P_T - P_M $	Т	Тор
				< 0.14		
Else if	$0.47 \le P_B < 0.54$	-	$0.47 \le P_T < 0.54$	-	V	Pure V
Else if	$> P_M$	-	$> P_M$ and $> P_B$	-	VT	V-Top
Else if	$> P_M$ and $> P_T$	-	$> P_M$	-	VB	V-Bottom
Else if	-	$P_B < P_M < P_T$	-	$P_M - P_B$	TC	Top-Central
				$\geq 0.14$		
Else if	-	$P_B > P_M > P_T$	-	$P_M - P_T$	BC	Bottom-Central
				$\geq 0.14$		
Else if	-	-	$P_M > P_T > P_B$	$P_T - P_B$	CT	Central-Top
				$\geq 0.14$		
Else if	$P_M > P_B > P_T$	-	-	$P_B - P_T$	CB	Central-Bottom
				$\geq 0.14$		
Else	-	-	-	-	TS	Transition

TABLE II. CLASSIFICATION CRITERIA OF SEGREGATION PATTERN

The second mechanism is a convection or sometimes also known as convection rolls. Indeed, the friction at the sidewalls of the container induces the convection rolls in a vertically vibrated system [20]. This can be explained by the fact that the mixtures become compacted and shear forces due to the friction at the sidewalls are developed for the whole system during the container move upward. On the contrary, during the downward motion of the container, the system is more expanded. This leads to the particles close to the sidewalls encountering stronger shear forces than those at the center of the container. Based on the combination of such the two phases, the particles move downward into the bottom of the container along the sidewalls in a thin-steam layer and the particles migrate upward at the center of the container [21, 22]. This is what we observed in the case of BNE, i.e. the larger particles shift to the top of the container by the convection rolls. In general, the convection consists of two symmetrical rolls, which can be separated into two patterns. The first pattern is normal convection, where the particles rise at the center of the container and move downward into the bottom of the container [23]. The second pattern is reverse convection, where the particles move downward at the center of the container and rise along the sidewalls [24].

Influences of particle shape and dimensionless acceleration on segregation phenomena are then systematically analyzed through the coefficient of segregation and the segregation pattern.

# A. Segregation Coefficient (H)

The video during experiments was exported as a sequence of photos. An open-source software named "*ImageJ*" [25] was employed to extract a position of the centroid for each particle. These centroids were then calculated a vertical height of each particle. The average vertical height of particle for each species j, which can be characterized by

$$Z_j = \frac{l}{n_p} \sum_{i}^{n_p} Z^i \tag{2}$$

where  $n_p$  is a total number of particles and  $Z^i$  is a vertical position of the centroid of each particle *i*. A segregation coefficient (*H*) was applied to determine the degree of segregation of binary mixtures in a stationary state [26], [27], which can be expressed by

$$H = 2\left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)$$
(3)

In this study, we defined  $Z_1$  for the particle having the higher angularity and  $Z_2$  for the particle having the lower angularity. Note that the factor 2 is a multiplication factor arising from the equal solid fraction of each species, which limits the segregation coefficient to -1 < H < 1. In this manner, the mixtures is in a mixing state when H = 0 and the mixtures is in a completely segregated state when H = 1 or H = -1. At the initial configuration, we prepared the granular mixtures in a random state, thus leading to the segregation coefficient approaches to zero.

## B. Segregation Pattern

Indeed, the segregation coefficient can quantify only the degree of segregation but cannot characterize a segregation pattern of the mixtures. In this study, *Three Thirds Segregation Indices Set* (TTSIS), proposed by Coletto *et. al* [28] in 2016, was thus employed with some modifications of the conditions for a classification of the segregation pattern. Table II presents the classification criteria of the segregation pattern. This classification criteria are based on three indicators: bottom indicator  $(P_B)$ , middle indicator  $(P_M)$ , and top indicator  $(P_T)$ . These indicators are defined by the ratio of a number of particles of interested species  $\alpha$  in a considered part  $(n_\alpha)$ to the total number of particles of interested species in the samples  $(n_p)$ , which can be expressed by

$$P_a = \frac{n_a}{n_p} \tag{4}$$

It must be noted that the granular mixtures enclosed by a square area was split into 3 considered parts as a function of the container height measured from the bottom. In addition, the particle having the higher angularity was only considered for the classification of the segregation pattern.

## IV. EXPERIMENTAL RESULTS

In this section, the effects of the particle shape and the dimensionless acceleration on segregation phenomena are analyzed by the segregation coefficient and the segregation pattern.

#### A. Segregation Coefficient (H)

Fig. 3 shows the values of the segregation coefficient for each tested granular sample as a function of the dimensionless acceleration. In order to eliminate the effect of particle rearrangement at the initial configuration, the segregation coefficient in the analysis was determined by a subtraction between the value of the segregation coefficient at the final configuration and that value at the initial configuration. It was found that the values of the segregation coefficient are close to zero for all cases. In addition, it was observed that the samples started to reorganize when  $\Gamma > 2$ . On the contrary, when  $\Gamma$  $\leq 2$ , the particles inside the mixtures moved in the manner of a cluster. In other words, the vertical vibration cannot destroy a packing of the granular samples. Fig. 4 shows the segregation coefficient as a function of time for  $\Gamma = 5$ . It is clearly seen that the values of the segregation coefficient are quite uniform and close to zero for all cases, except for CT, HT, and ST cases. This can be explained by an occurrence of convection rolls in those cases. In order to better understanding, an example of the particle displacement tracking of two particles is presented in Fig. 5. The normal convection rolls were clearly observed, thus leading to a small fluctuation of the segregation coefficient for CT, HT, and ST cases (see in Fig. 4). It can be said that the convection mechanism occurred in this study does not induce the segregation phenomena. In conclusion, it can be said that the segregation does not occur due to the effects of the particle shape and the level of the vibration.



## B. Segregation Pattern

To ensure that the segregation does not occur in the present study, the segregation pattern is employed to characterize the distribution of a cluster of the interested species inside the granular mixtures. Table III presents the segregation pattern at the final configuration as a function of the dimensionless acceleration. The analysis was performed only for the particle species having the higher angularity. The results show that only three different segregation patterns were found: Pure uniform (PU), Uniform (U), and Transition (TS). These three segregation patterns have a similar feature, i.e. numbers of the centroid of the particle in each considered part are nearly the same. Note that the transition pattern indicates an attempt to a change of the distribution into other segregation patterns. The segregation pattern observed here was mostly uniform for all cases (see in Figs. 6-a to 6-d), except for some cases that the transition pattern was found (see in Figs. 6-e and 6-f). Finally, these results obtained in this study enable us to conclude that no segregation phenomena occurred in the binary granular mixtures due to the effects of the particle shape and the level of the vibration.



Figure 4. The segregation coefficient (*H*) as a function of time (*min*) for  $\Gamma$ =5



Figure 5. Example of the particle displacement tracking normalized by the container width for TS case at  $\Gamma$ =5

	Γ = 1	$\Gamma = 2$	$\Gamma = 3$	$\Gamma = 4$	$\Gamma = 5$
СН	U	U	U	PU	U
CS	U	U	U	U	U
СТ	U	U	TS	U	U
HS	U	U	U	U	U
HT	U	U	U	U	TS
ST	U	U	U	U	TS

TABLE III SEGREGATION PATTERN AS A FUNCTION OF DIMENSIONLESS ACCELERATION ( $\Gamma$ ) at the Final Configuration. Note that the Pattern with Grey Background is Uniform Pattern.

any dimensionless accelerations ( $\Gamma$ ). Furthermore, convection rolls were observed when  $\Gamma$  is greater than 2, thus leading to the beginning of particles reorganization. However, such the convection rolls do not induce an occurrence of segregation in the mixtures. It was also observed that the distribution of a collection of the particles having the higher angularity inside the mixtures was only "Uniform" and "Transition" segregation patterns. In this manner, these results clearly indicate that the particle shape and the dimensionless acceleration do not induce an occurrence of the segregation phenomena



Figure 6. Final configuration of each tested binary granular mixtures: a) CH, b) CS, c) CT, d) HS, e) HT, and f) ST for  $\Gamma = 5$ 

## V. CONCLUSIONS

Segregation is one of unwanted phenomena found in many industrial processes relevant to handling and transportation of granular materials. This is a reason why segregation phenomena have been investigated by many researchers since long time ago. In this study, the effect of particle shape on segregation phenomena in 2D binary granular mixtures under various levels of a vertical vibration was experimentally investigated. Four different cross-sectional shapes of particle, i.e. circle, triangle, square, and hexagon, were employed to prepare the granular samples by mixing between two particle shapes. This leads to six different tested cases in this study. Each sample composed of 1000 particles (500 particles for each species) was randomly deposited in a rectangular container. After vibrations applied, the segregation was systematically analyzed through the coefficient of segregation and the segregation pattern. The results showed that the values of the segregation coefficient (H)were uniform and close to zero for all tested cases under

in 2D binary granular mixtures subjected to vertical vibrations. It is interesting to also note that convection rolls were found in the case of the mixtures containing triangular particles.

More generally, this study opens prospects for the effect of particle shape on segregation phenomena in granular media, which is a completely unexplored topic. The effects of particle with much more complex shape and friction would be interested to study further.

#### CONFLICT OF INTEREST

The authors declare no conflict of interest.

## AUTHOR CONTRIBUTIONS

T. Binaree established the concept of this research, performed the experiments, analyzed the data, and prepared the manuscript; I. Preechawuttipong and P. Jongchansitto provided some advices and suggestions for the experiments as well as contributed to the manuscript

revision; All authors have approved the final version of [18] the manuscript

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