Significance Analysis of Connections in a Multivariable Mathematical Model of an Unmanned Underwater Vehicle for Common Motion Trajectories

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***Abstract*—Unmanned underwater vehicles often are to move effectively in all six degrees of freedom during their missions. To control such motion effectively it is necessary to synthesize an automatic system of motion control which would manipulate thrusters of underwater vehicle considering interactions between all degrees of freedom. Synthesizing such control system is a complex problem requiring sophisticated methods of synthesis, which becomes more complex if it is necessary to consider uncertainty of systems parameters and its non-linearity of its elements during the synthesis procedure. This makes a problem of reducing a number of interconnections between manipulated parameters of underwater vehicle motion highly relevant. Authors propose a method of assessing a significance of each interconnection on a base of simulation modeling of underwater vehicle motion along most common trajectories used during mission planning. This will allow to replace a full-dimensional control system with a multimodal control system consisting of a set of low-dimensional control systems controlling the motion along of each common trajectory. This will lead to significant simplification of synthesizing controllers. The method is based on original models of elements of a motion control system. Further research will be dedicated to software implementation of the assessment method and testing it on an unmanned underwater vehicle prototype.**

***Index Terms*—mathematical modeling, underwater vehicle, motion control system, MIMO-system, simulation modeling**

I. Introduction

Unmanned underwater vehicles (UUV) are widely used to perform various missions of defense, industrial, ecological, rescue matters [1-5]. In order to operate effectively, they are required to move automatically along complex trajectories in variable rigid conditions and in all six degrees of freedom. In the simplest case, when motion in each degree of freedom is controlled by separate thruster, the motion control system (MCS) of full dimensionality includes six control channels and each of them is connected with others; so, it consists of thirty six interconnections. Structure of such control system is shown in the Fig. 1.

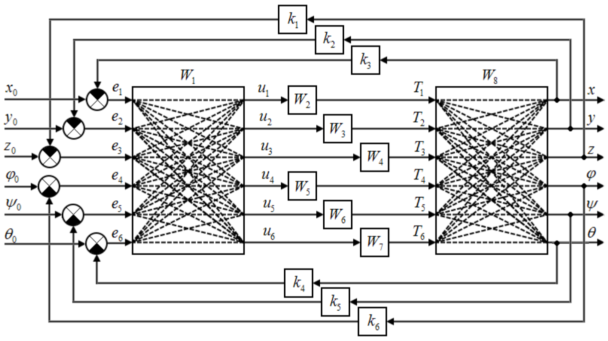

Figure 1. Structure of the full-dimensional UUV MCS.

Following designations are used in the figure 1: x_0, x – are setpoint and actual value of UUV coordinate along x axis; y_0, y – are setpoint and actual value of UUV coordinate along y axis; z_0, z – are setpoint and actual value of UUV coordinate along z axis; φ_0, φ – are setpoint and actual value of a heading angle; ψ_0, ψ – are setpoint and actual value of a trim angle; θ_0, θ – are setpoint and actual value of a roll angle; $e_1 - e_6$ – are control errors of manipulated variables; W_1 – is a transfer matrix of a multivariable controller; $u_1 - u_6$ – are control signals; $W_2 - W_7$ – are transfer functions of thrusters; $T_1 - T_6$ – are output signals of thrusters; W_8 – is a transfer matrix of a process of a UUV motion process; $k_1 - k_6$ – are transfer coefficients of measuring devices. From the Fig. 1 it is clear, that synthesizing such controller is a complex problem especially in conditions of parametric uncertainty and non-linearity of some of UUV MCS elements. Excluding some interconnections from this system would significantly simplify procedure of controller synthesis without losing any control quality as if every degree of freedom was controlled by a

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separate channel [6-8]. Let us now formulate the problem of the research.

II. Formulating a Problem

Significance of interactions between different degrees of freedom depends on hydrodynamic and hydrostatic features of UUV, which are determined by its type and construction of propulsion and steering system (PSS). There are two basic types of UUVs: autonomous underwater vehicles (AUV) and remotely operated vehicles (ROV). ROV usually have low hydrodynamic characteristics due to the shape of its hull which is often irregular and depends on payload of ROV. ROV are usually equipped by several thrusters, which provide from one to three degrees of freedom simultaneously, and move along simple planar trajectories like zigzag, meander or cycloid [5,6]. In the research let us consider AUV. Their hydrodynamic cigar-shaped hull is usually well streamlined; they are usually equipped with one or two main thrusters and several steering blades allowing to vary direction of a thrust vector [9,10]. AUV usually move along smooth complex tridimensional trajectories, so interaction between control channels in their MCS is more important to consider, than in ROV MCS. Typical AUV motion is immersing or rising along spiral trajectory.

Considering all aforementioned information, let us now formulate the main aim of the research as follows: to develop a method of assessing significance of interconnections in a mathematical model of UUV MCS in order to reduce its dimensionality and simplify synthesis procedure. Following objectives must be accomplished in order to reach the aim of the research:

A mathematical model of a process of AUV motion in a viscous fluid must be derived;

A mathematical model of a PSS must be derived;

A mathematical model of a whole AUV MCS must be composed;

A criterion of assessing a significance of each interconnection in a model must be developed and the assessment algorithm must be formulated.

Let us now consider accomplishing these objectives consequently.

III. Deriving Mathematical Model of Elements of a Motion Control System for an Autonomous Underwater Vehicle

General structure of UUV MCS includes controller, actuators – main thrusters and steering blades, control object – model of an AUV motion process and measuring devices – gyroscopes and accelerometers. Let us consider inertness of measuring devices insignificant and describe them with their transfer coefficients. Also, as far as the synthesis of a controller is not the problem of the research considered, we will use a nominal controller with manually adjusted parameters. Derivation of models of other elements will be considered further.

A. *Deriving a Mathematical Model of a Propulsion and Steering System*

Propulsion and steering system consists of six elements: two main thrusters and four steering blades. Two vertical steering blades allow varying a heading angle of an AUV; two horizontal steering blades allow varying trim of an AUV.

While modeling a thruster with propeller it is necessary to consider, that the thrust and moment of propellers resistance to rotation depends on rotation frequency according to following expressions [11,12]:

$$T = K_1 \cdot \rho \cdot \nu^2 \cdot D^4; M = K_2 \cdot \rho \cdot \nu^2 \cdot D^5,$$

where T – is a thrust; M – is a propellers moment of resistance to rotation; ρ – is a water density; ν – is a rotation frequency; D – is a diameter of a propeller; K_1, K_2 – are transfer coefficients of a propeller by thrust and rotation resistance moment. Also, we know, that a differential equation of an electric motor can be written as follows:

$$T_e \cdot T_m \cdot \frac{d^2}{dt^2} \cdot \omega + T_m \cdot \frac{d}{dt} \cdot \omega + \omega =$$
$$= K_{e1} \cdot u + K_{e2} \cdot (T_e \cdot \frac{d}{dt} \cdot M + M),$$

where T_e, T_m – are electric and mechanical time constants of the motor; ω – is motor rotation speed; K_{e1} – is transfer coefficient between control signal and rotation speed; u – is control signal; K_{e2} – is transfer coefficient between the moment of rotation resistance and rotation speed; M – is moment of rotation resistance.

On a base of these three expressions, structure of a model of a thruster can be described as shown in the Fig. 2.

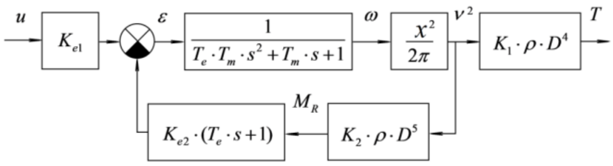

Figure 2. Structure of the thruster model.

The model of the thruster shown in the Fig. 2 includes a non-linear element and must be linearized in order to be applicable for synthesizing a controller [13-15]. It should be noticed, that for the research considered it is enough to make the model applicable for simulation modeling, so it will not be linearized.

The model shown in the Fig. 2 allows calculating thrust; now let us derive expressions, allowing to calculate its projections provided by steering blades. As far as steering blades allocation is symmetrical in relation to vertical-longitudinal plane of the AUV, let us consider their influence on an AUV roll insignificant. In this case, forces and moments provided by steering blades can be calculated as follows:

$$T_x = T_0 \cdot \cos(\delta) \cdot \cos(\gamma);$$
$$T_y = T_0 \cdot \cos(\delta) \cdot \sin(\gamma);$$
$$T_z = T_0 \cdot \sin(\delta) \cdot \cos(\gamma);$$
$$M_y = T_z \cdot a;$$
$$M_z = T_y \cdot b;$$

where T_x, T_y, T_z – are projections of a thrust on x, y, z axes provided by steering blades; T_0 – is an initial value of thrust directed along the longitudinal axis of a thruster; δ – is an angle of vertical steering blade rotation; γ – is an angle of horizontal steering blade rotation; M_y, M_z – are moments of thrust projections in relation to y, z axes; a – is a distance between axis of a vertical steering blade rotation and center of masses of an AUV; b – is a distance between axis of a horizontal steering blade rotation and center of masses of an AUV.

B. Deriving a Mathematical Model of an Autonomous Underwater Vehicle Motion Process

Let us now consider deriving a mathematical model of process of AUV motion in a viscous fluid. In order to simplify this problem, let us assume that the sums of hydrostatic forces and moments are equal to zero, so an AUV is affected only by governing forces and moments provided by PSS and hydrodynamic forces and moments.

Before developing a model, let us define input and output data for it. To make it compatible with the model of PSS, we will use forces and moments, performed by PSS as an input signal. In order to be able to assess significance of interconnections in the model, manipulated parameters of the system considered will be defined as output parameters. In this case, the structure of a model of AUV motion in a viscous fluid can be composed as shown in the Figure 3.

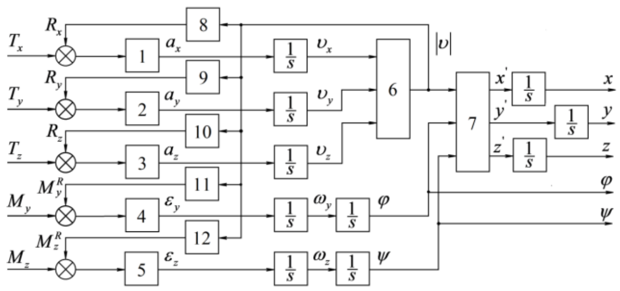

Figure 3. Structure of the AUV motion process model.

Following designations were introduced in the Fig.3: R_x, R_y, R_z – are forces of water resistance to AUV motion along x, y, z axes; M_y^R, M_z^R – are moments of water resistance to AUV motion around y, z axes; a_x, a_y, a_z – are linear accelerations of AUV along x, y, z axes; $\varepsilon_y, \varepsilon_z$ – are angular accelerations of AUV around y, z axes; $\upsilon_x, \upsilon_y, \upsilon_z$ – are linear velocities of AUV along x, y, z axes; ω_y, ω_z – are angular velocities of AUV around y, z axes; $|\upsilon|$ – is a module of an AUV velocity vector; φ – is an angle of AUV heading; ψ – is an angle of AUV trim. Let us now consider each of numbered blocks in the Fig. 3.

Blocks 1-5 calculate accelerations of an AUV and can be described via following expressions:

$$a_x = \frac{T_x + R_x}{m + \lambda_{11}};\ a_y = \frac{T_y + R_y}{m + \lambda_{22}};\ a_z = \frac{T_z + R_z}{m + \lambda_{33}};$$
$$\varepsilon_y = \frac{M_y + M_y^R}{J_y + \lambda_{55}};\ \varepsilon_z = \frac{M_z + M_z^R}{J_z + \lambda_{66}},$$

where λ_{ij} – are added masses of water. Then, after accelerations are calculated, they arrive at inputs of integrators, which calculate linear and angular velocities. Then angular velocities arrive at the second layer of integrators, which calculate values of heading angle and trim angle. Projections of AUV linear velocity arrive at the input of block 6, which can be described with the following expression:

$$|\upsilon| = \sqrt{\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2}.$$

Then, values of velocity module, heading angle and trim angle arrive at the input on the block 7, which calculates derivatives of AUV coordinates according to the following formulas [16-20]:

$$x' = |\upsilon| \cdot \cos(\varphi) \cdot \cos(\psi);$$
$$y' = |\upsilon| \cdot \sin(\psi);$$
$$z' = |\upsilon| \cdot \sin(\varphi) \cdot \cos(\psi).$$

Then, another layer of integrators calculates AUV coordinates.

Feedback in the model considered is organized according to the following expressions describing blocks 8-12:

$$R_x = 0.5 \cdot c_x \cdot \rho \cdot V^{2/3} \cdot |\upsilon|^2 \cdot [-sign(T_x)];$$
$$R_y = 0.5 \cdot c_y \cdot \rho \cdot V^{2/3} \cdot |\upsilon|^2 \cdot [-sign(T_y)];$$
$$R_z = 0.5 \cdot c_z \cdot \rho \cdot V^{2/3} \cdot |\upsilon|^2 \cdot [-sign(T_z)];$$
$$M_y^R = 0.5 \cdot m_y \cdot \rho \cdot V \cdot |\upsilon|^2 \cdot [-sign(M_y)];$$
$$M_z^R = 0.5 \cdot m_z \cdot \rho \cdot V \cdot |\upsilon|^2 \cdot [-sign(M_z)],$$

where c_x, c_y, c_z, m_y, m_z – are coefficients of hydrodynamic forces and moments depending on a form of AUV hull; V – is a water displacement of an AUV.

All the models described previously can be combined in a model of an AUV MCS considering non-linearity of some of its elements. Let us now consider developing a criterion to assess significance of interconnections in the model developed.

IV. Developing an Assessment Criterion

Simulation modeling of the AUV motion along any trajectory will provide us two sets of data: one will describe desired trajectory in terms of position and orientation and the other will describe simulated trajectory considering inertness of the AUV.

After finding a transfer matrix, which links datasets of desired trajectory and simulated trajectory, it is proposed to equate Laplace operator to zero and find static coefficients of elements of the matrix. Then, in each row the matrix all non-zero elements must be divided by element of the main diagonal from the row. If result of such division is less, than some predefined constant, than according connection is considered insignificant and can be excluded from the model. Considering this, the algorithm of assessment can be formulated as follows:

1) Find values of parameters of UUV considered: hydrodynamic coefficients, mass, water displacement, moments of inertia, characteristic of propellers of thrusters, maximal angles of steering blades rotation;

2) Substitute them to previously derived mathematical model and compose a model of the whole UUV MCS;

3) Define a motion trajectory to examine;

4) Perform simulation modeling and find transfer matrix *M(s)* describing dependency between desired and simulated trajectory;

5) Find $k_{ij} = \dfrac{M(0)_{ij}}{M(0)_{ii}}; i, j \in [1; n],$ where n – is a number of rows and columns in *M(s)*, define the lower limit for $k_{ij}^{\min}$;

6) exclude all interconnections, where $k_{ij} \le k_{ij}^{\min}; i, j \in [1; n]$, by substituting according manipulated parameters with their estimated interval values.

After applying the algorithm, the mathematical model of the system considered will include a set of separate control channels excluded from the *M(s)* by introducing interval parameters in their transfer functions and a transfer matrix, including remaining interconnections, whose significance was higher, than the predefined lower limit. This will allow to introduce more interval parameters to the model in order to describe parametric uncertainty of the system and synthesize controllers with the help of synthesis methods developed previously by authors [6].

V. Conclusion

The research resulted in an assessment algorithm of significance of interconnection in mathematical models of various underwater vehicles. Mathematical models of elements of UUV MCS can be easily modified to describe UUV of any construction and type with various kinds of propulsion and steering systems. As far as the method is based on identification of transfer matrices between sets of data describing real and desired trajectory, it is computationally feasible and easily applicable.

Further research will be dedicated to software implementation of the method and testing it on a problem of decomposing a mathematical model of an underwater vehicle prototype.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Gayvoronskiy Sergey developed a mathematical model of a propulsion and steering system and conducted the research.

Ivan Khozhaev developed a mathematical model of an autonomous underwater vehicle motion process and an assessment criterion, wrote the paper.

Tatiana Ezangina developed mathematical model of elements of a motion control system for an autonomous underwater vehicle and an assessment criterion, analyzed the data.

All authors had approved the final version.

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