Solution of the Problem about Speeds and Special Positions of Spherical Parallel Manipulator

Tung Vo Dinh
Institute of Engineering, Ho Chi Minh City University of Technology (HUTECH), Ho Chi Minh City, Vietnam
Email: vd.tung@hutech.edu.vn

Sergey Kheylo
Department of Theoretical Mechanics, The Kosygin State University of Russia
Email: sheilo@yandex.ru

Abstract— **In recent years the number of studies of spherical parallel mechanisms has grown. Manipulators of a parallel structure are increasingly used in various fields of technology, since there is a need for mechanisms with increased performance in terms of load capacity and accuracy. These devices are used as propulsion, measuring, technological and testing systems. Spherical manipulators are used in devices for orienting antennas, telescopes, and in test benches.**

The article considers the study of the kinematics problem for the spherical mechanism of parallel structure with three degrees of freedom, with three kinematic chains. A solution to the problem of speeds is presented and special positions of the mechanism are found by screw calculus. Examples of calculating the direct and inverse velocity problems are given.

Index Terms—**parallel structure mechanism, speed problem, special positions, crew calculus**

I. Introduction

The theory of screws and screw calculus apparatus, is used to calculate manipulators of a parallel structure. algorithms for analyzing mechanisms, but also to obtain qualitative characteristics associated with special positions, accuracy, and pressure angles. Since the screw approach operates with geometric images of a higher order than ordinary vectors, in some cases this makes it possible to generalize and obtain a result without resorting to complex calculations. The ancestor of the theory of screws is R. Ball [1]. The first theory of screws in the theory of mechanisms, was applied by F.M. Dimentberg [2, 3]. The relevance of applying the theory of screws and screw calculus is constantly growing, since the schemes of mechanical systems of robots are becoming more complicated. In particular, manipulators that perform spherical movements can be built on the basis of different design solutions. However, they all have one property - they can be represented by schemes in which the axes of the kinematic pairs intersect at one point. This corresponds to a closed three-membered group of screws [4].

The class of spherical manipulators is wide. It is advisable to extend the method of screw calculus for the entire class of spherical mechanisms.

It should be noted that the method of differentiation of the equations of constraints made it possible to solve the problem of the velocities of a spherical manipulator [5, 6]. However, this calculation causes difficulties in determining its special positions. As a result of this, a need arose for the development of a screw approach, which is still not sufficiently used in relation to manipulators of this type. In particular, the applicability of this approach to the analysis of kinematics and special positions has not been sufficiently studied. The development of this method will allow more efficiently solving problems associated with the functionality of devices of this type. This work is intended to help solve this problem. Here, using the theory of screw calculus, the structure of a spherical mechanism with three degrees of freedom is investigated, algorithms for solving problems of speeds and special positions are given. In the future, the presented results will allow us to solve the problems of dynamics and control mechanisms of this class.

II. Formulation of Problem

Spherical manipulators of a parallel structure are designed for orienting movements of the working body [7-9]. For example, fig.1a shows a spherical manipulator with three degrees of freedom [7], consisting of three kinematic chains with intersecting axes of pairs, the output link is a platform rotating around three axes. Another spatial spherical manipulator [8] (Fig. 1b) with three degrees of freedom consists of a base, an output link, three kinematic chains with coincident axes of drive and non-drive pairs of different kinematic chains, which simplifies the solution of position problems, but complicates the design. Schemes of spherical mechanisms can be applied to manipulators of a more complex design, for example, with six degrees of freedom (Fig. 1c) [10]. The analysis of the rotational movements of the links of such manipulators is reduced to spherical mechanisms [11–13].

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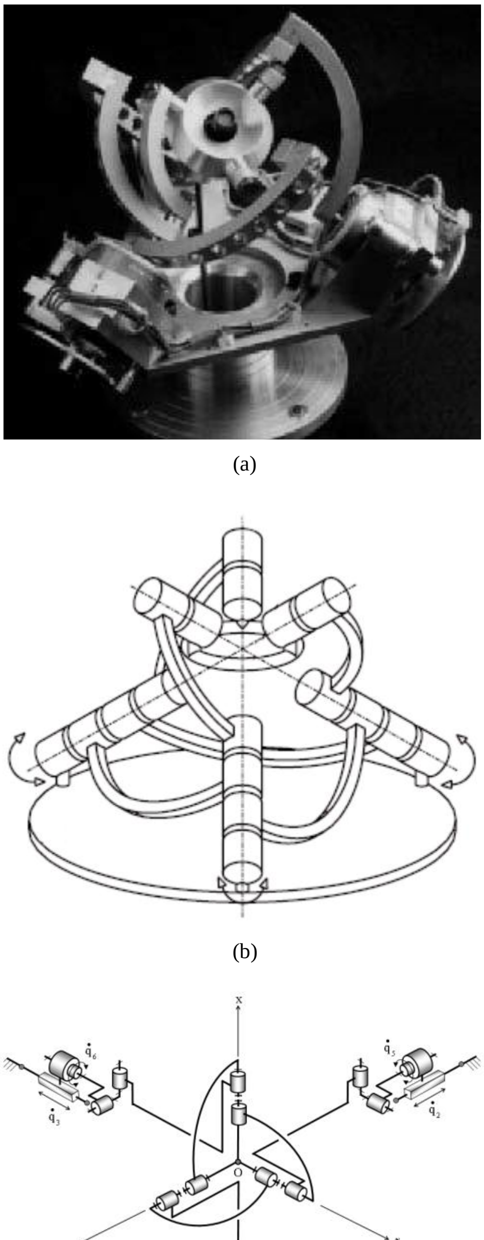

Figure 1. Spherical manipulator with three degrees of freedom

In this paper, we consider a spherical manipulator of a particular form (Fig. 2), in which the angles between the axes of adjacent kinematic pairs are 90°. This arrangement of kinematic pairs has great practical meaning, since for such a mechanism the solution of the problem of speeds and positions is simplified, and thus the structure can be optimized. In addition, a whole series of manipulators [9], which have a different design scheme, also belong to a similar design scheme.

Each input link of the circuit is connected to a motor. The output link is a platform that rotates around three axes intersecting at point O. The output coordinates are the rotation angles of the platform α, β, γ around the x, y, z axes, respectively. The generalized coordinates are the angles φ_{11}, φ_{21}, φ_{31} – respectively, the rotation angles of the input links of the first, second and third kinematic chains.

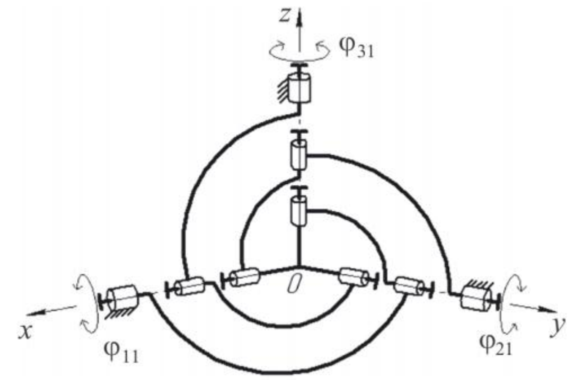

Figure 2. Kinematic diagram of spherical manipulator

In this paper, we solve the problem of determining the velocities of the input and output links of the manipulator and its special positions.

The initial data includes the positions of the input or output links of the manipulator. To solve the problem of speeds using screw methods, it is necessary to find power and kinematic screws for each kinematic chain and determine their relative moment. The system of equations composed of relative moments allows us to solve the direct velocity problem, i.e. determine the speed of the output links at known speeds of the input links, and the inverse problem, as well as identify the conditions for the emergence of special provisions of the manipulator.

III. Solving Inverse Speed Problem Using Screw Calculus

The inverse problem means the determination of the speeds of the links of the input link at known speeds of the output link. To solve the speed problem and determine the special positions of the manipulator, it is necessary to solve the position problem, that is, set the rotation angles of the output link and determine the positions of the input links, φ_{11}, φ_{12}, φ_{21}, φ_{22}, φ_{31}, φ_{32} from them. The solution to this problem was considered in [12].

In particular, the position of the output link α=1 rad, β=1 rad, γ=1 rad. correspond to the rotation angles of the input links:

φ_{11}=0.242 rad; φ_{12}=1.265 rad; φ_{21}=1.237 rad; φ_{22}=0.472 rad; φ_{31}=0.081 rad; φ_{32}=0.472 rad. The position of the output link is given the same as in solving the problem of determining the velocities by differentiating the constraint equations [12].

Using the screw calculus, it can be written that the angular velocity of the output link is equal to the sum of the angular velocities of the hinges of the links of one chain. For the first kinematic chain, we compose the equations:

$$\omega_x = \omega_{11} \cdot x_{11} + \omega_{12} \cdot x_{12} + \omega_{13} \cdot x_{13}$$
$$\omega_y = \omega_{11} \cdot y_{11} + \omega_{12} \cdot y_{12} + \omega_{13} \cdot y_{13} \quad (1)$$
$$\omega_z = \omega_{11} \cdot z_{11} + \omega_{12} \cdot z_{12} + \omega_{13} \cdot z_{13}$$

Where ω_x, ω_y, ω_z are the projections of the angular velocity of the output link on the *x, y, z* axis; ω_{11}, ω_{12}, ω_{13} – angular velocities of the first, second and third joints of the first chain, respectively; (x_{11}, y_{11}, z_{11}) – the Plücker coordinates of the unit vector e_{11} located along the axis of the first pair; (x_{12}, y_{12}, z_{12}) – the Plücker coordinates of the unit vector e_{12}, determined by the product of the rotation matrix around the ox axis by the coordinates of the vector located along the axis of the second pair in its initial position; (x_{13}, y_{13}, z_{13}) – the Plücker coordinates of the unit vector e_{13}, determined by the product of the rotation matrix of the output link by the coordinates of the vector located along the axis of the third pair in its initial position (the values of the Plücker coordinates are given in the Appendix).

We set the speeds of the output link, the same as in solving the problem of determining the velocities by differentiating the coupling equations [12]: ω_x=1 rad/s, ω_y=1 rad/s, ω_z=1 rad/s, and compare the results of calculating the speeds of the input links

We compose the velocity equation for the second kinematic chain

$$\begin{cases} \omega_x = \omega_{21} \cdot x_{21} + \omega_{22} \cdot x_{22} + \omega_{23} \cdot x_{23} \\ \omega_y = \omega_{21} \cdot y_{21} + \omega_{22} \cdot y_{22} + \omega_{23} \cdot y_{23} \\ \omega_z = \omega_{21} \cdot z_{21} + \omega_{22} \cdot z_{22} + \omega_{23} \cdot z_{23} \end{cases} \quad (2)$$

Where ω_{21}, ω_{22}, ω_{23} are the angular velocities of the first, second, and third joints of the second chain, respectively; (x_{21}, y_{21}, z_{21}) - the Plücker coordinates of the unit vector e_{21} located along the axis of the first pair; (x_{22}, y_{22}, z_{22}) - the Plücker coordinates of the unit vector e_{22}, determined by the product of the rotation matrix around the *oy* axis by the coordinates of the vector located along the axis of the second pair in its initial position; (x_{33}, y_{33}, z_{33}) - the Plücker coordinates of the unit vector e_{23}, determined by the product of the rotation matrix of the output link by the coordinates of the vector located along the axis of the third pair in its initial position (the values of the Plücker coordinates are given in the Appendix).

For the third kinematic chain, the velocity equations will be as follows:

$$\begin{cases} \omega_x = \omega_{31} \cdot x_{31} + \omega_{32} \cdot x_{32} + \omega_{33} \cdot x_{33} \\ \omega_y = \omega_{31} \cdot y_{31} + \omega_{32} \cdot y_{32} + \omega_{33} \cdot y_{33} \\ \omega_z = \omega_{31} \cdot z_{31} + \omega_{32} \cdot z_{32} + \omega_{33} \cdot z_{33} \end{cases} \quad (3)$$

Where ω_{31}, ω_{32}, ω_{33} are the angular velocities of the first, second, and third joints of the third chain, respectively; (x_{31}, y_{31}, z_{31}) are the Plücker coordinates of the unit vector e_{31} located along the axis of the first pair; (x_{32}, y_{32}, z_{32}) – the Plücker coordinates of the unit vector e_{32}, determined by the product of the rotation matrix around the oz axis by the coordinates of the vector located along the axis of the second pair in its initial position; (x_{33}, y_{33}, z_{33}) – the Plücker coordinates of the unit vector e_{33}, determined by the product of the rotation matrix of the output link by the coordinates of the vector located along the axis of the third pair in its initial position (the values of the Plücker coordinates are given in the Appendix).

Solving the system of equations, we obtain ω_{31}=0.532 rad/s; ω_{32}=1.078 rad/s; ω_{33}=1.029 rad/s, ω_{11}=1.316 rad/s, ω_{12}=1.21 rad/s, ω_{13}=2.43 rad/s, obtain ω_{21}=1.312 rad/s; ω_{22}=1.274 rad/s; ω_{23}= –0.785 rad/s.

Thus, the speeds of the input links of the spherical manipulator are determined.

IV. Solving Direct Problem Of Speeds By Using Screw Calculus

Solving the direct problem of speeds by the method of screw calculus

By solving a direct problem is meant determining the speed of the output link at known speeds of the input link. When considering the direct speed problem, it is necessary to determine the power and kinematic screws [4]. $\mathbf{R}_i$ power screw with coordinates $\left(r_{ix}, r_{iy}, r_{iz}, r_{ix}^0, r_{iy}^0, r_{iz}^0\right)$ is reciprocal to two unit vectors of axes e_{i2}, e_{i3} of non-drive pairs. This screw $\mathbf{R}_i$ is balanced by a set of screws - reactions in pairs corresponding to the vectors e_{i2}, e_{i3}

The relative moment mom ($\mathbf{R}_i$, $\mathbf{\Omega}_i$) is the sum of the scalar products of the vector of the first screw at the time of the second relative to some point and the vector of the second screw at the time of the first relative to the same point, where $\mathbf{\Omega_i}$ is the kinematic screw of the output link with coordinates (V_x, V_y, V_z, ω_x, ω_y, ω_z), V_x, V_y, V_z are the linear velocities of the output link, m /s.

The kinematic screw of the output link is equal to the sum of the kinematic screws of the chain links $\mathbf{\Omega}_i=\mathbf{\Omega}_{i1}+\mathbf{\Omega}_{i2}+\mathbf{\Omega}_{i3}$, where $\mathbf{\Omega}_{i1}$, $\mathbf{\Omega}_{i2}$, $\mathbf{\Omega}_{i3}$ are the kinematic screws of the first, second, third links with coordinates respectively.

$$\left(x_{i1}, y_{i1}, z_{i1}, x_{i1}^0, y_{i1}^0, z_{i1}^0\right) \cdot \omega_{i1}$$
$$\left(x_{i2}, y_{i2}, z_{i2}, x_{i2}^0, y_{i2}^0, z_{i2}^0\right) \cdot \omega_{i2}$$
$$\left(x_{i3}, y_{i3}, z_{i3}, x_{i3}^0, y_{i3}^0, z_{i3}^0\right) \cdot \omega_{i3}$$

Then mom($\mathbf{R}_i$, $\mathbf{\Omega}_i$)=mom($\mathbf{R}_i$, $\mathbf{\Omega}_{i1}$ + $\mathbf{R}_i$, $\mathbf{\Omega}_{i2}$ + $\mathbf{R}_i$, $\mathbf{\Omega}_{i3}$).

Since the power screw is reciprocal to the unit vectors of non-drive pairs, the relative moments mom($\mathbf{R_i}$, $\mathbf{\Omega}_{i2}$)=0, mom($\mathbf{R_i}$, $\mathbf{\Omega}_{i3}$)=0. Therefore, we can write that mom ($\mathbf{R_i}$, $\mathbf{\Omega}_i$)=mom($\mathbf{R_i}$, $\mathbf{\Omega}_{i1}$). Substituting the coordinate values of the power and kinematic screws, we obtain the equations of relative moments:

$$\mathrm{mom}\left(\mathbf{R}_i, \mathbf{\Omega}_i\right) = \omega_x r_{ix}^0 + \omega_y r_{iy}^0 + \omega_z r_{iz}^0;$$
$$\mathrm{mom}\left(\mathbf{R}, \mathbf{\Omega}_{i1}\right) = \omega_{i1}(x_{i1} r_{ix}^0 + y_{i1} r_{iy}^0 + z_{i1} r_{i1z}^0)$$

Where (x_{i1}, y_{i1}, z_{i1}) are the Plücker coordinates of the unit vectors ei1 located along the axes of the first pairs, $\mathbf{r}_i^0$ - moment part of the power screw with coordinates $r_{1x}^0, r_{1y}^0, r_{1z}^0$

We compose a system of equations for three kinematic chains:

$$\begin{aligned}
\omega_x r_{1x}^0 + \omega_y r_{1y}^0 + \omega_z r_{1z}^0 &= \omega_{11}(x_{11} r_{1x}^0 + y_{11} r_{1y}^0 + z_{11} r_{1z}^0) \\
\omega_x r_{2x}^0 + \omega_y r_{2y}^0 + \omega_z r_{2z}^0 &= \omega_{21}(x_{21} r_{2x}^0 + y_{21} r_{2y}^0 + z_{21} r_{2z}^0) \\
\omega_x r_{3x}^0 + \omega_y r_{3y}^0 + \omega_z r_{3z}^0 &= \omega_{31}(x_{31} r_{3x}^0 + y_{31} r_{3y}^0 + z_{31} r_{3z}^0)
\end{aligned} \tag{4}$$

For the first kinematic chain, the coordinates of the moment part of the power screw will be calculated as follows:

$$\mathbf{r_1^0} = \mathbf{e_{12}} \times \mathbf{e_{13}} \tag{5}$$

Substituting the coordinate values of unit vectors, we obtain the Plücker coordinates of the power screw: $r_{1x}^0 = 0{,}301$; $r_{1y}^0 = 0{,}229$; $r_{1z}^0 = -0{,}926$.

For the second and third kinematic chains, the coordinates of the moment part of the power screws and are determined, respectively:

$$\mathbf{r_2^0} = \mathbf{e_{22}} \times \mathbf{e_{23}} \tag{6}$$

$$\mathbf{r_3^0} = \mathbf{e_{32}} \times \mathbf{e_{33}} \tag{7}$$

The values of the plucker coordinates will be equal to: $r_{2x}^0 = -0{,}149$, $r_{2y}^0 = 0{,}889$, $r_{2z}^0 = 0{,}430$, $r_{3x}^0 = 0{,}037$, $r_{3y}^0 = -0{,}454$, $r_{3z}^0 = 0{,}891$.

We set the values of the input link velocities obtained previously ω_{11}=1.316 rad/s, ω_{21}=1.312 rad/s, ω_{31}=0.532 rad/s. Substituting the found values of the coordinates of the moment part of the power screws into equation (4), we obtain the values of the speeds of the output links: ω_x=1 rad/s, ω_y=1 rad/s, ω_z=1 rad/s.

The obtained values of velocities by a screw calculus coincide with the results obtained by differentiating the constraint equations [12], which indicates the reliability of the calculations.

V. Solving Problem of Special Positions by Screw Calculus for Each Kinematic Chain

To determine the special position in the first kinematic chain, we substitute the Plücker coordinates $(x_{13}, y_{13}, z_{13})'$ in the velocity equations (1) unit vector e_{13}, determined by the product of the rotation matrix of the output link around the first axis — the *ox* axis, then the second axis — the *oy* axis and the coordinates of the vector located along the axis of the third pair in its initial position (the values of the Plucker coordinates are given in the Appendix).

We transform the Plücker coordinates in equations (1) to the matrix form:

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\sin\phi_{12} \\ 0 & \cos\phi_{11} & \cos\phi_{12}\cdot\sin\phi_{11} \\ 0 & -\sin\phi_{11} & \cos\phi_{11}\cdot\cos\phi_{12} \end{pmatrix} \tag{8}$$

Find the angle φ_{12}, in which the determinant of the matrix becomes equal to zero. The determinant of the matrix does not depend on the angle φ_{11}, and is determined by the angle φ_{12}. The determinant is zero for φ_{12}=90° and φ_{12}=180°, in this case, the location planes of the first and second chains coincide (Fig. 3).

The special position of the mechanism is determined by the loss of the degree of freedom, since three kinematic pairs lie in one plane - all rotations can occur around an axis lying in one plane, and all rotations around an axis perpendicular to this plane are impossible. Special positions for the second and third kinematic chains are defined similarly.

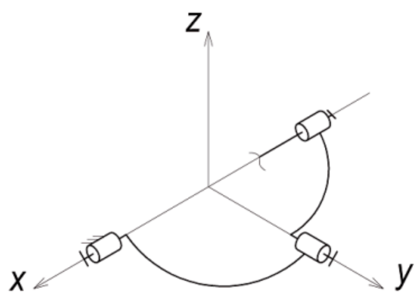

Figure 3. Special position in the first kinematic chain

For the second kinematic chain, we substitute the Plücker coordinates of the unit vector e_{23} in the velocity equations (2), denoting them $(x_{23}, y_{23}, z_{23})'$. They are determined by the product of the rotation matrix of the output link around the first axis - the *oy* axis, then the second axis - the *oz* axis and the coordinates of the vector located along the axis of the third pair in its initial position (the values of the Plücker coordinates are given in the Appendix).

We transform the coordinates in equations (1) to the matrix form:

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0 & -\sin\phi_{21} & \cos\phi_{22}\cdot\cos\phi_{21} \\ 1 & 0 & \sin\phi_{22} \\ 0 & \cos\phi_{21} & \cos\phi_{22}\sin\phi_{21} \end{pmatrix} \tag{9}$$

The determinant of the matrix does not depend on the angle φ_{21}, and is determined by the angle φ_{22}. The determinant is zero for φ_{22}=90° and φ_{22}=180°, in this case, the planes of the first and second chains coincide.

For the third kinematic chain, substitute the Plücker coordinates in equation (3) $(x_{33}, y_{33}, z_{33})'$ unit vector e_{33}. They are determined by the product of the rotation matrix of the output link around the first axis — the oz axis, then the second axis — the *ox* axis and the coordinates of the vector located along the axis of the third pair in its initial position (the values of the Plucker coordinates are given in the Appendix).

The coordinates in the velocity equations are written in matrix form:

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0 & \cos\phi_{31} & -\cos\phi_{32}\cdot\sin\phi_{31} \\ 0 & \sin\phi_{31} & \cos\phi_{31}\cdot\cos\phi_{32} \\ 1 & 0 & \sin\phi_{32} \end{pmatrix} \tag{10}$$

The determinant of the matrix does not depend on the angle φ_{31}, and is determined by the angle φ_{32}. The determinant is zero for φ_{32}=90° and φ_{32}=180°, in this case, the planes of the first and second chains coincide

VI. Solution of Problem of Special Positions by Screw Calculus for Mechanism

To determine the special positions of the whole mechanism, it is necessary to study the system of equations (4) for three kinematic chains.

For the first kinematic chain, the coordinates of the moment part of the power screw have the following meanings: $r_{1x}^0 = \cos\varphi_{12}$; $r_{1y}^0 = \sin\varphi_{11} \cdot \sin\varphi_{12}$; $r_{1z}^0 = \cos\varphi_{11} \cdot \sin\varphi_{12}$.

For the second and third kinematic chains, the moment parts of the coordinates of the power screws are determined:

$r_{2x}^0 = -\cos\varphi_{21} \cdot \sin\varphi_{22}$; $r_{2y}^0 = -\cos\varphi_{22}$; $r_{2z}^0 = -\sin\varphi_{21} \cdot \sin\varphi_{22}$;

$r_{3x}^0 = -\sin\varphi_{31} \cdot \sin\varphi_{32}$; $r_{3y}^0 = \cos\varphi_{31} \cdot \sin\varphi_{32}$; $r_{3z}^0 = \cos\varphi_{32}$.

We compose the matrix **R** from the moment part of the Plücker coordinates of the power screws (given in the Appendix).

The matrix composed of Plücker coordinates degenerates, i.e. the determinant of the matrix is zero for the following combinations of angle values:

1) φ_{12}=0°, φ_{22}=90°, φ_{32}=90°;
2) φ_{12}=0°, φ_{22}=90°, φ_{32}=0°;
3) φ_{12}=0°, φ_{22}=0°, φ_{32}=90°;
4) φ_{12}=90°, φ_{22}=0°, φ_{32}=90°;
5) φ_{12}=90°, φ_{22}=90°, φ_{32}=0°.

This corresponds to such positions in which the planes have at least two parallel norms (Fig. 4).

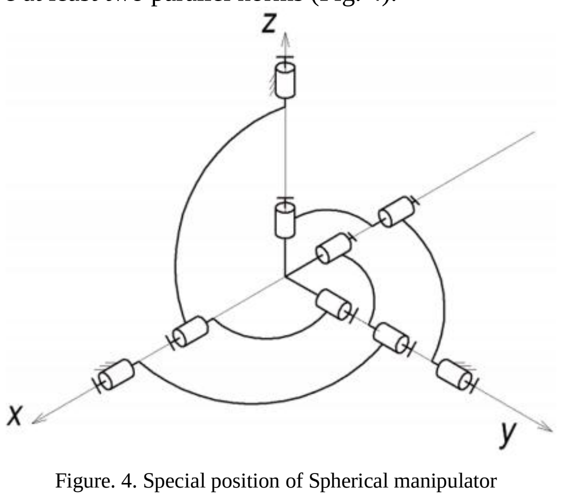

Figure. 4. Special position of Spherical manipulator

The loss of controllability of the manipulator is determined by the fact that the three power screws have become coplanar, i.e. are parallel to one plane, and rotation around a vector perpendicular to the power screws becomes uncontrollable.

VII. Conclusion

This paper presents the development of the theory of spherical mechanisms of parallel structure based on the application of screw calculus. This applies to solving direct and inverse speed problems and determining the special positions of the manipulator. In this case, power screws are determined that are reciprocal to the unit vectors of the axes of the non-drive pairs, of each kinematic chain.

Based on the found power screws, equations are compiled expressing the speeds of the spherical mechanism of a parallel structure. These equations are solved as applied to the direct and inverse velocity problems.

It was shown that the loss of controllability is due to the linear dependence of the power screw system, and the loss of one or more degrees of mobility is associated with the degeneration of the kinematic screw systems of the connecting chains.

The proposed algorithms based on the apparatus of screw calculus can be used to solve problems of optimizing the parameters of manipulators. These results can also be used to analyze the functionality of manipulators.

Appendix

A is a matrix describing the transition of the output link from the moving coordinate system to the fixed.

$$A = \begin{pmatrix} \cos\gamma \cdot \cos\beta & \cos\gamma \cdot \sin\beta \cdot \sin\alpha - \cos\alpha \cdot \sin\gamma & \sin\alpha \cdot \sin\gamma + \cos\alpha \cdot \cos\gamma \cdot \sin\beta \\ \cos\beta \cdot \sin\gamma & \cos\alpha \cdot \cos\gamma + \sin\alpha \cdot \sin\beta \cdot \sin\gamma & \cos\alpha \cdot \sin\gamma \cdot \sin\beta - \cos\gamma \cdot \sin\alpha \\ -\sin\beta & \cos\beta \cdot \sin\alpha & \cos\beta \cdot \cos\alpha \end{pmatrix}$$

Plucker power screw matrix:

$$\mathbf{R} = \begin{pmatrix} r_{1x}^0 & r_{1y}^0 & r_{1z}^0 \\ r_{2x}^0 & r_{2y}^0 & r_{2y}^0 \\ r_{3x}^0 & r_{3y}^0 & r_{3z}^0 \end{pmatrix}.$$

Conflict of Interest

The authors declare no conflict of interest

Author Contributions

Tung Vo Dinh counducted the research and wrote the paper.

Sergey Kheylo analyzed data and checked paper.

All authors had approved the final version.

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