Finite Element Modeling of Bottom Hole Assembly Stress-strain State

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***Abstract*—The results of the numerical solution of the problem of static loading of the bottom-hole assembly (BHA) by axial load are given, taking into account the contact interaction with the borehole wall. The numerical solution of the presented problem is important in the process of preparing for drilling various types of wells. The finite element method is used as the main computational research method. A finite element model of a special type, designed to simulate the contact interaction of a drill string with a borehole wall, is presented. A computational algorithm for numerical simulation of nonlinear deformation of a drillstring based on the use of linear beams and nonlinear rod finite elements is proposed. The developed algorithm allowed the calculation of the forces acting on the bit, as well as the calculation of the drillstring displacements inside the wellbore in a two-dimensional formulation. The results of computational experiments are presented.**

***Index Terms*—Bottom hole assembly, weight on bit, side force coefficient, bottom hole assembly side force determination, finite element**

I. Introduction

The bottom hole assembly (BHA) is the part of the drill column (Fig. 1), which affects the trajectory of the end cutting part of the tool (hereinafter, bit) and, therefore, forms the geometry of the wellbore [1].

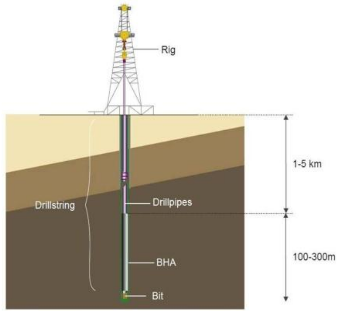

Figure 1. Schematic model of the drill column.

Predicting the trajectory along which the well formation process will take place is a difficult task. Many different, affecting on each other, parameters of the BHA determine the trajectory of the entire drill column. The first attempts to obtain a mathematical model of the BHA and its movements were carried out by researchers H.B.Woods and A. Lubinski [2]. They presented the first analytical model of the BHA, which allowed to take into account the influence of the stabilizers positions on the forces arising on the bit. The main drawback of the created model was the lack of consideration for the occurrence of contact between the drill pipes and the wellbore during loading of the bottom hole assembly with axial force.

Later in the work of F.J. Fisher [3] a finite difference model of the BHA was presented, which allowed for the occurrence of a contact to be taken into account and produce more accurate modeling. The finite difference model of F.J. Fisher also had shortcomings: the diameter of the drill pipe was the same throughout the entire study area, and their shape and stiffness characteristics were assumed to be the same.

The model of a nonlinear contact interaction of drill strings with a well is described in sufficient detail in [4]. In [5], due to the use of the finite element approximation of the BHA, the researcher Millheim K. succeeded to build a model of the BHA with the use of drill pipes of various diameters and shapes. The advantages of using the finite element method, which makes it possible to build a three-dimensional dynamic model that takes into account large deformations, to analyze the loading of a drill string with an axial load are described in detail in [6], which presented a dynamic analysis of the BHA loading.

Trajectory along which the drillstring will move in the process of loading it is influenced by different factors, namely: internal stresses in individual drill pipes and the resulting oscillations [7]. For example, in [8–10], the results of numerical simulation of the drilling process are presented, taking into account the effect on the motion path of the oscillations of the drill. A similar approach can be used for more accurate modeling of the movement of the bit inside the wellbore.

The modeling process should also take into account the nonlinear nature of the buckling of the BHA and frictional forces that occur in the contact areas of the drill pipe with the borehole [11]. The use of the finite element method can also be justified by the frequent use of this approach in various fields of engineering [12].

Despite the existence of well-developed mathematical models in the field of modeling the BHA and its movement [4–6], most of the numerical results were obtained using commercial software [13]. The development of accessible expandable software in the studied area remains topical. This work is part of a

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complex of software development in the field of numerical modeling of the BHA motion in a well, considering various factors. A finite-element mathematical model of a multisectional BHA is presented, which takes into account the nonlinear nature of the contact interaction with the well under the action of static loading of the BHA by axial load.

II. Mathematical Model of Bottom Hole Assembly

To solve the problem of analyzing the stress-strain state of the BHA under the action of a static axial force, finite-element approximation of bottom hole assembly was constructed (Fig.4), which included finite elements of various types. A beam finite element was selected as an element for approximation of drillstrings (Fig.5). To account for the possible occurrence of contact between the wellbore and the drill column, a rod finite element is used (Fig.6). (See figures in Appendix A)

Mathematical model of beam finite element:

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \frac{EI_z}{l^3} \begin{bmatrix} \frac{Al^2}{I_z} & 0 & 0 & -\frac{Al^2}{I_z} & 0 & 0 \\ 0 & 12 & 6l & 0 & -12 & 6l \\ 0 & 6l & 4l^2 & 0 & -6l & 2l^2 \\ -\frac{Al^2}{I_z} & 0 & 0 & \frac{Al^2}{I_z} & 0 & 0 \\ 0 & -12 & -6l & 0 & 12 & -6l \\ 0 & 6l & 2l^2 & 0 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} \quad (1)$$

where S_1 - axial force acting along the axis of the element applied to the first node of the element; S_2 - shear force acting perpendicular to the axial direction applied to the first element node; S_3 - bending moment applied to the first element node; S_4 - axial force acting along the axis of the element applied to the second node of the element; S_5 - shear force acting perpendicular to the axial direction applied to the second element node; S_6 - bending moment applied to the second element node; A - element cross-sectional area; E - element stiffness modulus; l - element length; I_z - polar element moment, relative to the axisz; u_1 - displacements of the first element node along the axial direction; u_2 - displacements of the first node of the element along the direction perpendicular to the axial; u_3 - angular displacements of the first node; u_4 - displacements of the second element node along the axial direction; u_5 - displacements of the second node of the element along the direction perpendicular to the axial; u_6 - angular displacements of the second node.

The mathematical model of a one-dimensional rod element (Fig. 4) with a linear shape function has the following form [14]:

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (2)$$

where S_1 - axial force acting along the axis of the element applied to the first node of the element; S_2 - axial force acting along the axis of the element applied to the second node of the element; A - element cross-sectional area; E - element stiffness modulus; l - element length; u_1 - displacements of the first element node; u_2 - displacements of the second element node.

Mathematical models of rod and beam elements are presented in detail in the work [14].

III. Non-linier Mathematical Model of Rod Finite Element

One-dimensional rod finite element was used for modeling the occurrence of contact between drill pipes and wellbore. The model which was based on model (2), for which the element stiffness had a non-linear dependence on its deformation:

$$E = \begin{cases} E_1, \varepsilon < 1; \\ E_2, \varepsilon \geq 1; \end{cases} \quad (3)$$

where $E_1 \sim 1$, $E_2 = 10^8$, $\varepsilon = \frac{u_1 - u_2}{l}$ (Fig.2).

In the modeling process, the deformation of each rod element should be calculated and, depending on its value, the stiffness of the corresponding element should be discretely changed. The advantages of using such special rod elements to account for the occurrence of contact between the drill pipes and wellbore are presented in [15].

In case if the deformations of the element are in the range $\varepsilon \in [-1; 1]$, the stiffness of the element should take values of zero order (during the computational experiment, the stiffness was zero), which means there is no contact between the drill pipe and borehole walls. A specific element that is outside the allowable range, that is, $-1 > \varepsilon, \varepsilon > 1$, the element stiffness should be greatly increased, which allows modeling the reaction forces acting from the side of the wellbore on the drillstrings.

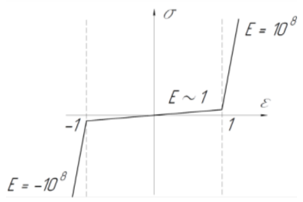

Figure 2. The dependence of internal stresses on the strain for the rod element.

IV. Accounting Self-weight Forces Acting on the Column in the Modeling Process

Modeling the BHA loading involves determining the vector of external loads acting on the drillstrings. Conventionally, the vector of forces $\vec{F}$ acting on the BHA can be divided into two components: the axial load $\vec{F}_{WOB}$ which applied at the upper point of the BHA or to the first node of the uppermost (zero) element of the finite element approximation of the BHA and the force of the own weight of the drill pipe $\vec{F}_{grav}(x)$, depending on the x coordinate [16]. The force of the own weight of the drill pipe at a specific point of the BHA is determined by the weight of the drillstring located above the BHA and increases as it approaches the lower point of the BHA.

So, if there is no additional applied external load at the upper point of the BHA ($\vec{F}_{WOB} = \vec{0}$), value of the axial load at the upper point of the BHA $S_1^0$is determined only by the weight of the drill pipe (boundary condition, which it is necessary to use in the finite element method):

$$S_1^0 = \left|\vec{F}_{WOB}\right| + \left|\vec{F}_{grav}(0)\right| \tag{4}$$

where x = 0 defines the coordinate of the BHA top point.

Taking into account the applied beam finite element with three degrees of freedom at each node, contribution to vector of external forces acting on element $\{S\}^e$, due to the own weight of the drill pipe, will have the following form [16]:

$$\{F_{grav}\}^e = \left\{\frac{P\cdot\cos(\alpha)}{2} \quad \frac{P\cdot\sin(\alpha)}{2} \quad 0 \quad \frac{P\cdot\cos(\alpha)}{2} \quad \frac{P\cdot\sin(\alpha)}{2} \quad 0\right\}^T \tag{5}$$

where $\{F_{grav}\}^e$- is the vector of self-weight forces acting on the beam finite element; P - is the own weight of the drill pipe, which is determined based on the pipe size, and characteristics of the material, from which pipe is made; α- inclination angle.

V. Algorithm of Drill Column Gradual Loading with Axial Load

Process of modeling BHA loading with axial load is complicated by using of a non-linear rod element with variable stiffness. In order to correctly take into account the contacts arising between the drillstrings and borehole, it is necessary to implement the process of gradual loading of the BHA with an axial load.

The algorithm is based on a gradual increase of axial force at each iteration, determining the rod element with the largest deformations and calculating the maximum axial force for this element, at which the deformations still fall within the allowable range of values $\varepsilon \in [-1; 1]$. Once such a force is found, it is necessary to change the stiffness of this element according to (3) and continue the process of further loading the BHA. At each iteration of the modeling process, the deformations of the rod elements are calculated, as well as the stresses generated inside the drill pipe, approximated by the beam finite elements. Flowchart of the developed algorithm, which takes into account the peculiarities of modeling the BHA loading with axial force, is presented in Fig.7 (See appendix B).

To implement the presented algorithm, programming language C ++ was used.

VI. Computational Experiment and Its Key Features

To test the developed algorithm, the task was to numerically analyze the stress-strain state of BHA with the parameters presented in Table I.

As a result of the computational experiments, the forces acting on the bit, as well as the displacements of drill pipes inside borehole, are presented, which are shown in Fig.3. The lateral force on the bit was 2200 N. According to the obtained numerical results, the closest contact of the drill pipes with borehole is at a distance of 50 meters from the bit.

To assess the correctness of the results obtained using the developed software, calculations comparison was made for a typical configuration of the BHA with the analysis of the stress-strain state in existing software packages. The comparison of results are presented in Fig.8 (See appendix C).

Analyzing the resulting displacements of the drill pipe using the created software package and the developed algorithm, we can conclude that the development presented in the article can be used to numerical modeling the deformation of the BHA under the action of an axial load, taking into account the contact interaction with the borehole walls.

TABLE I. The Values of the BHA Parameters in the Computational Experiment

Name	Value	Name	Value
Drillstring material	Steel	Outer diameter of drillstring	0.1524 [m]
Drillstring length	0.5 [m]	Bit diameter	0.224 [m]
Amount of drillstrings	100	Inclination angle	30 [degree]
Inner diameter of drillstring	0.0508 [m]		

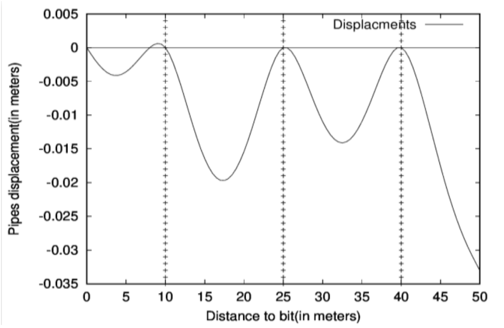

Figure 3. Displacements of the drillstrings inside the wellbore for axial load component of 15 kN with three stabilizers in the BHA configuration at the distance of 10, 25 and 40 meters from the bit, respectively.

VII. Conclusion

As a result of the work, an algorithm was developed that allows the analysis of influences of the loading of the BHA by axial load on the formation of the wellbore trajectory. As a mathematical model of a BHA, its finite element approximation using beam and rod finite elements is used. The developed algorithm made it possible to numerically determine the displacements of drillstring column inside the wellbore, as well as the loads occurring on the bit, taking into account the possible occurrence of contact between the drill pipes and the borehole.

A promising direction for the development of the developed algorithm for analyzing the stress-strain state of a BHA is predicting the well trajectory taking into account the forces arising on the bit in a common three-dimensional setting. On displacements of the BHA is influenced by a variety of parameters, from which we can distinguish such parameters as bit characteristics, rock characteristics and reaction forces arising on the bit [17-18]. It is planned to further develop the developed models and software for solving the problems of:

- Automated design of BHA for given inclinometric parameters of the bore hole at the stage of planning the wellbore trajectory;
- Creation of real-time recommendations on the composition of the BHA, which provides the maximum rate of penetration (ROP), no deviations from the planned borehole trajectory and safety of equipment use.

To create such an expandable and maintainable software, it is planned to use a graph-oriented approach [19].

The study was carried out on an initiative basis.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

D.O. Zaharov developed the mathematical model of a drillstring, all authors developed the algorithm, which presented in Appendix B. D.O. Zaharov implemented the developed algorithm on C++ programming language and conducted numerical experiments. All authors wrote the paper and had approved the final version.

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Appendix A Finite Element Models

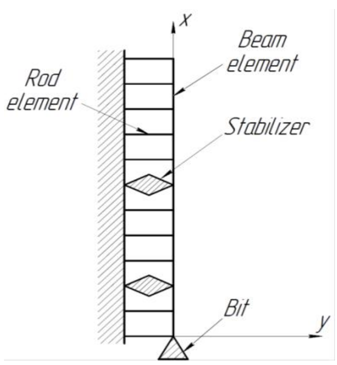

Figure 4. Finite element model of bottom hole assembly

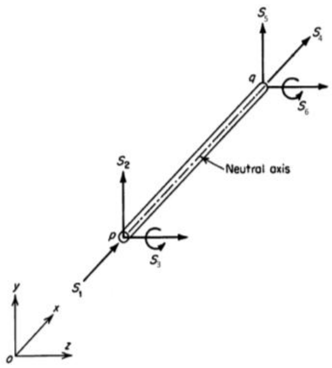

Figure 5. Schematic model of the used beam finite element

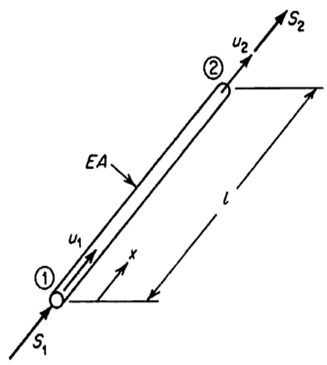

Figure 6. Schematic model of the used rod finite element

APPENDIX B ALGORITHM FLOWCHART

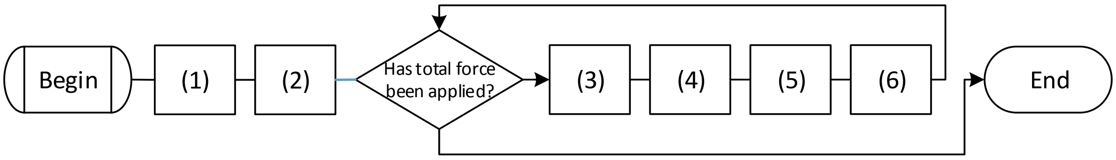

Figure 7 Flowchart of the algorithm for modeling the loading of the drill string with axial load

Description of the flowchart: (1) construction of finite element model of the BHA using two types of finite elements; (2) determining the boundary conditions, namely: taking into account the proper weight of the drill pipe and the initial axial load on the upper configuration element; (3) calculation of displacements of drill pipes inside the wellbore for the current load iteration taking into account "partially applied" boundary conditions; (4) calculation and preservation of internal stresses of drill pipes for the next iteration of the loading process; (5) calculation of the stiffness of the rod elements depending on their current deformations; (6) increasing applied axial load, as well as an adjustment of the vector of external forces depending on the internal stresses occurring inside the drill pipe.

APPENDIX C COMPARISION OF EXPIRIMENT RESULT

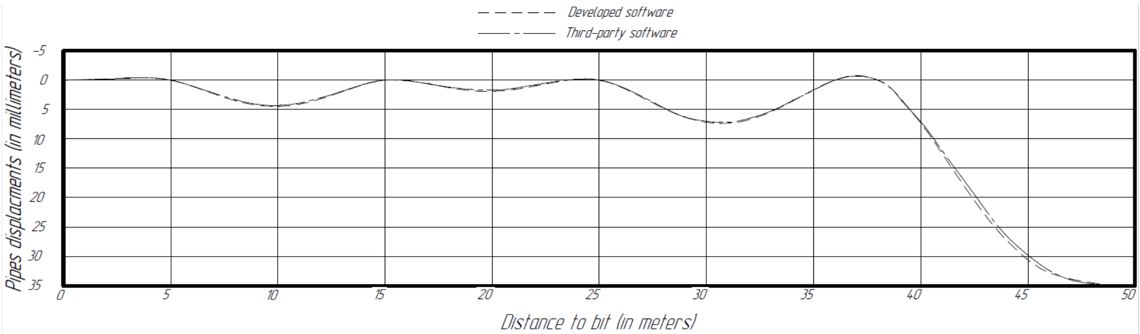

Figure 8. The results of numerical simulation of the displacements of drillstrings inside the wellbore with axial load component of 15 kN with four stabilizers in the BHA configuration at a distance of 5, 15, 25 and 38 meters from the bit, respectively, obtained using: a) developed software and b) third-party software