The Approach to Determine the Elastic Characteristic of the Contact of Rough Surfaces

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***Abstract*—Contact zone has a major effect on machine stiffness. For an accurate calculation, the contact stiffness should be known. This paper presents the approach to determine the elastic characteristic of the contact layer formed by two rough surfaces. Factors affected the shape of contact surfaces and the existing methods of its modelling are put into consideration and a new model of contact surface profile is proposed. The determination of elastic characteristic was made by computing microscale simulation of contact zone unit volume deformation under a compressive force. The simulation was performed using the Finite Elements Method. The average contact properties were obtained as the result of multiple randomized simulations. The simulation of contact layer compression shows that the contact stiffness almost linearly decreases with the increasing of the mean spacing of profile irregularities and the decreasing of the mean waviness height.**

***Index Terms*—elastic rough contact, numerical property identification, contact detection**

I. INTRODUCTION

A. The Problem of Mechanical Interaction between Jointed Parts of Machines.

For producing the high-quality machines it's necessary to know its mechanical behaviour under both static and dynamic load [1-4]. The most common method for estimation of mechanical behaviour of construction is the Finite Elements Method (FEM), which easily allows calculating deformation of solid bodies. But since machines are made of a lot of different parts, the estimation of its mechanical behaviour cannot be done without taking into account the features of the deformation characteristics of a contact layer between the machine parts [5]. Usual contact layer can be replaced by the third body with equivalent parameters when making the simulation [6]. And there is a problem to arouse: an elastic characteristic of the contact layer is supposed to be determined correctly.

B. Existed Approaches for Simulation of Contact Layer Behaviour

Since 1930s a great number of works have been dedicated to the problem of contact stiffness. E.g. those by D. N. Reshetov and Z. M. Levina [7,8] are based on an equation for calculation of contact convergence:

$$\delta = cp^m, \tag{1}$$

where δ – contact convergence, µm; p – contact pressure, MPa; m – exponent; c – empirical coefficient, $\mu\text{m} \cdot \text{MPa}^{-m}$. This equation was firstly proposed by A. P. Sokolovskiy in , which stated that convergence occurs due to the deformation of roughness peaks and contact stiffness would change with the increasing of normal load F_N. Physical model of contact convergence shown in Fig. 1.

A disadvantage of this approach is that it needs to conduct tests for defining parameters values in every irregular case.

Later, A. S. Ivanov developed a model contact deformation, which considers the effect of surface roughness and elastic modulus of contact parts [10-12]:

$$\delta = Ra\, c_0 \varepsilon \sqrt{\frac{p}{E^*}}, \tag{2}$$

where Ra – average deviation of roughness profile, c_0 - coefficient, which considers the direction of contact surfaces roughness, ε – scale factor, which consider surface waviness and form error, $E^* = ((1-\mu_1^2)/E_1 + (1-\mu_2^2)/E_2)^{-1}$ – equivalent modulus of elasticity, in which E – Yang modulus and μ – Poisson's ratio of each of parts, p – contact pressure in the contact layer. This work was an attempt to build the model acceptable on the design stage. But this model has almost the same drawback as the approach based on the usage of (1) – it has a small accuracy without clarification of scale factor ε for new irregular cases of surfaces roughness and waviness of contact parts.

In addition to these works, those by N. B. Demkin and V. V. Izmailov [13-17], M. M. Matlin [18-21] and others, which are described in detail in , should also be mentioned. But all of them have empirical or semiempirical character, when elastic characteristic defined by an equation, in which coefficients get from the natural or model experiments. In this regard, it's necessary to develop a method to determine the elastic characteristic of rough contact, which would be acceptable in the design stage and will not need additional tests conduction.

C. Aim and Objectives of the Investigation.

In this investigation, the contact layer between two parts connected by a bolted joint was considered. The aim of the investigation is the development of the method allowing to determine an elastic characteristic of the

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contact layer. For accomplishing this particular task, the following objectives were stated:

1. Development of the numerical procedure of elastic properties determination took into account the mechanical properties of contacted parts and real geometry of their surfaces.
2. Development of the model of the rough surface profile.
3. Carrying out a simulation of contact layer deformation under the compressive load.

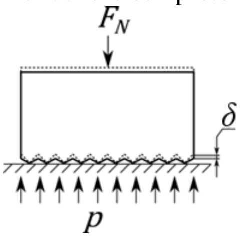

Figure 1. Physical model of contact convergence.

II. Description of the Methods Used and Algorithm of Elastic Characteristic Determination

A. The Geometry of the Real Surface

There are 4 types of geometric deviation:

- *The error of the form*: widely spaced single deviations of the real surface from the nominal surface caused by errors and elastic deflections of the part or machining equipment;
- *Waviness*: periodical regular deviations of surface shape in the form of peaks and valleys with a height from 0.3 to 500μm and a repeating period from 0.8 to 10 mm, which are typically caused by machine vibrations;
- *Roughness*: a complex of deviations with spacing from 2 to 800μm and height from 0.01 to 400μm, which general source is the geometry of cutting tool and strategy of machining;
- *Subroughness*: the irregular surface shape deviations of small size (2–20nm) that usually result from the inherent action of the production process or material condition.

Parameters that characterize roughness (Fig. 2) and waviness (Fig. 3) are standardized in . In the present work the following parameters were used:

- *Ra – the average deviation of the roughness profile*, which describes roughness magnitude and is defined by:

$$Ra = \frac{1}{L}\int_0^{L_w} h(l)dl \approx \frac{1}{n}\sum_{i=1}^{n} h_i$$

- *RSm – the mean spacing of profile irregularities*, which describe the roughness period of repeating and is defined by:

$$RSm = \frac{1}{n}\sum_{i=1}^{n} Sm_i.$$

- *Wt – the mean waviness height*, which describes waviness magnitude and is defined by:

$$Wt = \frac{1}{n}\sum_{i=1}^{n} Wt_i.$$

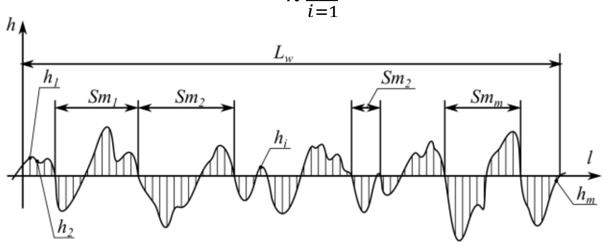

Figure 2. Illustration of calculation of roughness parameters

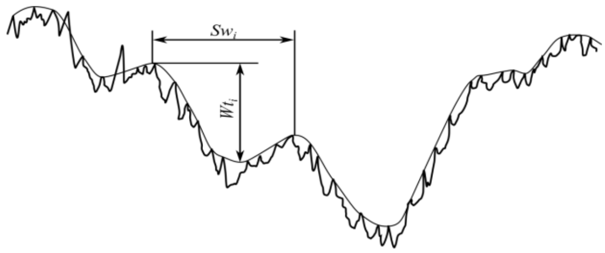

Figure 3. Illustration of calculation of waviness parameters

- *RSw – the average waviness spacing*, which describes the waviness period of repeating and is defined by:

$$RSw = \frac{1}{n}\sum_{i=1}^{n} Sw_i.$$

B. Assumptions

The approach presented in this paper was made under the following assumptions, which are caused by the features of the bolted joints:

1. only elastic deformation considered because bolted joints undergo a “training” process, which leads to crumpling of the roughness peaks;
2. surfaces of contact parts machined by milling, turning or planing usually, which define a certain profile of the surface;
3. surfaces with the direct lay of the texture, which allow considering of the flat contact problem;
4. the surface profile contains three levels of the deviations: low-frequency deviations (waviness and error of form), mid-frequency deviations (roughness) and high-frequency deviations;
5. in the investigation, the “equivalent rough surface” was used, proposed in , which allow replacing contact of two rough surfaces by contact of rough and flat surfaces.

C. FE Formalization of the Contact Problem

Determination of contact layer elastic properties was carried out by simulation of contact interaction using the finite element analysis. The mentioned contact problem represents a flat FEM problem with kinematic loading. A calculation scheme of the problem is shown in Fig. 4.

The main equation of FEM is

$$\mathbf{K\delta} = \mathbf{F},$$

where **K –** global stiffness matrix, **δ –** vector of mesh nods displacements and **F –** vector of external loads. The procedure of matrix **K** and vector **F** assembling described in detail in . Finite element mesh was being built to be coherent, which allow finding contacted surface points by simple comparison:

$$y_i^{c1} > y_i^{c2},$$

where y_i^{c1} and y_i^{c2} is vertical coordinates of nodes of bottom and top contact surfaces. After the contact points

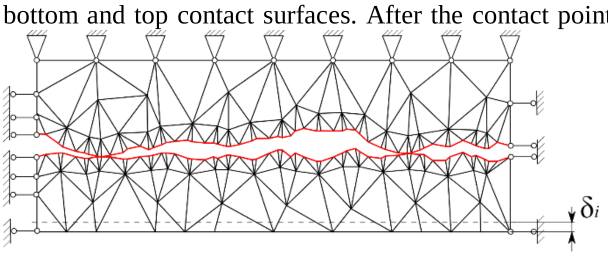

Figure 4. Calculation scheme of the considered contact problem.

were found, the stiffness matrix **K** must be modified because their displacements should be compatible. For achieving the compatibility of connected nodes displacements, the penalty function method was used. According to this method, a “penalty” element is generated between nodes, which should be constrained (see Fig. 5). This element can be expressed by the following relations:

$$\begin{bmatrix} w & -w \\ -w & w \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = \begin{Bmatrix} f_i \\ f_j \end{Bmatrix},$$

where w – stiffness of the “penalty” element; i and j – indices of constrained nodes in the global stiffness matrix. The global stiffness matrix should be modified as:

$$\tilde{k}_{ii} = k_{ii} + w;$$
$$\tilde{k}_{jj} = k_{jj} + w;$$
$$\tilde{k}_{ji} = k_{ji} - w;$$
$$\tilde{k}_{ij} = k_{ij} - w;$$

And a reaction caused by compression of the penalty element should be added to the forces vector:

$$\tilde{f}_i = f_i - w\Delta y_{ij};$$
$$\tilde{f}_j = f_j + w\Delta y_{ij};$$

where Δy_{ij} – distance between the nodes in projection on the axis through which the constraint is generated. If penetrations are not allowed, a big number, as 10^{15}, should be taken as the stiffness of the “penalty” element.

A flow chart of the full algorithm of the elastic characteristic determination of the contact layer is shown in Fig. 6.

D. Model of the Rough Surface

Relying on the described structures of the surface profile, its general equation can be defined by:

$$h(l) = h_l^s(l) + h_l^r(l) + h_m^s(l) + h_m^r(l) + h_h^r(l),$$

where $h_l^s(l)$ – a systematic component of low-frequency deviations, $h_l^r(x)$ – a random component of low-frequency deviations, $h_m^s(l)$ – a systematic component of mid-frequency deviations, $h_m^r(l)$ – a random component of mid-frequency deviations, $h_h^r(l)$ – a component of high-frequency deviations (contain only random component).

The random component of the surface profile deviations can be obtained by cubic interpolation of random points which correspond to the β-distribution. A distance between the point is defined by spacing parameters, and magnitude of distribution is defined by the height parameters.

Systematic component of waviness can be expressed by sine function:

$$h_l^s(l) = W_t \sin\left(\frac{2\pi}{RSw} l + \phi_0\right),$$

where ϕ_0 – initial waviness phase, which can be taken as zero.

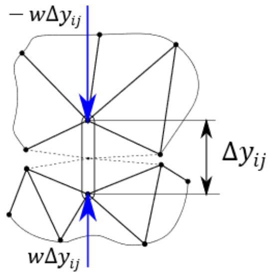

Figure 5. Illustration of multi-point constraints applying.

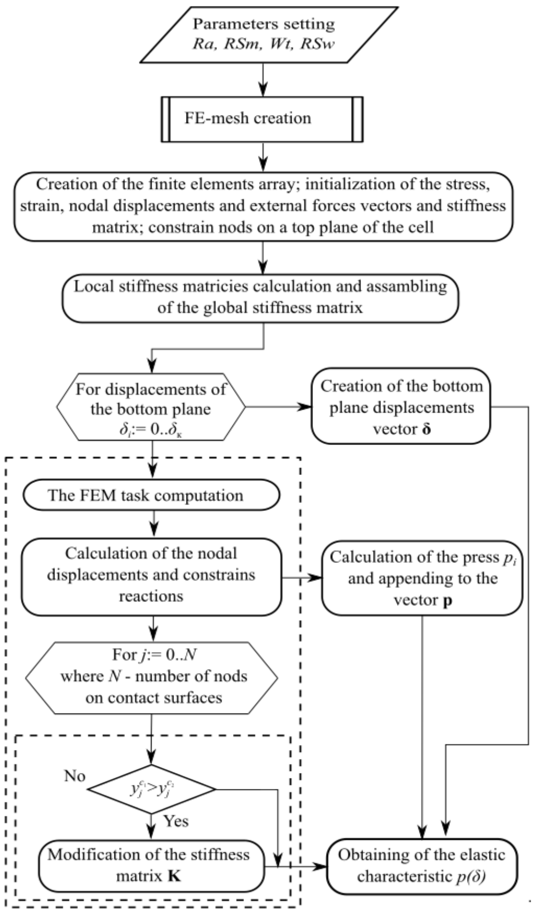

Figure 6. Flow chart of the contact layer elastic characteristic determination algorithm.

For the systematic component of roughness the following equation was used:

$$h_m^s(l) = \frac{k \cdot Ra}{\max\limits_{[0;L_w]} f(l) + \min\limits_{[0;L_w]} f(l)} f(l) - \frac{\left(\max\limits_{[0;L_w]} f(l) + \min\limits_{[0;L_w]} f(l)\right)}{2},$$

where k – coefficient, which equals to 5.5 for considered machining methods and $f(l)$ – chosen roughness form function, defined by

$$f(l) = \sum_{i=0}^{n} \exp(-|l - i \cdot RSm|).$$

An example of a real surface profile measured by electronic profilometer and corresponding simulated surface profile are shown in Fig.7 (a, b). Good convergence of profiles and their bearing area curve (c) show the correctness of the chosen model.

III. Numerical Simulation of the Contact Layer Compression

A. Numerical Calculation

The algorithm of the elastic characteristic determination of the contact layer was implemented by means of Python for fast matrix calculation, *scipy* for SLE solving, *matplotlib* for plotting the results of calculations and *triangle* from *meshpy* for FE-mesh generation. Calculations were performed on a representative cell of the contact layer. The length of the cell was chosen to ensure acceptable calculation time and accuracy and was equal to 1.6 mm. The height of the cell was chosen by the recommendation of the as 10 values of the average deviation of the roughness profile *Ra*. Elements size in the contact area was chosen as 2 μm by the results of pretest and had been increasing near the bottom and top sides of the cell for decreasing the calculation time. An example of the generated representative cell of the contact layer is shown in Fig. 8.

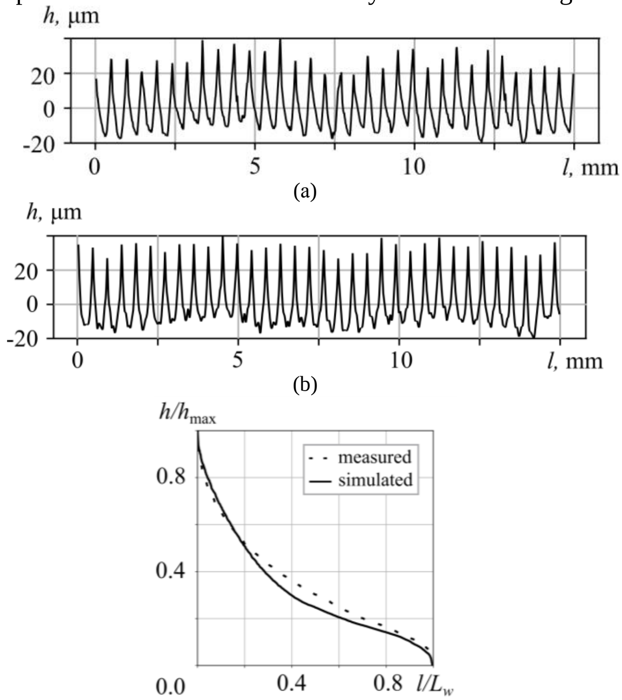

Figure 7. A profile example of measured (a) and simulated (b) planned surfaces and their bearing area curve (c).

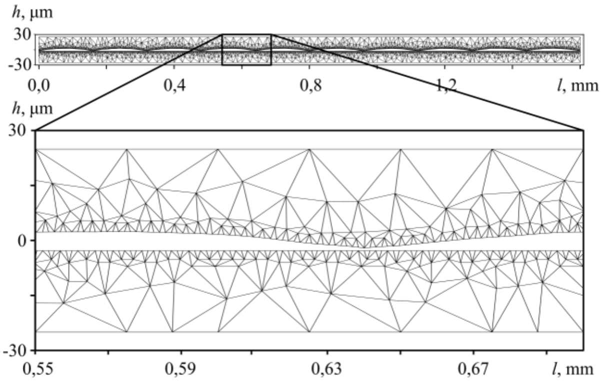

Figure 8. Example of a representative cell of the contact layer

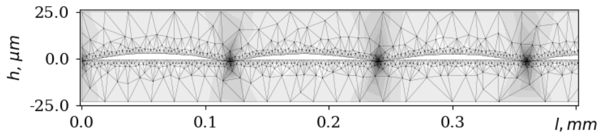

Figure 9. Example of stress field calculation of contact cell

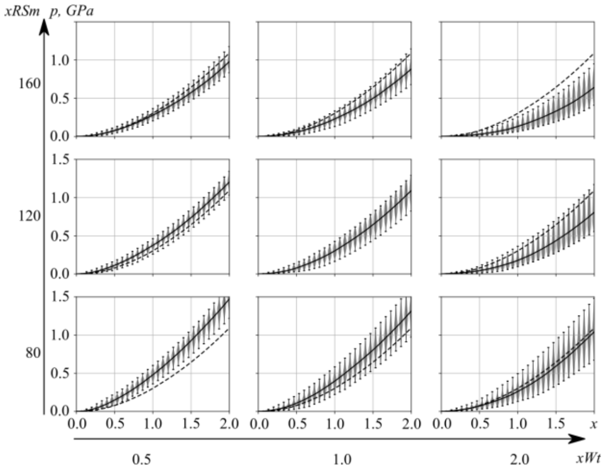

Figure 10. Calculation results of the average elastic characteristics of the contact layer (dashed line – characteristic obtained with *xWt*=1.0 and x*RSm*=120)

The value of the iteration was chosen for the reason of accuracy by the results of pretests and was equal to *Ra*/15.

According to π-theorem, the number of calculations can be reduced if we introduce a dimensionless displacements x and dimensionless analogues of surface profile parameters *xRSm* and *xWt*:

$$x = \delta / Ra\,;$$

$$xRSm = RSm / Ra\,;$$

$$xWt = Wt / Ra.$$

B. Calculation Results

An example of the stress field calculated in the last iteration of compression of the contact cell is shown in Fig. 9.

Tests were repeated thirty times for each combination of parameters used. The results of the calculation are shown in Fig. 10 as the matrix of mean elastic characteristics with deviation. On the plots the higher characteristics correspond to the contact layer with the higher stiffness.

Obtained characteristics were approximated by (2) with c_0 as the variable parameter. The results of approximation are presented in Table I.

IV. Analysis of the Results

As far as the calculations demonstrate, the changing of the average waviness spacing *Sw* does not take a matter on the elastic characteristics of the contact layer and is not shown in Fig. 10. The presented results vividly show that stiffness of the contact layer decreases with te increasing of the waviness height *Wt* and the decreasing of the roughness spacing *RSm*. The results of the approximation show that parameter c_0 linearly relies on the roughness spacing *RSm.* It also needs to be mentioned that the deviation of the characteristics increases with the increasing of the waviness height which could lead to the calculation errors.

TABLE I. Values of the Model Parameter c_0

xWt / *xRSm*	0.5	1.0	2.0
80	58.27	55.95	47.04
120	52.95	50.20	42.51
160	46.92	44.42	36.15

V. Conclusion

Within the frames of this work the approach to determine the elastic characteristic of the contact layer used the real profile of the contact surfaces, which allow avoiding the natural compressive experiments, was proposed. For designing calculations of contact compression, in case we do not have information about the profile of contact surfaces, the developed model of rough surface profile based on the description of surface texture by standard parameters and their distribution character can be used, which was confirmed by comparison of experimental data.

The numerical simulation of contact layer compression shows that the contact stiffness almost linearly decreases with the increasing of the mean spacing of profile irregularities and the decreasing of the mean waviness height. Accounting of these parameters increases the calculation accuracy up to 30%.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Formulation of the aim and objectives of the investigation, development of the algorithm of elastic characteristic determination and result analysis were made by Mikhail N. Zakharov.

Development of the rough surface profile model, the contact cells generating, conducting of numerical calculation and preparing the paper were made by Mikhail S. Kuts.

All authors have approved the final version.

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