Optimize the Feed Rate and Determine the Joints Torque for Industrial Welding Robot TA 1400 Based on Kinematics and Dynamics Modeling

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Abstract— **This paper focuses on optimization of the feed rate parameter for the TA 1400 industrial welding robot with 6 degrees of freedom based on solving the inverse kinematics problem and the optimal algorithm in the parametric domain. The position, velocity, acceleration, and jerk of joints are determined from the given parametric curve. These results are used to calculate the value of the joint torques through the inverse dynamics problem which is solved effectively by using the algorithm for adjusting the increment of generalized vector for the redundant system. The optimal algorithm for the feed rate parameter is performed with the kinematics constraints of the robot. The feed rate values are increased gradually through each loop until the kinematics constraints are broken and constantly change according to the weld seam profile. Each optimum value corresponds to a position on the given weld seam. Robot dynamics equations are constructed using the Lagrange equations. The research results play an important role in optimizing the production process through time reduction and productivity improvement machining.**

Index Terms— **optimal feed rate, welding robots, inverse kinematics, inverse dynamics, joints torque**

I. INTRODUCTION

Nowadays, industrial robots are widely used in manufacturing such as welding, cutting, printing 3D plastic, and additive manufacturing metal because of their flexibility. Robots are designed and controlled with a greater number of degrees of freedom (DOF) than Computer Numerical Control machines. This advantage allows the robot to be more flexible than the CNC machine and they can be used to manufacture complex objects or significantly replace manual labor. Although using robots in manufacturing, the demand to increase production efficiency is always an important issue that needs to be addressed in order to increase competitiveness in the production industry. One of the most effective ways to increase productivity is to optimize production processes, reduce machining time based on optimizing the feed rate of robots, especially for complex toolpaths constantly changing. However, the position accuracy of the tool tip and kinematics limits such as velocity, acceleration, and jerk of joints have to ensure preventing the robots from overloading, avoiding unwanted vibrations and toolpath errors.

The inverse kinematics problem always plays an important role in analyzing dynamics problems and designing the control system of robots. These problems become more complicated for the redundant system because of its multiplicity. Some methods are developed to solve the inverse kinematics problems such as Pseudoinverse [1], Jacobian Transpose [2], [3], Damped Least Squares [4], Quasi-Newton and conjugate gradient [5], [6], Closed-loop inverse kinematics (CLIK) [7], [8], [9], [10], Offset Modification (OM) method [11], Neutron network algorithm [12], [13], The Quick IK algorithm [14]. Aydun [15] proposed the inverse kinematics solution of 6-dof industrial manipulator with Euler’s wrist using the quaternion vector pair method. The new solution method to avoid joint limitation, singularities and obstacles for redundant robots are introduced in [16]. Kinematics modeling analyzing for welding and cutting robots are mentioned in [17], [18], [19], [20]. The inverse dynamics problems of the redundant robot are mentioned in [21], [22], [23]. The researches on optimizing the feed rate for industrial robots are mainly addressed in the works [24], [25], [26], [27]. The method of feed rate planning using machining robots was concerned in [24] based geometric parametric interpolation. FIR filter was used to ensure the smooth feed rate with the limited jerk of toolpath. My and et al in [25], [26] and [27] built a calculation model to optimize the feed rate for a 5-axis CNC machine in the parametric domain based on the machine's kinematics characteristics. The study was conducted when considering a 5-axis CNC machine as a combined system of two robotic manipulators, the kinematics problem was solved in the

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parametric domain. The mathematical models of the system play an important role in the construction of the toolpath, the optimal design for the multi-axis CNC machine, machining robots, the optimization of technological parameters.

This paper presents the results of optimal calculation for the feed rate parameter and joints torque values of the industrial welding robot TA 1400 through the kinematics, dynamics modeling and the algorithm for optimizing feed rate in the parametric domain. Based on the given weld seam profile and the algorithm for adjusting the increments of generalized vector, the position, velocity, acceleration and jerk values of joints are determined. These results and the kinematics constraints from the structure of the robot are input data to the optimal algorithm. The values of the torque of the joints are calculated through solving the inverse dynamics problem.

II. Kinematics and Dynamics Modeling for TA 1400 Robot

Consider the kinematics model of industrial welding robot TA 1400 with 6-DOF as shown in Fig. 1. The fixed coordinates system is $(OXYZ)_0$ located at point O_0 and $(OXYZ)_i, (i = 1 \div 6)$ are the local coordinate systems attached link i.

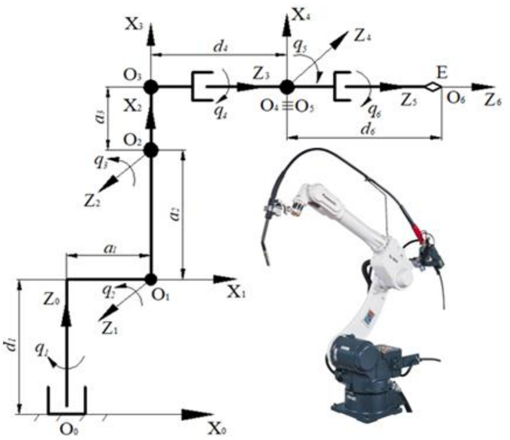

Figure 1. Kinematics model of the welding robot TA 1400

Table I describes the kinematic parameters according to the D-H rule [2], [28]. Accordingly, the transformation homogeneous matrices $\mathbf{H}_i, (i = 1 \div 6)$ are determined.

TABLE I. Kinematics Parameters D-H

Links	DH parameters			
	$\theta_i = q_i$	d_i	a_i	α_i
1	q_1	d_1	a_1	$\pi/2$
2	q_2	0	a_2	0
3	q_3	0	a_3	$\pi/2$
4	q_4	d_4	0	$\pi/2$
5	q_5	0	0	$-\pi/2$
6	q_6	d_6	0	0

The position and direction of the end-effector point (point E) from the $\mathbf{D}_6$ matrix following the fixed coordinate system are determined as follows [2], [28]. In this paper, the tip point of the welding torch is the end-effector point.

$$\mathbf{D}_6 = \mathbf{H}_1\mathbf{H}_2\mathbf{H}_3\mathbf{H}_4\mathbf{H}_5\mathbf{H}_6 \tag{1}$$

Define the generalized vector of robot is $\mathbf{q} = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6]^T$ and $\mathbf{x} = \begin{bmatrix} x & y & z \end{bmatrix}^T$ is the coordinate vector of end-effector point following fixed coordinate system. The forward kinematics equations can be written as

$$\mathbf{x} = f(\mathbf{q}) \tag{2}$$

Where, f is a vector function representing the robot forward kinematics. Derivative (2) with respect to time, the relation between generalized velocities is obtained as

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \tag{3}$$

Where, $\mathbf{J}(\mathbf{q})$ is the Jacobian matrix with size 3×6. The acceleration of the end-effector point can be given by derivation (3)

$$\ddot{\mathbf{x}} = \dot{\mathbf{J}}\dot{\mathbf{q}} + \mathbf{J}\ddot{\mathbf{q}} \tag{4}$$

Derivative continuously (4), the jerk of the end-effector point is determined as

$$\dddot{\mathbf{x}} = \ddot{\mathbf{J}}\dot{\mathbf{q}} + 2\dot{\mathbf{J}}\ddot{\mathbf{q}} + \mathbf{J}\dddot{\mathbf{q}} \tag{5}$$

The inverse kinematics equations of robots are written as

$$\mathbf{q} = f^{-1}(\mathbf{x}) \tag{6}$$

The values of vector $\mathbf{q}$ have been determined from (6), the joints velocity is determined as

$$\dot{\mathbf{q}} = \mathbf{J}^{+}(\mathbf{q})\dot{\mathbf{x}} \tag{7}$$

Where, $\mathbf{J}^{+}(\mathbf{q})$ is the pseudo-inverse matrix of $\mathbf{J}(\mathbf{q})$ matrix and is defined as [2], [28]

$$\mathbf{J}^{+}(\mathbf{q}) = \mathbf{J}^{T}(\mathbf{q})\left[\mathbf{J}(\mathbf{q})\mathbf{J}^{T}(\mathbf{q})\right]^{-1} \tag{8}$$

The joints acceleration is calculated from (7)

$$\ddot{\mathbf{q}} = \mathbf{J}^{+}(\mathbf{q})(\ddot{\mathbf{x}} - \dot{\mathbf{J}}\dot{\mathbf{q}}) \tag{9}$$

Similarly, the joints jerk also is determined from (9)

$$\dddot{\mathbf{q}} = \mathbf{J}^{+}(\mathbf{q})(\dddot{\mathbf{x}} - 2\dot{\mathbf{J}}\ddot{\mathbf{q}} - \ddot{\mathbf{J}}\dot{\mathbf{q}}) \tag{10}$$

For the given $\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}, \dddot{\mathbf{x}}$ vectors and using the algorithms for adjusting the increments of generalized vector which was proposed in [29], the approximately joint variables value can be determined exactly. Given a geometric trajectory such as a toolpath in parametric domain $\mathbf{x}(u)=\begin{bmatrix} x(u) & y(u) & z(u) \end{bmatrix}^T, u=[0,1]$. Define $f(t)=\dot{s}(t)$ is the feed rate along the toolpath [24], where s is the arc length of curve $\mathbf{x}(s)=\begin{bmatrix} x(s) & y(s) & z(s) \end{bmatrix}^T$. The inverse kinematics equation in parametric domain is rewritten as

$$\mathbf{q}(u)=f^{-1}(\mathbf{x}(u)) \tag{11}$$

Assume that the value of generalized vector $\mathbf{q}(u)$ is calculated by using the method mentioned above. The velocity, acceleration and jerk of joints need to be determined in parametric domain. From (7), we have:

$$\dot{\mathbf{q}}=\mathbf{J}^{+}\frac{d\mathbf{x}}{ds}\frac{ds}{dt}=\mathbf{J}^{+}\mathbf{x}'_{s}\dot{s} \tag{12}$$

Where, $ds\cong\left|\frac{d\mathbf{x}}{du}\right|du$ and,

$$\mathbf{x}'_{s}=\frac{d\mathbf{x}}{ds}=\frac{d\mathbf{x}/du}{ds/du}=\frac{d\mathbf{x}/du}{\left|d\mathbf{x}/du\right|}=\frac{\mathbf{x}'_{u}}{\left|\mathbf{x}'_{u}\right|} \tag{13}$$

The generalized velocity vector can be given as

$$\dot{\mathbf{q}}=\mathbf{J}^{+}\frac{\mathbf{x}'_{u}}{\left|\mathbf{x}'_{u}\right|}f \tag{14}$$

The acceleration and jerk of joints are determined in parametric domain as [27]

$$\ddot{\mathbf{q}}=\mathbf{J}^{+}\left(\left(\frac{\mathbf{x}''_{u}}{\left|\mathbf{x}'_{u}\right|^{2}}+\frac{\mathbf{x}'_{u}\left((\mathbf{x}'_{u})^{T}\mathbf{x}''_{u}\right)}{\left|\mathbf{x}'_{u}\right|^{4}}\right)f^{2}+\frac{\mathbf{x}'_{u}}{\left|\mathbf{x}'_{u}\right|}\dot{f}-\dot{\mathbf{J}}\dot{\mathbf{q}}\right) \tag{15}$$

And,

$$\dddot{\mathbf{q}}=\mathbf{J}^{+}\left(\begin{array}{l}\frac{\mathbf{x}''_{u}}{\left|\mathbf{x}'_{u}\right|^{3}}f^{3}+\frac{\mathbf{x}''_{u}\left((\mathbf{x}'_{u})^{T}\mathbf{x}''_{u}\right)+\mathbf{x}'_{u}\left((\mathbf{x}''_{u})^{T}\mathbf{x}''_{u}+(\mathbf{x}'_{u})^{T}\mathbf{x}'''_{u}\right)}{\left|\mathbf{x}'_{u}\right|^{5}}f^{3}\\ +\frac{\mathbf{x}''_{u}}{\left|\mathbf{x}'_{u}\right|^{2}}+\frac{\mathbf{x}'_{u}\left((\mathbf{x}'_{u})^{T}\mathbf{x}''_{u}\right)}{\left|\mathbf{x}'_{u}\right|^{4}}\dot{f}f-2\dot{\mathbf{J}}\ddot{\mathbf{q}}-\ddot{\mathbf{J}}\dot{\mathbf{q}}\end{array}\right) \tag{16}$$

From (14), (15) and (16), values of $\dot{\mathbf{q}},\ddot{\mathbf{q}}$ and $\dddot{\mathbf{q}}$ depend mainly on the geometric characteristics of the toolpath trajectory $(\mathbf{x}(u),\mathbf{x}'(u),\mathbf{x}''(u),\mathbf{x}'''(u))$, feed rate $(f,f^{2},f^{3},\dot{f})$ and the kinematics structure of the robot $(\mathbf{J},\dot{\mathbf{J}},\ddot{\mathbf{J}})$.

The inverse dynamics problem is built to determine the forces and torques acting on the robot according to the given motion characteristics. It is necessary to build a system of dynamic equations to solve this problem. The dynamics equations show the relationship between forces and torques with the motion characteristics of robots such as joint position $\mathbf{q}$, velocity $\dot{\mathbf{q}}$, joint acceleration $\ddot{\mathbf{q}}$. The dynamics equations of the robot are described as follows

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})=\boldsymbol{\tau} \tag{17}$$

Where, $\mathbf{M}(\mathbf{q})$ is the mass matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is Coriolis matrix, $\mathbf{g}(\mathbf{q})$ is the gravity vector, $\boldsymbol{\tau}$ is the joints torque vector. The components of (17) are determined similarly in [29]. The generalized vectors $\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}}$ and $\dddot{\mathbf{q}}$ are calculated from solving the inverse kinematics problem and performing the optimal feed rate algorithm. So, the values of joints torque are determined clearly following (17).

III. Optimize the Feed Rate Parameter

In this section, the feed rate of the welding torch along the weld seam is calculated optimally based on the kinematics constraints of the robot. The initial feed rate value is given and will increase gradually with each increment of the loop. This increase only stopped when the constraints conditions were broken. Thus, each position in the parametric trajectory will have a corresponding optimal feed rate value. Define some symbols as follows: f_{ini} and $f_{\max}$ are the initial and maximum feed rate value, generalized vectors $\dot{\mathbf{q}}_{\max},\ddot{\mathbf{q}}_{\max}$ and $\dddot{\mathbf{q}}_{\max}$ are the maximum velocity, acceleration and jerk vector of joints. The optimal function is given as

$$f_{optimal}=\max_{u\in[0,1]}f(u) \tag{18}$$

The constraints conditions for optimal problem can be defined as

$$\left|\dot{\mathbf{q}}\right|\leq\dot{\mathbf{q}}_{\max},\left|\ddot{\mathbf{q}}\right|\leq\ddot{\mathbf{q}}_{\max},\left|\dddot{\mathbf{q}}\right|\leq\dddot{\mathbf{q}}_{\max},f\leq f_{\max} \tag{19}$$

The algorithm diagram is described as Fig. 2

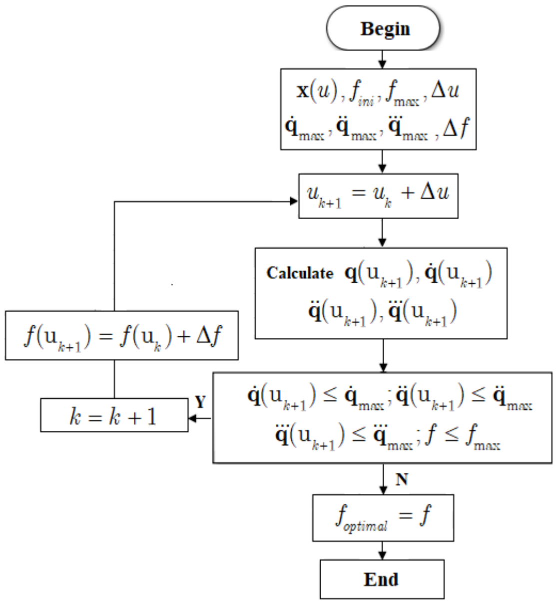

Figure 2. The algorithm diagram for feed rate optimization

IV. Numerical Simulation Results

This section presents the numerical simulation results for welding robot TA 1400 with a complex weld seam. Some parameters of the system can be showed as

$$d_1 = 0.42(m), a_1 = 0.15(m), a_2 = 0.56(m), a_3 = 0.13(m),$$
$$d_4 = 0.6(m), d_6 = 0.325(m), f_{\max} = 1.2(m/s), m_1 = 50(kg),$$
$$m_2 = 39(kg), m_3 = 46(kg), m_4 = 20(kg), m_5 = 3.2(kg),$$
$$m_6 = 2.7(kg), g = 9.81(m/s^2),$$
$$\dot{\mathbf{q}}_{\max} = \begin{bmatrix} 2.97 & 3.32 & 3.32 & 6.64 & 6.54 & 10.5 \end{bmatrix}^T,$$
$$\ddot{\mathbf{q}}_{\max} = \begin{bmatrix} 7.4 & 8.3 & 8.3 & 16.2 & 16.4 & 26.3 \end{bmatrix}^T,$$
$$\dddot{\mathbf{q}}_{\max} = \begin{bmatrix} 37 & 41.5 & 41.5 & 81 & 82 & 131.5 \end{bmatrix}^T,$$

The weld seam is defined as follows

$$x(u) = 0.4 + 0.3(1 + \sin(2u)\cos(u));$$
$$y(u) = 0.4 + 0.3(1 + \sin(2u)\sin(u)); z = 0.8$$

Note that the motion of the robot depends on the kinematics structure and the welding technology parameters such as voltage, welding current and wire output speed. In fact, the weld seam is conducted with feed rate value much smaller than the maximum value. Therefore, based on the actual welding, the maximum speed limit for the algorithm is redefined as follows

$$f_{ini} = 0.01(m/s), \Delta f = 0.005(m/s), f_{\max} = 0.3(m/s),$$
$$\Delta u = 0.01, \mathbf{q}_0 = \begin{bmatrix} 0.78 & 1.34 & 0.23 & 0.15 & 1.22 & 0 \end{bmatrix}^T$$

Fig. 3 describes the numerical calculation steps.

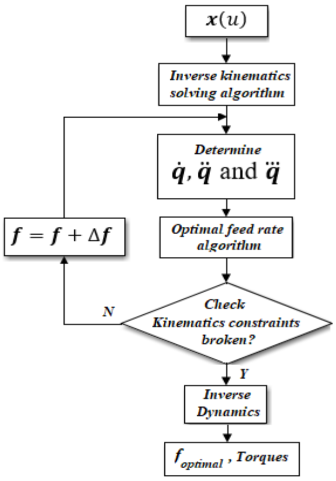

Figure 3. Calculation steps diagram

Fig. 4 and Fig. 5 depict the robot model and the weld seam in the workspace using MATLAB software. Figures from 6 to 9 present the results of the inverse kinematics problem in the parametric domain including position, velocity, acceleration and jerk joints, respectively.

These values are within the allowable kinematics limits and are used for simulation in Fig. 4 and Fig 5. The error values of the end-effector point between the desired and actual weld seam are shown in Fig. 10. The optimal values of feed rate through the optimization algorithm present in Fig. 11. According to the given weld seam profile, feed rate values change accordingly.

Fig. 12 shows the torque values of the joints through solving the inverse dynamics problem. Based on the given weld seam profile and the dynamics parameters of the robot, it was found that joint 2 has the maximum torque value.

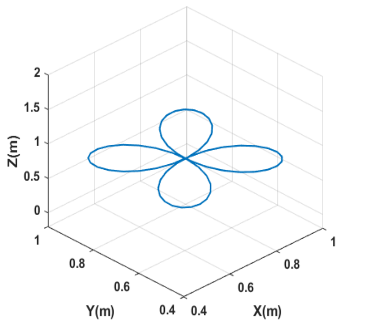

Figure 4. Results simulation the weld seam

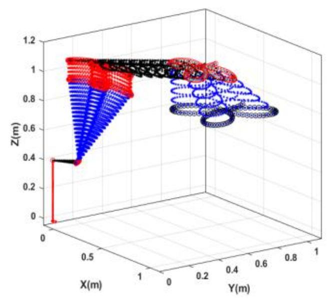

Figure 5. Model of the robot in MATLAB

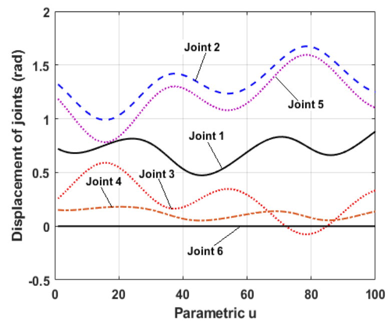

Figure 6. The values of joints position

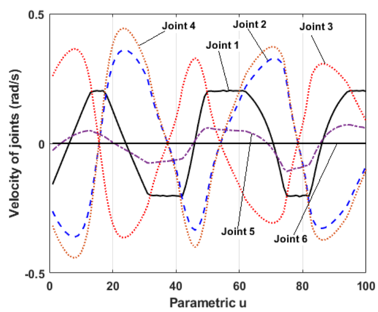

Figure 7. The values of joints velocity

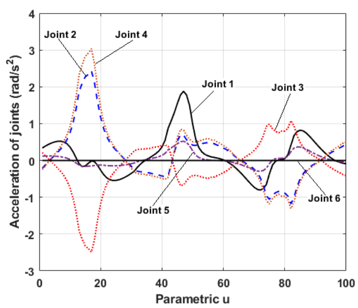

Figure 8. The values of joints acceleration

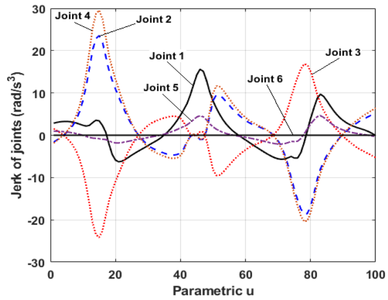

Figure 9. The values of joints jerk

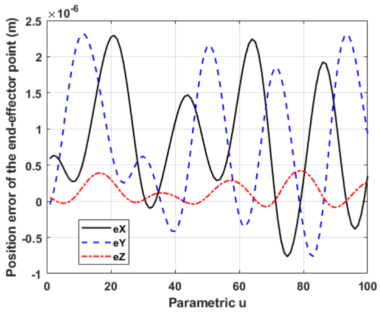

Figure 10. The position error of the end-effector point

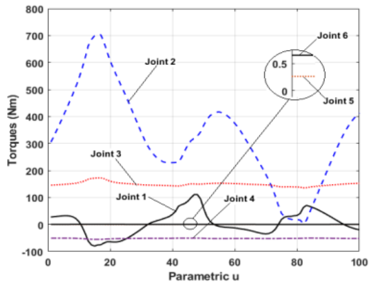

Figure 11. The values of joints torque

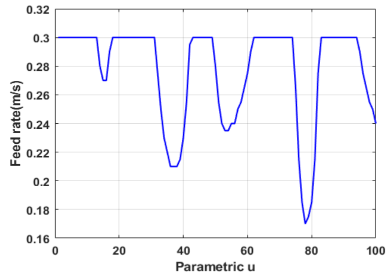

Figure 12. The values of optimal feed rate

V. Conclusions

In general, the optimal feed rate value for each position on the given weld seam for the industrial welding robot TA 1400 has been determined using the kinematics, dynamics modeling and the optimal algorithm in the parameter domain. The algorithm for adjusting the increments of the generalized vector is used to effectively solve the inverse kinematics problem for the redundant system. The position, velocity, acceleration, and jerk of the joints are calculated within the kinematics limits of the robot and ensuring weld seam errors in the workspace. In addition, the torque value on each joint of the robot is also determined by solving the inverse dynamics problem. The results of this study show that the efficiency of the algorithm for adjusting the increments of the generalized vector applies to the redundant system. This optimal algorithm can be applied to the other machining robots such as cutting metal, 3D printing. In addition, the optimal solution is also a method to reduce machining time, improve industrial production efficiency. Experiments supporting the results of this paper will be presented in another article in the near future.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Xuan Bien Duong and Ngoc Anh Mai conducted the research; Anh Tuan Phan, Duy Nhat Do and Xuan Hiep Dang analyzed the data; Xuan Bien Duong and Khanh Nghia Truong wrote the paper; All authors had approved the final version.

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