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Abstract—When performing works on improvement in the city line, a variety of road and construction machines are used, however, in case of inaccurate operation of such machines, previously laid communications can be damaged, which leads to disturbances in the operation of urban systems affecting thousands of people. In this article estimation of the accuracy of the position of the cutting edge of the bucket is given considering the temperature expansion of the links, the variable cutting forces, the delay of the hydraulic drive, gaps and elastic deformations in kinematic pairs. Based on the calculations carried out, recommendations are proposed for developing methods to improve the accuracy of digging.

Index Terms—shovel excavator, mathematical model, control system, service area, accuracy

I. INTRODUCTION

When performing works on improvement in places of historical and cultural value, or when carrying out work in the city, there is a need to perform a significant part of them using construction equipment. If these works are not accurately carried out, it is possible to damage the cultural layer or previously laid communications.

II. LITERATURE REVIEW

One of the most versatile construction machines is a single bucket shovel excavator. The task of investigating the accuracy of production and the search for possible ways to improve accuracy is important for the developers of such machines [1, 2]. According to the building codes and rules in force in Russia, the deviation of the marks when working with a single bucket excavator is allowed not more than 0.05 m [3].

Modern excavators are equipped with a control system that allows you to quickly assess the position of the cutting edge of the bucket, the initial and processed profiles of the ground surface and inform the operator of errors in operation [4, 5]. However, most modern systems operate in automatic mode only on finishing operations, while untimely embedment of the cutting edge is possible not only on the finishing operations, which can lead to the breakage of existing communications.

III. THE MATHEMATICAL MODEL OF THE WORKING MECHANISM OF THE EXCAVATOR

The purpose of this work is to analyze the accuracy of the possible production of excavation works, produced by single-bucket excavators.

For calculations we chose the Russian crawler excavator CHETRA EGP-230 with the "backhoe" equipment (Fig. 1) produced by “CHETRA—Industrial machines” (Cheboksary).

Figure 1. Single-bucket r excavator CHETRA EGP-230

The position of the working mechanism, as well as the initial and processed profiles of the ground surface, are characterized only by the coordinates along the x and y axes, with the rotation of the working mechanism relative to the caterpillar base not considered because at that moment no longitudinal profile of the bottom of the trench. Kinematic diagram of the working mechanism of the caterpillar excavator Chetra EGP-230, is presented in Fig. 2.

Figure 2. Kinematic scheme of the working mechanism, where 1 — boom; 2 — handle; 3 — bucket; 4, 5, 6 — cylinders; 7, 8, 9 — valve cylinders; 10 — rocker; 11 — thrust
The position of the cutting edge of the excavator bucket can be described by the function $f = f(s_1, s_2, s_3)$ of the generalized coordinates of the input action, where $s_1$, $s_2$, $s_3$ — linear coordinates characterizing the displacement of the rods of hydraulic cylinders [3, 6-9].

Thus, the kinematic diagram of the flat working mechanism of the excavator with the number of mobilities $W = 3$ has three generalized coordinates of the input action and three coordinates of the output action and can be conditionally represented as a connected control system (Fig. 3).

The mathematical model of the working mechanism on the basis of structural-kinematic relationships makes it possible to study the characteristics of the kinematic chain of the working mechanism, to evaluate the influence of each of the links on the position of the cutting edge of the bucket, and to estimate the accuracy of the position of the cutting edge as a function of the input generalized coordinates $s_1, s_2, s_3$.

IV. EVALUATION OF ERRORS IN THE MOVEMENT OF THE WORKING MECHANISM

To select the best methods for increasing the accuracy of excavation, the kinematic chain of the working mechanism was modeled taking into account the components of the cutting force and the reactions on the rods of the hydraulic cylinders that affect the accuracy of work and the position of the cutting edge of the bucket of the working mechanism.

A. Errors in the Valve Cylinders

For the compiled mathematical model, errors in the valve cylinders were taken into account, which has a significant effect on the position of the cutting edge of the bucket of the excavator [10]. The distribution of possible errors in the service area is shown on Fig. 4.

![Figure 4. Distribution of possible errors in position of the cutting edge of the bucket in the service area](image)

Taking into account the indicated errors of the position of the hydraulic cylinder rods, we can also evaluate the effect of the generalized coordinates of the input action $s_1, s_2, s_3$ on the coordinates of the output action $x, y, \gamma$ (Fig. 3). To assess the effect of the generalized coordinates $s_1, s_2, s_3$ on $x, y, \gamma$ it is necessary to set the change of all three generalized coordinates $s_1, s_2, s_3$ to each of the three coordinates $x, y, \gamma$ of the output action. This is a very difficult task because it leads to the need to create a four-dimensional diagram, and this presents difficulties not only in construction, but also mainly in the perception of data. In this case, the generalized coordinate $s_1$ is fixed, because this coordinate is farthest from the cutting edge of the bucket 3. The value $s_1 = 500$ mm for all the dependences of the change in the coordinates of the output action. The coordinates $s_2$ and $s_3$ change, varying the coordinate $s_3$ discretely from 0 to 700 mm in increments of 100 mm. The dependences of the change in the coordinates of the output action $x, y, \gamma$ on the generalized coordinates of the input action are presented, shown in Fig. 5.

![Figure 5. The dependence of the change in the coordinates of the output action $x, y, \gamma$ on the generalized coordinates of the input action $s_1, s_2, s_3$ with $s_1 = 500$ mm for a - $x$; b - $y$; c - $\gamma$](image)
B. Errors from the Temperature

In addition, the accuracy of the position of the cutting edge is affected not only by the geometric characteristics of the links, but also by the possible effect of temperature. The use of technology only in Russia suggests a wide range of possible ambient temperatures. To specify the temperature range for the middle zone of Russia: from minus 30 to plus 30°C. However, in the warm season at an ambient temperature of plus 30°C under direct sunlight, additional heating of the links of the working mechanism up to 50°C can occur.

The temperature influence of the environment on the links of the working mechanism affects simultaneously all the links of the kinematic chain, which affects the position of the cutting edge of the bucket. The possible change in the lengths of the links of the working mechanism of the excavator is calculated taking into account the values of the linear coefficient of temperature expansion as a function of temperature.

The trajectory of the cutting edge of the bucket of the working mechanism is simulated in the temperature range from minus 30 to plus 50 °C when the generalized coordinates of the input action $s_2$ and $s_3$ change from the minimum to the maximum values and the constant value $s_1 = 0 \text{ mm}$, the results are shown in Fig. 6.

Changing the lengths of links leads to a positioning error of 15 mm in the vertical coordinate in the presented case.

V. ASSESSMENT OF THE INFLUENCE OF DYNAMIC CHARACTERISTICS ON THE WORKING MECHANISM OF THE EXCAVATOR

To assess the possible errors of digging in the construction of a mathematical model, in addition to the geometry of the kinematic chain, it is necessary to take into account the time dependence of its parameters represented by generalized coordinates that are functions of time, $s_{1,2,3}=f(t)$.

The greatest influence is exerted by dynamic influences from variable cutting forces, which can not be determined in advance. On the entire trench digging process, according to the cyclogram of the working process (Fig. 7), it is possible to distinguish the stages when the process of untimely bucket penetration is possible.

The formation of the longitudinal profile of the bottom of the trench under development takes place at the stage 6, and hence the change in the generalized coordinate $s_3$ can have the greatest effect on the introduced errors in the position of the bottom of the trench. The digging process is treated as a process with one degree of freedom, at each moment only one generalized coordinate changes.

When the generalized coordinate $s_3$ is changed, the remaining generalized coordinates do not change.

A. Assessment of the Effect of Power Loading on the Operation of the Excavator

The digging process, taking into account the simplifications introduced in the calculations, is considered as a process with one degree of freedom, only one generalized coordinate changes at any time. When changing the generalized coordinate $s_3$, the remaining generalized coordinates do not change, which allows us to consider links that are not connected with the generalized coordinate $s_3$ to be conditionally fixed.

In view of the foregoing, the structural part of the mechanism is considered when the generalized coordinate $s_3$ changes, the diagram is shown in Fig. 8.

The mechanism in Fig. 8 is flat, single-moving and has $n=5$ moving links (link 2 is conditionally fixed), $p_1=7$ one-moving kinematic pairs, no two-moving kinematic pairs - $p_2=0$. For a flat mechanism according to the formula P.L. Chebyshev number of mobility $W=1$. The indicated part of the mechanism on the basis of calculations really has one mobility. The movement of this part of the mechanism is completely determined by the change in the generalized coordinate $s_3$, there are no redundant bonds, there are no deformations of the links. To determine the load transmitted to the hydraulic cylinder rod, kinetostatic methods can be used.
The calculation for \( s_3 \) is carried out from the output link 3 - the bucket with the mass concentrated at point \( Q \). External forces act on this link:

- \( G \) - gravity, which is the sum of the gravity of a bucket with soil. Filling the bucket with soil is considered as a transition between discrete states characterized by constant masses of the bucket with soil, defined by the formula:

\[
G = G_0 + \sum \Delta G_i
\]

where \( G_0 \) - own gravity bucket 3; \( \Delta G_i \) is the gravity from the increment in mass of link 3 during soil collection during the transition from one discrete state to another.

- \( P_{cut}^n \) - the normal component of the cutting force from soil destruction in the shear plane is determined depending on soil parameters, bucket width and chip thickness of the layer being removed (it is assumed that the chip thickness is constant during operation).

- \( P_{cut}^t \) - tangential component of the cutting force, which is actually a component of the friction force along the rear surface of the bucket. It is associated with the normal component.

- \( \Phi_3 \) - the inertial force of link 3 is determined through the mass of link 3 and the moment of inertia, and moment of inertia, and the moment of rotation relative to the point \( K \), that is, as a rod with a mass concentrated at the point \( Q \). It is considered as \( M^*_{S3} = -J_{S3} \cdot \eta_3 \).

The design scheme for determining the reactions in the kinematic pairs of link 3 is shown in Fig. 9.

To determine the shoulders of forces, write equations and their subsequent solution, all equations are given in projection on the \( x \) and \( y \) axis. Link 3 has four unknowns: \( R_{Ts}^0 \), \( R_{Ts}^0 \), \( R_{Ts}^0 \), \( R_{Ts}^0 \). For link 3, a system of three equations (3) is compiled, which allows one to determine unknown quantities.

\[
\begin{align*}
\sum F_x^3 &= P_{cut x}^i + P_{cut x}^n + \Phi_{3x} + R_{Ts}^3 + R_{Tk}^3 - 0; \\
\sum F_y^3 &= P_{cut y}^i + P_{cut y}^n + \Phi_{3y} + G + R_{Tk}^3 + R_{Tk}^3 = 0.
\end{align*}
\]

From the system of equations (3), all unknown components are determined \( R_{Ts}^0 \), \( R_{Ts}^0 \), \( R_{Ts}^0 \), \( R_{Tk}^3 \). The solution for link 11 is not given, by virtue of the above reasoning \( R_{Ts}^0 = -R_{Tk}^3 \), and in projections \( R_{Ts}^0 = -R_{Tk}^3 \), \( R_{Ts}^0 = -R_{Tk}^3 \). The reaction along the link 9 - the rod 9 of the hydraulic cylinder 6 - to determine the excess movement of the rod of the hydraulic cylinder must be projected onto the direction of the generalized coordinate \( s_3 \) and determined by the formula:

\[
R_{Ts}^0 = \frac{R_{Ts}^0}{\sin(NTy)}
\]

For further calculations, one should also consider the equilibrium of units 6 and 9 in order to determine the reactions in pair \( N \). The forces and moments acting in this pair of units are determined as well as for unit 3. The calculation scheme for determining the reactions in the kinematic pairs of units 6 and 9 is shown in Fig. 10.
Link group 6-9 has two unknowns: $R_{6N}^6$, $R_{6N}^9$. For links 6–9, a system of two equations (5) is compiled, which allows one to determine unknown quantities.

$$\sum M_T^{6,9} = -M_{\phi_{60}}^0 + T N_x^6 \cdot R_{6x}^6 + T N_y^6 \cdot R_{6y}^6 = 0;$$

$$\sum F_s^{6,9} = R_{6N}^6 + \Phi_{x} + R_{6x}^6 = 0.$$  \hspace{1cm} (5)

From the system of equations (5) unknown components are determined $R_{6N}^6$, $R_{6N}^9$, they will be needed for further calculations when changing the generalized coordinate $s_2$.

**B. Calculation of Errors in the Movement of the Working Body under Power Loading**

The calculation of dynamic characteristics allows us to estimate the possible deviation of the input generalized coordinate $s_3$, which is determined by the displacement of the rod of the hydraulic cylinder of the working mechanism, from the theoretical value specified by the control action. The calculation is performed at values of the generalized coordinates $s_1 = 0$ m, $s_2 = 0$ m, the results are shown in Fig. 11.

![Figure 11](image)

Figure 11. Deviations of the positions of the generalized coordinate $s_3$ with the dynamic characteristics of the working mechanism of the theoretical values $s_3^t$.

In addition, taking into account the indicated errors in the position of the rod of the bucket hydraulic cylinder (generalized coordinate $s_3$), we can also evaluate the effect of the generalized coordinate of the input action $s_3$ on the coordinates of the output action $x$, $y$, $\gamma$.

In the course of further calculations, the influence of the deviations of the positions of the generalized coordinate $s_3$ in time shown in Fig. 7 taking into account the dynamic characteristics of the working mechanism on the values of the output coordinates $x$, $y$, $\gamma$ of the working mechanism is studied, the results are shown in Fig. 12. Theoretical (calculated) values for the coordinates of the output are shown $x$, $y$, $\gamma$, the theoretical values of the coordinates of the output effect in the figures are indicated by “t”, and the coordinates, taking into account the dynamic characteristics, are indicated by “d”.

![Figure 12](image)

Figure 12. Deviations of the output coordinates $x$, $y$, $\gamma$ depending on the change in time for theoretical values and taking into account the dynamic characteristics for $a$ - $x$; $b$ - $y$; $c$ - $\gamma$.

Calculations were made for a fixed value of the cutting resistivity, which even for one category of soil can fluctuate significantly, which can lead to unpredictable deviations in the position of the cutting edge of the bucket from the required position. The random component of the cutting force $P_{cut}$ due to a change in the resistivity to cutting leads to an additional deviation of the generalized coordinate $s_3$. The simulation results are shown in Fig. 13 for the mean values of the generalized coordinate $s_3$. 

When dynamic characteristics of the working mechanism are taken into account, the difference between the set and received positions is 89.2 mm at the end of stage 6 of the work cycle, and when the random component of the cutting force is taken into account, the difference is 92.5 mm.

VI. ERRORS FROM THE HYDRAULIC DRIVE LAG, GAPS AND ELASTIC DEFORMATIONS IN KINEMATIC PAIRS

The accuracy of production is also affected by hydraulic drive lag, gaps and elastic deformations in kinematic pairs. The parameters presented in the literature, obtained experimentally, can be used to calculate the errors in the displacement of the working member with allowance for retardation, gaps in the movable joints in the kinematic pairs of the working mechanism. The method of accounting for the gaps was proposed by N. Alaadin [11].

In real rotational kinematic pairs, there is a gap between the elements of the pairs, which is the cause of the displacement of one link relative to another under the action of the load, and the introduction of the gap is considered as additional mobility. But in the ideal model of a kinematic pair, the origin of the coordinate systems associated with two adjacent links coincide with each other and with the ideal center of the kinematic pair. At the same time, there can be several options for possible displacements in the rotational kinematic pair, which can lead to different positioning errors. For a real kinematic pair, in which the gap is much smaller than the dimensions of the axis and the sleeve, the axis of the kinematic pair of diameter \( d \) can be replaced by a segment, in this case, the diameter of the hole in the sleeve will be equal to the double gap \( \Delta \) between the axis and the sleeve when they are coaxial. The gap pattern in the kinematic pair is shown in Fig. 14.

Calculations show that in kinematic pairs the error in determining the angle of rotation of the axis due to replacement of the gap in a real kinematic pair by the design scheme is very small and does not exceed 0.06%.

In this case, the direction of the relative displacement of the links of the rotational kinematic pair, which arises due to the presence of gaps in the pair, coincides with the direction of the reaction in the kinematic pair.

For different positions of the kinematic chain of the working mechanism, on the average, the positioning error along the vertical coordinate in the presented case is from minus 13 to plus 15 mm. In addition, it was indicated [11] that the elastic deformations of the links also lead to positioning errors, and the resulting deviations due to the presence of gaps and elastic deformations are approximately the same.

VII. TOTAL ERROR FROM THE WORKING MECHANISM

The total error in the position of the cutting edge of the bucket of the working mechanism can be determined as the sum of the errors:

\[
\Delta_{\text{wm}} = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5, \tag{6}
\]

where \( \Delta_{\text{wm}} \) – error of working mechanisms; \( \Delta_1 \) – dynamic component of error; \( \Delta_2 \) – hydraulic lag error; \( \Delta_3 \) – error of gaps in kinematic pairs; \( \Delta_4 \) – error of elastic deformations in kinematic pairs; \( \Delta_5 \) – technological component of error.

Taking into account the foregoing, the total error in the position of the cutting edge of the bucket of the working mechanism is 171.9 mm, which exceeds the required value of 50 mm by 3.4 times.

VIII. DISCUSSION

To assess the maximum error in the position of the cutting edge of the bucket of the excavator's working mechanism, the maximum values of the parameters were taken to estimate the greatest damage due to the submission of untimely signals about the deepening or burrowing of the bucket, which leads to the destruction of the cultural layer, as well as to the breakage of
communications and the need for finishing works on leveling the bottom of the trench. To solve this problem, it is necessary to develop a methodology for making changes in the control system of the working mechanism of the excavator [12-14].

In other words, the correspondence of all links to technical requirements does not guarantee compliance with the required accuracy of the displacement of the output link of the kinematic chain.

**IX. CONCLUSION**

The conducted researches allowed to draw the following conclusions:

- The constructed mathematical model allows to take into account accuracy issues;
- The working mechanism of the excavator has insufficient accuracy;
- Requirements for the management system have been formed.

To dramatically increase the accuracy of excavators working processes it is needed to consider the characteristics of the energy sources, gears and transmissions and properties of equipment which is in usage in complex with excavators. This evaluations may be made on base of works [15-20].

**CONFLICT OF INTEREST**

Author declares no conflict of interest.

**REFERENCES**


