The Features of Design Elements with Controlled Elastic Deformation

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Abstract — The article deals with the problem of formation of a mathematical model and algorithm of numerical study of flexible elastic elements in the form of thin-walled shells. The proposed algorithm allows us to quickly solve the problem of numerical synthesis of the structure. The main problem for the synthesis of these elements is to provide the required deformation. The deformation of shell elements is essentially a nonlinear process. The main relations of the version of the theory of thin axisymmetric shells are known. However, the study of the elements under consideration requires taking into account some peculiarities. The paper presents the derivation of the equations describing the axisymmetric deformation of thin-walled shells for several practically important cases, as well as reflects the features that must be taken into account for the cases under consideration. The proposed algorithm allows in the process of modeling the element to carry out a kind of programming of the properties of the future design and as a result provide the required performance characteristics in the process of numerical design. The article presents the results of the application of this technique for the design of real products based on flexible elements with controlled elastic deformation.

Index Terms — multi-parametric nonlinear task, nonlinear processes of deformation, controlled elastic deformation flexible elastic element, axisymmetric shell, working characteristic

I. INTRODUCTION

Modern designs in some cases acquire the properties of controlled changes in the structure and parameters. These properties provide additional opportunities for moving objects and output motion in vacuum, clean and (or) aggressive environments [1, 2]. The principle of controlled elastic deformation is used.

Actuators using the principle of controlled deformation are considered. The main goal is to implement a given trajectory by changing the shape of the element – its deformation. At the same time, the inherent deformation processes must provide large displacements, that is, displacements commensurate with the characteristic dimensions of the structure.

II. MATHEMATICAL MODEL FOR THE ANALYSIS OF LARGE DEFLECTIONS OF THE FLEXIBLE AXISYMMETRIC SHELLS

It is known that in thin-walled elements, even within the framework of relatively small elastic deformations, large displacements can be realized. In this case, the deformation process can be caused by a variety of external factors: force and kinematic effects, internal or external pressure, temperature changes, phase transformations in alloys with shape memory, etc.

To ensure the required performance in the process of numerical modeling and design of the element is required to conduct a kind of programming properties of the future design. Includes the ability to implement the desired nonlinear deformation processes. It should be noted that with such a provision of deformation, it becomes possible to plan the processes occurring by means of claps or jumps [3-5]. These elements are called elements of controlled elastic deformation in the literature.

For the studied structures the most adequate is the design scheme of axisymmetric thin-walled shell. This scheme allows us to reduce the problem of analysis of the deformation process to the solution of the nonlinear boundary value problem for the system of differential equations in ordinary derivatives.

Next, the article considers derivation of the basic relationships and the algorithm of numerical study for an extensive class of flexible thin-walled shells.

In the derivation we will use the general laws of geometric shape, loading and stress state, which provide these shells the ability to elastic displacement. Note that in this case, the elastic displacement is many times greater than the thickness of the shell.

In its form, the shells of these types are axisymmetric or close to the sector of the shell of rotation. In the second case, we will call the shell – axisymmetric shells unclosed in the circumferential direction. The stress state
for the main part of such a shell is assumed to be constant or slightly changing in the direction of the circumferential coordinate, which imposes certain restrictions on the law of change of the external load along the circumferential coordinate and requires special conditions for fastening and loading at the circumferential ends.

III. EQUATIONS DESCRIBING AXISYMMETRIC DEFORMATION OF THIN-WALLED SHELLS

In this part of the article the basic relations of the variant of the theory of thin axisymmetric shells preserving continuity with the equations proposed in [6 - 8] are given. The relations are prepared for the application of the method using the reduction of the nonlinear boundary value problem to the solution of the system of nonlinear equations and the Cauchy problem. Since there are some fundamental moments for further exposition, as well as some modifications associated with the expansion of the spectrum of the problems studied with their help, it is advisable to dwell on the derivation of the relations in more detail.

To describe the shape of an arbitrary shell of rotation, we use the reference surface associated with the shell. In a particular case, the equidistant or middle surface of the shell is used as the reference surface. The meridional section of the reference surface (Fig. 1) in the undeformed state is written in parametric form:

\[ x_0 = x_0(s_0); \ y_0 = y_0(s_0) \]

(1)

where \( s_0 \) is the independent coordinate from the pre-selected starting point to the current point – \( A_0 \) along the meridian arc (Fig.1); \( x_0; y_0 \) - Cartesian coordinates of the current meridian point.

![Figure 1. To the conclusion of the basic geometric relations.](image)

\( \theta_0 \) - the current angle of inclination of the tangent to the meridian in the undeformed state, counted by the X-Axis to the tangent counterclockwise. Note the validity of the following geometric relations:

\[ \frac{dx}{ds_0} = \cos \theta_0; \quad \frac{dy}{ds_0} = \sin \theta_0. \]

(2)

To denote the parameters related to the undeformed state of the shell, we will use the lower index “o”.

In the deformed state (Fig.1) the considered material point of \( A_o \) belonging to the reference surface, will move to a new spatial position \( A \), characterized by the coordinates \( x, y \) and \( s \), respectively.

The geometric relations are similar (2), but for the deformed state will be written in the form:

\[ \frac{dx}{ds} = \cos \theta; \quad \frac{dy}{ds} = \sin \theta. \]

(3)

The horizontal - \( u \) and vertical - \( v \) components of the movement of point \( A \), and the change in the angle of rotation of the normal \( \Delta \theta \) are determined by the relations:

\[ u = x - x_0; \quad v = y - y_0; \quad \Delta \theta = \theta - \theta_0 \]

(4)

For the main radius of curvature in the considered point in the initial (undeformed) and actual (deformed) states, the dependences are valid:

\[ \frac{1}{\rho_{m0}} = \frac{d\theta}{ds}, \quad \frac{1}{\rho_{m}} = \frac{d\theta}{ds}, \quad \frac{1}{\rho_{t0}} = \frac{\sin \theta}{x}, \quad \frac{1}{\rho_{t}} = \frac{\sin \theta}{x}. \]

The lower index "m" is used to denote the values corresponding to meridional direction and the index "t" are used to denote the values corresponding to circumferential direction respectively. The linear deformation of the reference surface element in the meridional direction for the current state will be equal to:

\[ \varepsilon_m = \frac{ds - ds_0}{ds_0}. \]

(5)

During the deformation of shells of revolution open in the circumferential direction (Bourdon spring) is changed the central angle of the shell \( \Phi_0 \) and \( \Phi \) for the initial and actual states respectively (see Fig.2).

The relative change of the central angle is determined by the expression:

\[ \chi = \frac{\Phi - \Phi_0}{\Phi_0}. \]

(6)

Linear deformation of the reference surface in the circumferential direction will be written in the form:

\[ \varepsilon_c = \frac{(x_0 + u)\Phi - x_0\Phi_0}{x_0\Phi_0} = \chi + \frac{u}{x_0} (1 + \chi). \]

(7)
To determine the change in the central angle of the axisymmetric shell of rotation unclosed in the circumferential direction

The complete change of the main curvatures of the reference surface element is connected with the change of the current angle of inclination of the tangent and the arc length by the following relations:

\[
\Delta \kappa_s = \kappa_s - \kappa_s^0 = \frac{d \theta}{ds} - \frac{d \theta}{s^0},
\]

\[
\Delta \kappa_{\theta} = \kappa_{\theta} - \kappa_{\theta}^0 = \frac{\sin \theta}{x} - \frac{\sin \theta}{x^0}.
\] (8)

A complete change of curvature in both directions is the sum of the change of curvature caused by the action of the load \( \Delta \kappa^F \), and measuring the curvature caused by the change of linear dimensions of the shell \( \Delta \kappa^L \).

\[
\Delta \kappa_s = \Delta \kappa^F_s + \Delta \kappa^L_s,
\]

\[
\Delta \kappa_{\theta} = \Delta \kappa^F_{\theta} + \Delta \kappa^L_{\theta}.
\] (9)

To describe directly the shell, which is a three-dimensional body, we introduce two front surfaces of the shell, separated from the reference surface at distances \( h(\cdot) (s) \) and \( h(\cdot) (s^0) \). Distances are measured in the direction of the outer normal to the undeformed reference surface. The shell thickness at the current point is a known coordinate function - \( s^0 \).

\[
h(s) = h(\cdot) (s) + h(\cdot) (s^0).
\] (10)

Normal coordinate \( \zeta \) current material point belonging to the shell varies \( -h(\cdot) (s^0) \leq \zeta \leq h(\cdot) (s^0) \). If the shell is thin-walled, then according to this assumption:

\[
\max \left| h(s) \right| \ll \min \left| \rho_{\theta 0} \right|, \left| \rho_{\theta 1} \right|
\] (11)

Under the assumption of the validity of the kinematic part of the Kirchhoff hypothesis [9-11], the material points belonging to the normal to the undeformed reference surface in the process of deformation pass to the normal to the deformed reference surface, maintaining a fixed distance \( \zeta = \zeta^0 \) to the deformed reference surface. Therefore, for linear deformations of an arbitrary element, in the small neighborhood of the material point with the coordinates \( s^0 \) and \( \zeta^0 \), the relations are fair:

\[
\varepsilon^s = \varepsilon^s + \xi \Delta \kappa^F_s, \quad \varepsilon^\theta = \varepsilon^\theta + \xi \Delta \kappa^F_{\theta}.
\] (12)

The consequence of these hypotheses is a linear distribution of deformations in the thickness of the shell, and the fact that these deformations will be fully determined if it is possible to calculate the changes in the curvature and deformation of the reference surface in the meridional and circumferential directions.

Therefore, it is possible to present the geometric relations of the considered variant of the equations of axisymmetric shells in the following form:

\[
\frac{du}{ds^0} = (1 + \varepsilon^s) \cos \theta - \cos \theta^0,
\]

\[
\frac{dv}{ds^0} = (1 + \varepsilon^s) \sin \theta - \sin \theta^0,
\]

\[
\frac{d\theta}{ds^0} = (1 + \varepsilon^\theta) \kappa^F_{\theta} + \frac{d\theta^0}{ds^0}.
\] (13)

Positive directions of power factors are shown in Fig.3.

Normal - \( N_m \) and transverse - \( Q \) forces in the site, the external normal of which coincides with the positive direction of the arc coordinate \( s^0 \) (shown in Fig.4 by dotted lines) associated with the horizontal \( U \) and vertical \( V \) components of internal efforts by using rotation matrix that depend on the current value of the angle \( \theta \):

\[
\begin{bmatrix}
N_m \\
Q
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
U \\
V
\end{bmatrix}
\] (14)
Figure 3. The positive directions of the forces acting on the shell element.

Force factors make sense of intensities, that is, referred to the unit length of the reference surface. Consider the equilibrium conditions of the shell element in the deformed state (Fig. 4). Composing the equilibrium equations of the shell element after the necessary transformations we obtain the relations:

\[
\frac{dU}{ds_u} = -(1 + \varepsilon_m) \left( \frac{\cos\theta}{x_0 + u} U - \frac{N_u}{x_0 + u} + q_u \right),
\]

\[
\frac{dV}{ds_v} = -(1 + \varepsilon_m) \left( \frac{\cos\theta}{x_0 + u} V + q_v \right),
\]

\[
\frac{dM_m}{ds_m} = -(1 + \varepsilon_m) \cdot \frac{\cos\theta}{x_0 + u} \left( M_m - M_t \right) - U \sin\theta + V \cos\theta
\]

(15)

where \(q_u\) and \(q_v\) are the intensity of the distributed load acting along the axes \(U\) and \(V\) respectively.

Following the algorithm described in [5, 6, 12], the system of nonlinear differential equations of the sixth order (13, 15) will be considered as the main system. Unknown quantities, derivatives of which are included in the equation, we call the main unknown. The vector \(\{X\} = \{u, v, \theta, U, V, M_m\}^T\) called the principal unknown vector in the current shell section. Other unknowns included in the equation (13, 15) we will call auxiliary. In solving the basic system of equations, the auxiliary unknowns are expressed through the basic equations by means of algebraic relations. The following are expressions for a number of practical important cases. For the isotropic axisymmetric shell under power loading, the middle surface of the shell is used as the reference surface. Linear deformation of the reference surface in the circumferential direction takes the form:

\[
\varepsilon_i = \frac{u}{x_0}. \quad (16)
\]

Coefficients \(D\) and \(B\) are defined by expressions:

\[
B = \frac{Eh}{1 - v^2}, \quad D = \frac{Eh^3}{12(1 - v^2)} \quad (17)
\]

In the result the following formulas to determine the auxiliary values:

\[
\varepsilon_u = \frac{1}{B} \left( U \cos\theta + V \sin\theta \right) - \frac{v}{x_0} \frac{u}{x_0},
\]

\[
\kappa_u = \frac{M_u}{D} - \frac{x_0 + u}{x_0} \frac{\sin\theta}{x_0} \left( -\frac{\sin\theta}{x_0} \right),
\]

\[
N_i = v\left( U \cos\theta + V \sin\theta \right) + \frac{Eh}{x_0} \frac{u}{x_0},
\]

\[
M_i = vM_m + \frac{Eh}{12} \left[ \frac{x_0 + u}{x_0} \left( \frac{\sin\theta}{x_0} \frac{\sin\theta}{x_0} \right) \right]. \quad (18)
\]

In the study of shells unclosed in the axial direction, following [13], the system of equations includes the value of the relative change of the central angle \(\chi\), determined by the expression (6). Depending on the loading conditions \(\chi\) can be set or be the desired value. To find a solution, the system should be supplemented with a condition imposed on the value of the resultant moments of internal forces in the cross section. From the equilibrium condition of the cut-off part, the resultant must be equal to the external moment \(M^*\) applied to the cut-off part.

\[
\int [M_u \sin\theta + N_u (x_0 + u)] ds_v = M^* .
\]

(19)

If there is no external moment, the resultant must be zero. We emphasize that the method considered is based on the additional assumption that the meridional section of the shell, unclosed in the circumferential direction,
remains flat. This is true for a number of thin-walled shells. For example it is manometric springs in areas sufficiently remote from the ends. For more thorough research, an open shell in the circumferential direction should be considered as a two-dimensional problem and solve the problem for a system of partial differential equations.

The auxiliary values are determined by formulas:

\[
\varepsilon_n = \frac{1}{B} \left( U \cos \theta + V \sin \theta \right) - \nu \left[ \chi + \frac{\mu}{x_0} \left( 1 - \chi \right) \right],
\]

\[
\kappa_n = \frac{M}{D} - \mu \frac{x_0 + u}{x_0} \left( \sin \theta \frac{\sin \theta}{x_0} \right),
\]

\[
N_i = \mu \left( U \cos \theta + V \sin \theta \right) + Eh \left[ \chi + \frac{\mu}{X_0} \left( 1 - \chi \right) \right],
\]

\[
M_x = \varepsilon M_n + \frac{Eh}{12} \frac{x_0 + u}{x_0} \left( \sin \theta \frac{\sin \theta}{x_0} \right).
\]

(20)

IV. EXECUTIVE ROBOTIC MECHANISMS WITH NON-TRADITIONAL METHODS OF MOVEMENT.

The intensive development of science and technology stimulates the development of robot designs or robotic systems. It is possible to mention actively developing today the direction of creation of the robots realizing the walking movement (step-by-step) of the person or animal [14]. Another direction is not only ensuring the required movement of the robot working element, but also the most effective realization of this movement [15-17]. In this case, it takes a lot of time to find a solution, because the law of motion, of course, depends on many initial parameters.

Modern medicine and technology have put the task of implementing a given mechanical movement or force to an object, while allowing the possibility of miniaturization. In modern conditions, it is not always possible to use effective systems because of their large size [18, 19]. Using the principle of controlled elastic deformation when designing technical systems, including constructions of micro-robots, will significantly increase their functionality and to reach a new technological level.

Using in actuators the deformable thin-walled elastic-elements offers a unique opportunity to minimize the adverse effects. External friction deteriorates the technological scope. It should be deleted especially when working near or inside the human body. Got the elements described in the literature [5, 8] name of the elements of the controlled elastic deformation.

The elements of the controlled elastic deformation, used for transporting and positioning process semi-finished electronic products when applying thin films have proved indispensable in constructing specific robotic mechanisms known as clean robots.

First clean robots that are designed to work in the electric vacuum and ultrahigh-purity areas, examined in the works of Aleksandrova A. T. [6]. Robots were a combined mechanisms that use bellows and flexible sealed tubular elements. One of these robots is shown in the Fig. 5.

In Fig. 5 schematically presented robot-manipulator with four degrees of freedom, designed to generate a horizontal movement, circular movements in the vertical and horizontal planes, as well as capture products. Inside the chassis, mounted on flange, adapter sleeve guides 9 moves hollow rod 1, inside which through the choke 2 is served. Rod tightly put into vacuum separation capacity by item 8 in the form of a bellows. At the end of a rod located in the atmosphere is placed in the groove of the roller cam 7 specifies the law of movement of the stem in the horizontal direction. In part located in a vacuum cavity rod tightly connected with a flexible tubular element sealed edges 3. Vertical and horizontal circular movement, carried out as a result of the elastic deformation of tubular elements 3 and 6. Total displacement generates spatial displacement grips 4. Items 3 and 6 are triggered simultaneously when applying compressed air through the socket 2. To engagements 4 compressed air enters through the tube 5.

The sequence of movements of the robot, in according to the work cycle diagram, installed cam, specifying the cycle of the work for the stock 1 and distribution spool. Fixation regarding products in extreme positions is achieved by installing special stops.

To address external friction (friction movement) in a vacuum it is expedient to apply mechanisms that move the executive bodies is a consequence of their elastic deformation under the influence of an external, not in contact with the energy carrier area. This condition is most fully satisfy the elements in the form of thin membranes: membranes and flexible bellows sealed tubular elements, known as Bourdon spring [5, 6]. This elements are known for a long time, and, in particular,
used to measure the pressure in the manometers, because of got the name manometric spring. The working part of the manometric elements under the action of internal pressure to deform elastically. In doing so, the working part commits the movement which with appropriate design can bring to a size commensurate with the geometrical dimensions of the element.

The main advantage of the considered flexible elements is to maintain the purity of the working environment [2, 6, 20]. If it is necessary to place a device, a device or a mini-robot inside a person, then the requirement of environmental safety becomes the most important [21]. With the help of the developed numerical technique, implemented as a package of PURGA application, studies of a number of promising designs, which should include new types of environmentally friendly actuators, technical devices and systems.

On the basis of the principle of controlled elastic deformation can be created qualitatively new designs of mobile mini and micro robots for special purposes. The application of such a natural principle for living organisms provides high efficiency of medical micro robots with peristaltic principle of movement.

![Image](image_url)

Figure 6. Layout of the mini-robot using the peristaltic principle of motion.

For Fig. 5 a working model of an endovascular mini-robot designed for medical operations in blood vessels is presented. The robot is a structure consisting of a series of connected bellows, which are associated with flexible rod elements. In accordance with the cyclogram of motion, pressure is applied to the inner cavity of the bellows. In this case, the flexible elements are deformed and consistently leaning against the walls, move the robot along the tubular channel.

V. CONCLUSION

The proposed algorithm of numerical design is based on the method of continuation of the decision on the parameter [5, 12] in combination with the reception of subspace change of control parameters [4]. In the design of nonlinear shell structures in active transformation operators with properties of controlled changes in the structure and parameters of the principle of controlled elastic deformation, which provides additional opportunities for moving objects and output motion in vacuum, clean and (or) aggressive areas [2, 21]. An important factor is also the low sensitivity of the proposed mechanisms to the effects of radiation, which is detrimental to electronic devices.

Deformation of the structure can be caused by pressure, temperature or shape memory effect. An important, fundamental feature of the designed devices, which significantly differ them from the known technical solutions, is the possibility of implementing a relay characteristic, that is, a discrete response to a monotonic change in external influence.

With the help of the developed numerical method implemented as a package of PURGA application programs [5], studies of a number of promising designs, which include new types of environmentally friendly actuators, technical devices and systems that do not pollute the environment and minimize human intervention in natural ecosystems.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

O.O. Baryshnikova conducted the research and wrote the paper.

N. T. Gavryushina has analyzed data and made a conclusions.

Both authors accepted the paper.

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