Differential Flatness Based LQR Control of a Magnetorheological Damper in a Quarter Car Semi-active Suspension System

Aneesh D. Diwakar and P. V. Manivannan
Mechanical Engineering Department, Indian Institute of Technology, Madras, Chennai, India
Email: aneeshdinesh7794@gmail.com, pvm@iitm.ac.in

***Abstract*——Semi-active suspension is widely used in automotive applications, as it is providing good ride quality at reasonably low cost. Among the various methods available for enhancing ride comfort, magnetorheological (MR) suspension has drawn much attention due to its robustness and fail-safe nature as compared to electrorheological suspension. In semi-active suspension, the damping coefficient of the MR damper is varied by modulating the current supplied to damper coil. In the present work, a quarter car with semi-active suspension system is modeled and its performance is studied through simulation under the MATLAB / Simulink environment. A hybrid control algorithm consisting of differential flatness along with LQR (linear quadratic regulator) is proposed to vary the current input for the magnetorheological damper for reducing the sprung mass acceleration, jerk and to improve ride quality. The simulation results also show that the performance of semi-active suspension that uses magnetorheological damper along with developed hybrid algorithm is superior to the LQR controlled suspension, when subjected to random road profiles.**

***Index Terms*—Magnetorheological, Differential flatness, LQR, semi-active suspension, ride comfort**

I. Introduction

Magnetorheological (MR) dampers (semi-active devices) have been successful in generating considerable interest in the last few decades. These are hydraulic dampers comprising of oil with metallic particles (MR fluid) and belong to the category of smart materials. The properties of MR fluid can be controlled according to the supplied electric current. When subjected to an external magnetic field, MR fluid is capable of changing its state from viscous to a semi-solid with controllable yield strength. In general, the vehicle suspensions are categorized as passive, active and semi-active, depending on the ability to adjust the damping force characteristics. Passive suspension can only store and dissipate the energy and their damping characteristics remain constant with time. However, the active suspension has additional actuator to generate force to compensate the ground reaction force. Despite providing good ride quality, the active suspension system has some disadvantages. Due to its high cost, complex design and high-power demand, the active suspension is less common and, therefore has very low usage rate in actual vehicles.

An intermediate technology between active and passive suspension system is a semi-active suspension. To vary the damping force in a semi-active suspension, a small amount of energy needs to be supplied to the damper (i.e. current to needs to be supplied to MR damper coil), which is very less compared to high power requirement of actuator of active suspension. In case the control system malfunctions, the semi-active suspension still acts like a passive one i.e. it is fail-safe [1]. For these reasons, the semi-active suspension dampers are a feasible solution to control the vertical vehicle dynamics with lower power consumption and reasonable cost [2]. Mathematical formulation of MR dampers is challenging, especially due to its nonlinear dynamics (typically hysteresis). Development of control strategies for vibration mitigation purposes through MR dampers becomes complex [3].

A number of control methodologies have been designed and implemented owing to the increased interest in the control of semi-active suspension systems. Mansor et. al. [4] developed a controller for MR semi-active suspension system. It used a non-parametric linearized data driven (NPLDD) model along with proportional integral (PI) as the inner loop which tracked the required force in combination with an improved PI controller in the outer loop for improving the vehicle response by minimizing the vehicle body displacement and acceleration. Raczka [5] used a weighted multitone optimal controller (WMOC) in order to reduce the vibration level in a vehicle. The sinusoidal disturbance vectors were utilized for evaluating the control signal. A mixed Skyhook-Power Driven Damper (SH-PDD) was proposed by Liu [6]. A switching law was defined by this algorithm which mixes SH and PDD, thereby carrying their advantages like minimizing the high jerk behaviors to improve the suspension performance. Pepe [7] carried out a deep investigation on nonlinear suspension system control by proposing to adopt variational feedback control (VFC). VFC reformulates the variational optimal control theory and can be applied to a wider spectrum of dynamical systems.

Manuscript received September 7, 2019; revised April 11, 2020.

doi: 10.18178/ijmerr.9.7.998-1006

Matt Hussin [8] used the fuzzy-skyhook controller on the MR damper, which was modelled using Spencers approach. Cuckoo search algorithm (CSA) was used for fine tuning the controller by optimizing the gain parameters. Ahmed Hafizal [9] proposed a Skyhook controller in combination with Particle swarm optimization (PSO) for a MR suspension system. The nonlinear optimization problem of finding the Skyhook controller gains was solved using the PSO algorithm. The controller was able to determine the desired damping force. Ahmed O. [10] proposed a hybrid-fuzzy and fuzzy-PID controller (HFFPID) for a semi-active suspension with a MR shock absorber. It comprised of a fuzzy self-tuned proportional integral derivative, fuzzy logic controller and a fuzzy selector.

The property of differential flatness is exhibited by many dynamical systems. It is equivalent to controllability property. A set of independent outputs, called flat outputs (Fliess et al. [11] and Sira-Ramirez et al. [12]) is used to completely parametrize every state variable and control input. Chavez et.al [13] combined sliding mode and differential flatness control techniques. In order to attenuate the vibration in a quarter car active suspension system the computed values of the vertical positions of the tire and car body were used.

In this paper, it is proposed to use a hybrid control algorithm consisting of differential flatness along with linear quadratic regulator (LQR) to reduce sprung mass acceleration and improve ride comfort. The differential flatness acts like the feedforward compensator of the controller, whereas LQR acts like the full state feedback controller. Numerical simulations are used to verify the dynamic system model and the hybrid controller performance under different grades of irregular road profiles

II. Modeling and Simulation

A. Quarter Car Dynamic Model

A schematic diagram of the quarter car model of a semi-active suspension system is shown in the Fig.1. In this model it is assumed that the load is distributed among the four wheels. In this model, the longitudinal and lateral dynamics of the wheel is not taken into the account. The governing equations of the vertical dynamics of the model is represented in (1) and (2) below:

$$m_s\ddot{x}_S + k_s\,(x_s - x_{us}) + F_D = 0 \tag{1}$$

$$m_{us}\ddot{x}_{us} + k_s\,(x_{us} - x_s) + k_t\,(x_{us} - x_r) - F_D = 0 \tag{2}$$

In the above model, a sprung mass (m_s) and an unsprung mass (m_{us}) is considered. The magnetorheological (MR) damper having damping constant b_1 and the spring with stiffness coefficient k_s represent the suspension between both masses. The damping force F_D of the MR damper depends on the electric current (controlled input) supplied and it is shows significant nonlinearity with respect to the motion of the suspension. The term k_t represents the stiffness coefficient of the tire, while x_s and x_{us} represent the vertical positions of the sprung and unsprung mass respectively. The unknown road disturbance is represented as x_r. The key element in the semi-active suspension system is the MR damper and it is important to model its nonlinearities and properties of its actuation with good accuracy, in order to design model-based controllers. As mentioned earlier, the damping force generated varies with the current i(t) supplied to the damper. The functioning of the damper is inspired from the parametric model of Guo [14] et.al and is represented by (3).

$$F_D = i(t) f_c\, \rho(t) + b_1\, \dot{x}_{def}(t) + b_2 x_{def}(t) \tag{3}$$

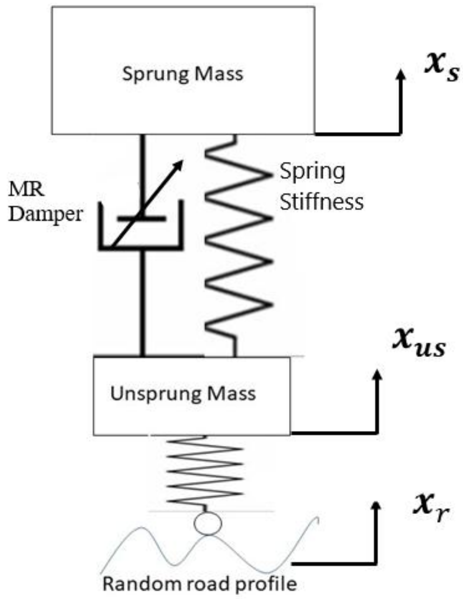

Figure 1. Quarter Car Semi-Active Suspension System

Here, fc is the dynamic yield force of Guo model. The term ρ(t) given by (4) represents the non-linearities in the shock absorber. The terms b_1 and b_2 are the viscous damping coefficient and stiffness coefficient of the Guo model respectively, whereas a_1 and a_2 represent the preyield viscous damping coefficient of Guo model. The suspension deflection x_{def}(t) is the difference in positions of the sprung and unsprung masses.

$$\rho(t) = \tanh[\, a_1\dot{x}_{def} + a_2 x_{def}] \tag{4}$$

The state space model for the quarter car is obtained by substituting (4) in (1) and (2). Defining the state variables as $x_1 = x_s$, $x_2 = \dot{x}_S$, $x_3 = x_{us}$ and $x_4 = \dot{x}_{us}$, the representation in the state space form is:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ez(t); \tag{5}$$

where $x(t) \epsilon R^{4X1}$, $A \epsilon R^{4X4}$, $E \epsilon R^{4X1}$, $B \epsilon R^{4X1}$ and the matrices are given below with $h = k_s + b_2$.

$$\begin{bmatrix} \dot{x_s} \\ \ddot{x_s} \\ \dot{x_{us}} \\ \ddot{x_{us}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -h/m_s & -b_{1/m_s} & h/m_s & b_{1/ms} \\ 0 & 0 & 0 & 1 \\ h/m_{us} & b_{1/m_{us}} & (-h+k_t)_{/m_{us}} & -b_{1/m_{us}} \end{bmatrix} \begin{bmatrix} x_s \\ \dot{x_s} \\ x_{us} \\ \dot{x_{us}} \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ -\rho(t)f_c/m_s \\ 0 \\ \rho(t)f_c/m_{us} \end{bmatrix} [i(t)] + \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_t/m_{us} \end{bmatrix} [x_r] \qquad (6)$$

B. Road Profile Generation

Road profiles are generated according to the ISO 8608 Standard consisting of various road roughness profiles characterized by different damage levels. This method does not produce a particular roadway, but it produces roadways that are sufficient for majority of the vehicle simulation purposes. A PSD function is used to represent the road profiles. The surface profile is measured with respect to a datum plane in order to compute the power spectral density function (PSD). Random road profiles are approximated by a PSD as:

$$G_d(\Omega) = G_d(\Omega_o).\,(\Omega/\Omega_0)^{-w} \qquad (7)$$

where, $\Omega = \frac{2\pi}{L}$ is in rad/m denotes the angular spatial frequency and L denotes the wavelength. Here Ω_o is reference wave number and takes the value 1 rad/m and w is the waviness index and its value is two for most of the road surfaces. The road surface is classified [15] based on the degree of roughness and the specifications are mentioned in Table I below.

TABLE I. ISO 8608 CLASSIFICATION OF ROAD ROUGHNESS

Road Class	Degree of Roughness $G_d(\Omega_o)$ $(10^{-6}\ m^3)$		
	Lower limit	Geometric mean	Upper limit
A (very good)	-		2
B (good)	2	4	8
C (average)	8	16	32
D (poor)	32	64	128
E (very poor)	128	256	512

The above road profiles are generated in MATLAB and is considered as the external disturbance for the suspension system. The different grades of road profile are shown in Fig. 2. The numerical values of the properties of the vehicle suspension system used for simulation are shown in Table II below.

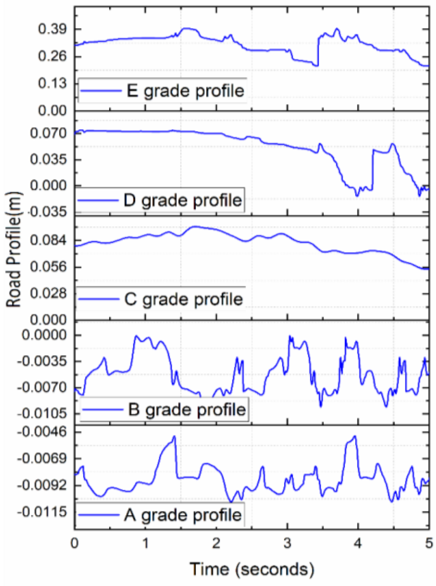

Figure 2. Random road profiles

TABLE II. VEHICLE SUSPENSION PROPERTIES FOR QUARTER CAR MODEL

Parameter	Value
Sprung mass (m_s)	630 [kg]
Unsprung mass (m_{us})	81.5[kg]
Spring stiffness (k_s)	43500[N/m]
Tire Stiffness (k_t)	230000[N/m]
Viscous Damping coefficient of Guo Model (b_1)	2830 [Ns/m]
Stiffness coefficient of Guo Model (b_2)	7897.2 [N/m]
Pre-yield viscous damping coefficient of Guo Model (a_1)	37.8 [s/m]
Pre-yield viscous damping coefficient of Guo Model (a_2)	22.1 [1/m]
Dynamic yield force of Guo model (f_c)	600.9 [N]

III. CONTROL OF SEMI-ACTIVE SUSPENSION SYSTEM

A. Differential Flatness Control

The flat output (FO) for a suspension system is given in terms of the displacements of the sprung and unsprung masses. For the system described in (5) and (6), the differential parameterization of all the state variables and control input are shown below. The derivatives of FO with respect to time up to the fourth order are formulated. We have assumed that $k_t x_r$ =0, thereby easing the analysis of the differential flatness for the suspension system.

$$FO = m_s\, x_1 + m_{us}\, x_3 \qquad (8)$$

$$FO^{(1)} = m_s x_2 + m_{us} x_4 \qquad (9)$$

$$FO^{(2)} = -k_t x_3 \qquad (10)$$

$$FO^{(3)} = -k_t x_4 \qquad (11)$$

$$FO^{(4)}=\frac{-k_s k_t}{m_{us}}(x_1-x_3)-\frac{k_t b_1}{m_{us}}(x_2-x_3)+\frac{{k_t}^2}{m_{us}}x_3+\frac{\rho f_c k_t}{m_{us}}\mathrm{i(t)} \tag{12}$$

The parametrized forms of the state variables and the control input in terms of the flat output are as follows:

$$x_1=\frac{1}{m_s}\left(FO+\frac{m_{us}}{k_t}FO^{(2)}\right) \tag{13}$$

$$x_2=\frac{1}{m_s}\left(FO^{(1)}+\frac{m_{us}}{k_t}FO^{(3)}\right) \tag{14}$$

$$x_3=\frac{-FO^{(2)}}{k_t} \tag{15}$$

$$x_4=\frac{-FO^{(3)}}{k_t} \tag{16}$$

$$i(\mathrm{t})=\frac{m_{us}}{\rho f_c k_t}FO^{(4)}+\frac{b_1 m^2{}_{us}}{\rho f_c k_t}FO^{(3)}+\left(\frac{1}{\rho f_c}+\frac{k_s m_{us}}{\rho f_c k_t m_s}-\frac{k_s}{\rho f_c k_t}-\frac{b_1}{\rho f_c k_t}\right)FO^{(2)}+\frac{b_1}{\rho f_c m_s}FO^{(1)}+\frac{k_s}{\rho f_c m_s}FO \tag{17}$$

B. Linear Quadratic Regulator

The LQR approach is a long-established and widely used control system in vehicle suspension system. The suspension properties are able to adapt to changes within a cycle of vibration in a semi-active system. LQR controller is able to attain both ride comfort and road holding refinements through the semi-active suspension system. The LQR has the advantage that the designed system will be stable as long as the system is controllable [16]. LQR is a state feedback regulator and its output is represented by (18) and K is the state feedback gain matrix. LQR is an optimal controller, which computes the control input u (current i(t) in this case) to the plant. LQR optimization consists of minimizing the cost function. The cost function describes the performance attribute requirement, as well as the limitation to the controller input. It calculates Q and R which are the weight matrices for states and inputs, respectively. For minimizing the cost function, the Riccati equation needs to be solved in order to achieve a suitable controller. The Riccati equation returns a matrix which is a unique positive definite solution.

$$u=-Kx(t) \tag{18}$$

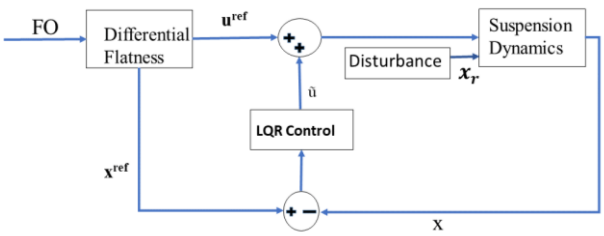

Figure 3. Hybrid control algorithm

C. Hybrid Control Algorithm

A hybrid control algorithm consisting of differential flatness along with linear quadratic regulator (LQR) is proposed in the current research work. The differential flatness acts like the feedforward compensator of the controller, whereas LQR acts like the full state feedback controller as shown in Fig.3. The objective of the controller is to minimize the sprung mass acceleration, when the vehicle is subjected to external road disturbances, thus ensuring ride comfort. This objective is met by controlling the electric current in the MR damper, which provides the damping force. Numerical simulations are used to verify the dynamic plant model and hybrid controller, under different grade of roads having irregular profiles.

The control system requires the measurements of the vertical displacements of the car and tires. Based on the formulation of the flatness based control, (17) can be written as:

$$\mathrm{i(t)}=\frac{1}{g}\left(FO^{(4)}+g_3 FO^{(3)}+g_2 FO^{(2)}+g_1 FO^{(1)}+g_0 FO\right) \tag{19}$$

where,

$$g_0=\frac{k_s k_t}{m_s m_{us}}$$

$$g_1=\frac{k_t b_1}{m_s m_{us}}$$

$$g_2=\frac{k_t}{m_{us}}+\frac{k_s}{m_s}-\frac{k_s}{m_{us}}-\frac{b_1}{m_{us}}$$

$$g_3=m_{us}b_1$$

$$g=\frac{\rho f_c k_t}{m_{us}}$$

We can observe from (19) that following input-output differential equation is satisfied by the flat output:

$$FO^{(4)}+g_3 FO^{(3)}+g_2 FO^{(2)}+g_1 FO^{(1)}+g_0 FO=g\,\mathrm{i(t)} \tag{20}$$

A fourth order linear system can then be used to describe the dynamics of the flat output:

$$\beta_1=FO \tag{21a}$$
$$\dot{\beta}_1=\beta_2 \tag{21b}$$
$$\dot{\beta}_2=\beta_3 \tag{21c}$$
$$\dot{\beta}_3=\beta_4 \tag{21d}$$

$$\dot{\beta}_4=-g_0\beta_1-g_1\beta_2-g_2\beta_3-g_3\beta_4+g\,\mathrm{i(t)} \tag{21e}$$

Let $FO^{(4)}=\mathrm{v}$ be the auxiliary control input. Then, the differential flatness based controller can be obtained from (20) as:

$$i(t)=\frac{v}{g}+\frac{1}{g}\left(g_0\beta_1+g_1\beta_2+g_2\beta_3+g_3\beta_4\right)$$

with:

$$v=-\alpha_3\beta_4-\alpha_2\beta_3-\alpha_1\beta_2-\alpha_0\beta_1$$

The closed loop dynamics of the controller is as follows:

$$FO^{(4)}+\alpha_3 FO^{(3)}+\alpha_2 FO^{(2)}+\alpha_1 FO^{(1)} +\alpha_0 FO=0 \tag{22}$$

We then get the closed loop characteristic polynomial as:

$$p(s)=s^4+\alpha_3 s^3+\alpha_2 s^2+\alpha_1 s+\alpha_0 \tag{23}$$

The design $\alpha_3, \alpha_2, \alpha_1$ and $\alpha_0$are chosen such that the correlated characteristic polynomial is Hurwitz. In order to get the corresponding design gains the following Hurwitz polynomial is used:

$$p(s)=(s^2+2\xi\,\omega_n s+\omega_n^2)^2 \tag{24}$$

where, ω_n is the natural frequency and ξ is the damping factor of the closed loop dynamics. Comparing the coefficients of (23) and (24), we get,

$$\alpha_0=\omega_n^4$$
$$\alpha_1=4\,\omega_n^3+\xi$$
$$\alpha_2=4\,\omega_n^2\xi^2+2\,\omega_n^2$$
$$\alpha_3=4\,\omega_n\,\xi$$

Linear Quadratic Regulator (LQR) is implemented using MATLAB, its output is represented by (18). The design of the Q and R matrices depend on the plant under control and are chosen to be diagonal matrices as a rule of thumb. They are chosen based on trial and error method through simulation to provide optimal control performance and chosen matrices for this work are given by (25) and (26).

$$Q=\begin{bmatrix}2&0&0&0\\0&100&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{25}$$

$$R=0.7 \tag{26}$$

Once the solution P of the Riccati equation is achieved, the gain K is calculated as given below:

$$K=R^{-1}B^TP \tag{27}$$

IV. Results and Discussion

The primary objective of designing the controller is to ensure ride comfort to the passenger. The comfort level can be maximized by minimizing the vertical body displacement, acceleration and jerk of the vehicle. The initial simulation for testing the algorithm was performed for a standard bump input and sprung mass displacements and vertical accelerations are observed. The simulation is performed in the MATLAB/Simulink environment. The bump input given as the external disturbance is represented by (28):

$$z=\frac{z_m\,(1-\cos 8\pi t)}{2} \tag{28}$$

Here, z is the road disturbance and z_m is a constant which takes values as shown in Table III below. The bump signal input generated by the above function is shown in the Fig. 4. The key parameters that affect the ride comfort are minimum body (sprung mass) displacement and acceleration. The plots of the sprung mass displacement and sprung mass acceleration are shown in the Fig. 5 and Fig.6 respectively.

TABLE III. VALUE OF z_m

Time(t) in seconds	Value of z_m
$0.5 \le t \le 0.75$	0.11 [m]
$3.0 \le t \le 3.25$	0.055 [m]
Otherwise	0[m]

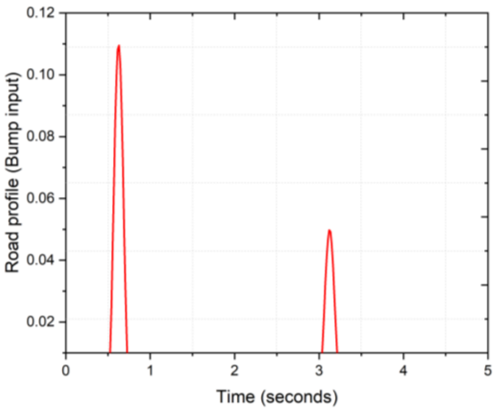

Figure 4. Road disturbance (Bump input)

From the results, we can see that the hybrid controller has better performance as compared with the LQR controller. This hybrid controller has lower settling time and overshoot. The settling time for the sprung mass displacement is around 0.25 seconds once it traverses over the bump.

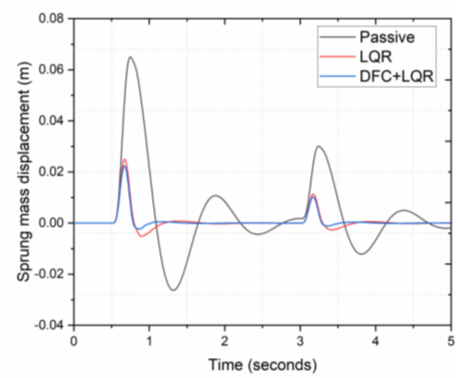

Figure 5. Sprung mass displacement

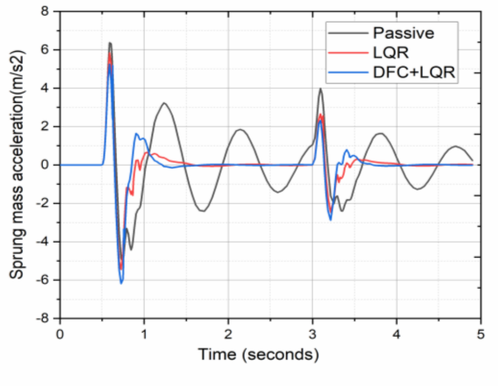

Figure 6. Sprung mass acceleration

Subsequent simulations are carried by inputting the disturbance in the form of different grades of random road profiles (ranging from Class A to Class E). The simulation is performed in the MATLAB/Simulink environment and is carried out on a vehicle travelling speed of 60km/h. For evaluating the performance of the hybrid control algorithm i.e. differential flatness-based control (DFC) along with linear quadratic regulator (LQR), the simulations were conducted with LQR controller alone and also with passive suspension as baseline system.

For simulation in MATLAB/Simulink, time step used is variable step type and the solver used is ode45. Fig.8 to Fig.12 shows the sprung mass responses like displacement and acceleration for various grade of roads, ranging from class A (very good) to class E (very poor). It may be noted that the sprung mass displacement and acceleration is minimum with hybrid controller; hence ensuring a smooth ride. The RMS error of sprung mass acceleration is minimum.

Acceleration is the rate of change of velocity and is an unwanted vibration for the vehicle. Minimum vertical displacement and acceleration is achieved by controlling the input current in the magnetorheological damper. The average of the control parameter (current) over time is the control effort and it is found that the control effort required for the hybrid controller is less as compared to LQR controller. Also, the Root Mean Square (RMS) of the errors in the desired states is calculated for each road profile. From the results presented in Table IV, it is observed that the differential flatness based LQR controller is able to reduce sprung mass vertical acceleration, thereby its displacement (having minimum error in the desired states) and is very stable. The road profiles used for testing the performance of the controller is shown in Fig.7 below.

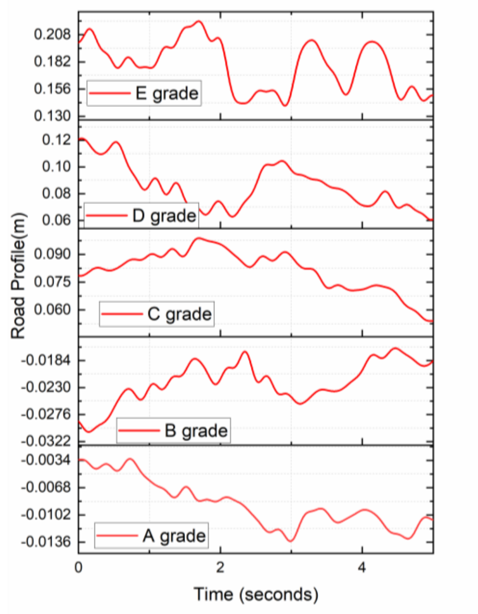

Figure 7. Random road profiles (Testing purpose)

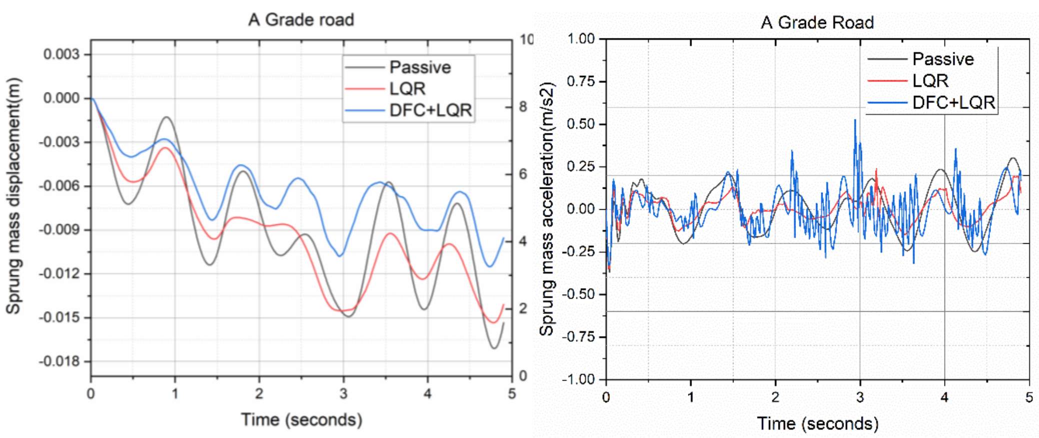

Figure 8. Sprung mass response for class A road

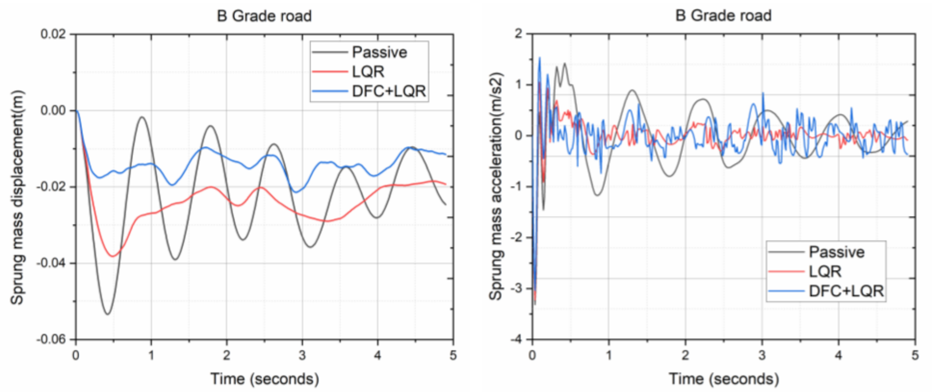

Figure 9. Sprung mass response for class B road

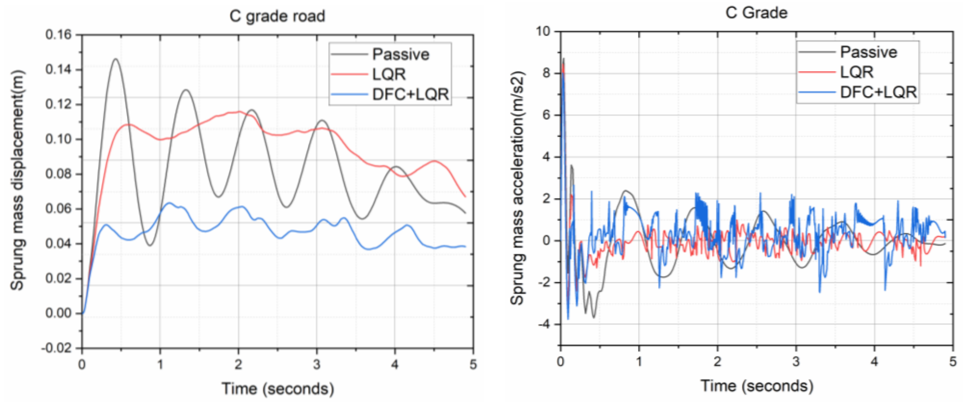

Figure 10. Sprung mass response for class C road

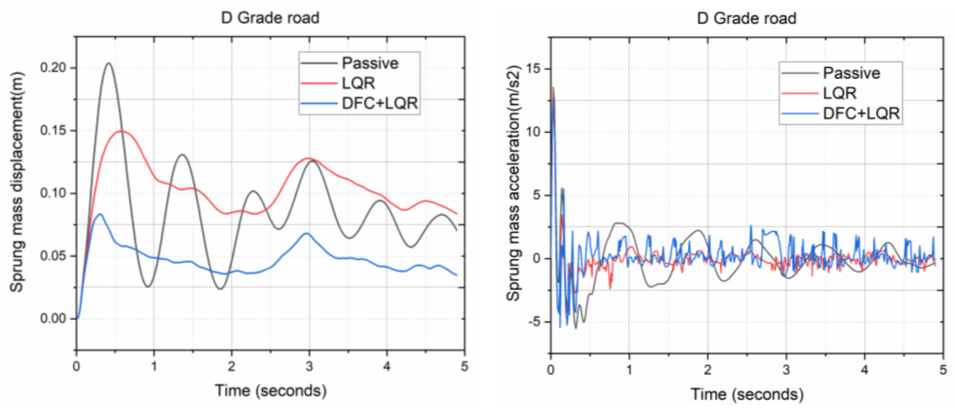

Figure 11. Sprung mass response for class D road

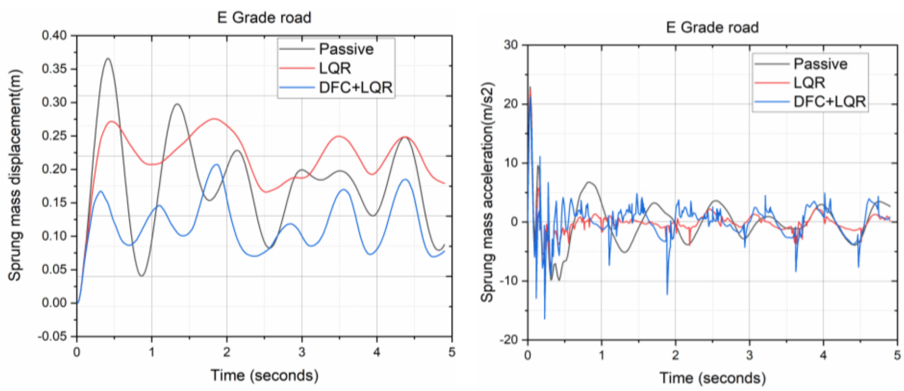

Figure 12. Sprung mass response for class E road

TABLE IV. COMPARISON OF RMS STATE ERROR AND AVERAGE CONTROL EFFORT (CURRENT)

Road Class (ISO 8608)	LQR		DFC+LQR	
	Average Control Effort (ampere)	RMS State Error (Acceleration)	Average Control Effort(ampere)	RMS State Error (acceleration)
A(very good)	0.154	0.137	0.138	0.08
B(good)	0.074	0.383	0.061	0.33
C(average)	1.80	1.052	0.94	0.64
D(poor)	2.35	1.682	2.22	1.20
E(very poor)	2.69	3.88	2.46	2.82

V. CONCLUSION AND FUTURE SCOPE

A hybrid control algorithm consisting of differential flatness and linear quadratic regulator was proposed for minimizing the vertical displacement and acceleration of the sprung mass. The performance of the algorithm was verified using simulations in MATLAB/Simulink environment. It can be concluded that this algorithm was successful in achieving its goal of minimizing vertical displacement and acceleration when the vehicle was subjected to disturbances in the form of different grades of random road profiles. The performance of the algorithm was also compared with LQR controller alone and it was found to perform better as well as display better stability.

This algorithm can also be implemented on half-car and full-car models having greater number of degrees of freedom. The proposed hybrid algorithm can also be adopted for optimizing the suspension of vehicles like military tracked vehicles and other all-terrain vehicles.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

The research work is carried out by Aneesh Dinesh Diwakar under the guidance and support of P.V. Manivannan. Both the authors have equally contributed for this paper. While the simulation, collection of data and writing the paper part is done by Aneesh Dinesh Diwakar evaluation, analysis and review is done by P.V. Manivannan. Both authors have approved the final version.

REFERENCES

[1] W. G. Ata and A. M. Salem, “Semi-active control of tracked vehicle suspension incorporating magnetorheological dampers,” *Vehicle System Dynamics*, vol. 55, no. 5, pp. 626-647, 2017.

[2] B. T. Fijalkowski, *Automotive Mechatronics: Operational and Practical Issues*, vol. 2, Springer, 2011.

[3] J. D. Carlson, "6.6 magnetorheological fluid Actuators," *Adaptronics and Smart Structures: Basics, Materials, Design, and Applications,* 2013, 1808.

[4] M. S. F. Mansor, et al., "Controller development for magneto-rheological semi-active suspension system to improve vehicle safety," *Journal of the Society of Automotive Engineers Malaysia, vol.* 3, no. 2, 2019.

[5] R. Waldemar, M. Sibielak, and J. Konieczny, "Active vehicle suspension with a weighted multitone optimal controller: Considerations of energy consumption," *Structural Health Monitoring, Photogrammetry & DIC*, vol. *6*. Springer, Cham, pp. 195-197, 2019.

[6] Y. L. Liu, and L. Zuo, "Mixed skyhook and power-driven- damper: a new low-jerk semi-active suspension control based on power flow analysis," *Journal of Dynamic Systems, Measurement, and Control*, vol. 138, no. 8, 2016, 081009.

[7] G. Pepe, and A. Carcaterra, "VFC–variational feedback controller and its application to semi-active suspensions,” *Mechanical Systems and Signal Processing*, vol. 76, pp. 72-92, 2016.

[8] A. Talib, M. Hussin, M. H. H. Zulkeflee, and H. M. Yatim, "Fuzzy-skyhook controller with cuckoo search algorithm for a semi-active suspension system with magnetorheological damper," *Jurnal Mekanikal* , vol. 42, no. 1, 2019.

[9] Yamin, A. H. Mohd, Et Al. "Semi-active suspension system using MR damper with particle swarm optimization–skyhook controller," *International Journal of Advanced Control and* Automation *System (IJACAS*), pp. 32–37, 2019.

[10] Bashir, O. Ahmed, X. T. Rui, and J. S. Zhang, "Ride comfort improvement of a semi-active vehicle suspension based on hybrid fuzzy and fuzzy-PID controller," *Studies in Informatics and Control* , vol. 28, no. 4, 2019, pp. 421-430.

[11] M. Fliess, J. Lévine, P. Martin, and P. Rouchon, “Flatness and defect of non-linear systems: Introductory theory and examples,” *International Journal of Control*, vol. *61*, *no.* 6, pp. 1327-1361, 1995.

[12] H. Sira-Ramirez and S. Agrawal, *Differentially Flat Systems*, Marcel Dekker, New York (2003).

[13] Conde, E. Chávez, et al. "Robust control of active vehicle suspension systems using sliding modes and differential flatness with MATLAB," *MATLAB for Engineers-Applications in Control, Electrical Engineering, IT and Robotics*. IntechOpen, 2011.

[14] S. Guo, S. Yang, and C. Pan, “Dynamical modeling of magnetorheological damper behaviors,” *Journal of Intelligent Material* Systems *and Structures*, vol. 17, pp. 314, 2006.

[15] M. Agostinacchio, D. Ciampa, and S. Olita, "The vibrations induced by surface irregularities in road pavements–a Matlab® approach," *European Transport Research Review*, vol. 6, no. 3, p. 267, 2014.

[16] H. P. Geering, “Optimal control with engineering applications,” 1st Edition, Zurich, Switzerland, Spinger- Verlag Inc.,Berlin, Heidelberg, pp. 3-15, 2007.

Aneesh Dinesh Diwakar completed his B.Tech in Mechanical Engineering in 2016 from Amrita School of Engineering, Coimbatore, Tamil Nadu, India. Aneesh is currently a M.S. research scholar in Mechanical Engineering Department at IIT Madras, India. Currently, he is working on condition monitoring and control of gearbox and suspension system of vehicles. His research focuses on developing novel algorithms and verifying its behaviour and performance through simulations in commercially available softwares.

P. V. Manivannan received his Ph.D. in Control System for SI Engines from IIT Madras, India and Master in Applied Electronics from College of Engineering, Anna University, Chennai, India. Currently, he is an Associate Professor at Department of Mechanical Engineering, Indian Institute of Technology Madras, Chennai, India. He has also taught summer term course "Mechatronic Systems" in University of Nebraska, Lincoln, USA and University of Kaiserslautern, Germany as a visiting faculty He is also a recipient of prestigious DAAD fellowship and ERASMUS MUNDUS Teaching fellowship. His teaching research interests include: Mechatronics, Robotics, Automotive Control Systems, Embedded System Design, Sensor Network.