Dynamic Modelling and Analysis of a Quadrotor Based on Selected Physical Parameters

Moad Idrissi* and Fawaz Annaz
Faculty of Computing, Engineering and the Built Environment
Birmingham City University
Birmingham, UK
Email: Moad.Idrissi@mail.bcu.ac.uk, Fawaz.Annaz@bcu.ac.uk

***Abstract*—Over the past decade, control techniques have been widely implemented on quadrotors to achieve the desired positions within the coordinate system. However, ensuring that the dynamics are correct and that similar results to a physical model can be obtained has been a question of interest. In this paper, the quadrotor dynamics are thoroughly analysed in simulation without using any controllers. Specifically, suitable actuators and propellers have been selected to generate ideal thrusts that will enforce the unmanned aerial vehicle (UAV) to lift. By using kinematics approach, one can analyse the expected motion of the UAV after a certain thrust is applied on all motors. Hence, the dynamics of the proposed quadrotor are recognised and verified through numerical simulations, leading to presenting the motions of the physical model. The results attained have illustrated promising results in which a comparative study between experimental and theoretical methods have presented little to no errors.**

***Index Terms*—UAV, Quadrotor, Dynamic Modelling, Automation**

I. Introduction

In recent years, researchers have dramatically increased efforts in improving the reliability and safety of UAVs. The control of such vehicles provides ease of maneuverability and quick orientation capabilities depending on the selected model, quadrotors in particular have been widely used in many applications due to their capability in vertically taking off and landing (VTOL). Additionally, hovering at certain altitudes has encouraged commercial and non-commercial applications demand more reliability and efficiency to optimize the endurance and performance of these systems within complex environments [1]. With regards to VTOL and Horizontal take-off/landing (HTOL) UAVs, research has been greatly undertaken into improving the flight performance by modifying the architectural structure of these systems, some of which are extremely small UAVs that could perhaps be the size of ‘small particles’ weighing around 0.1Kg while others could be as large as a conventional piloted aircraft weighing over 150Kg [2, 3]. Thus, the vast majority of these changes and modifications has resulted in the implementation of these drones on wider applications worldwide. While the mechanical architecture development is rapidly enhancing, control techniques has now become a major topic for researchers to carry out on certain drone operations. Doing so meant that the dynamic model for the UAV must be taken into great consideration without neglecting any parameters as that will characterize the performance of the control law [4].

With regards to analysing the quadrotor dynamics, little research has been considered into analysing the motions of the UAV by applying thrusts without using any control techniques. For instance, the authors in [5, 6, 7] have specifically analysed the quadrotors performance based on motion capabilities due to thrust generation. The authors in [5] have presented similar work in terms of the background study of the UAV. A controller was applied directly into the dynamics without illustrating any parameters that represent the model. Hence, the response of the vehicle may not be ideal in terms of representing a specific quadrotor model. The authors in [6] have presented the mathematical dynamics of a quadrotor as well as the selected actuators whereby, applying an input voltage to the motors will generate a thrust that will enforce the UAV to lift. The results show the behaviour of the system in various angles respectively due to the change in the excitation voltage. The authors mentioned that a good understand of the response from the vehicle has been achieved making it very helpful in the development of a suitable controller. However, although the overall results were satisfactory. The proposed quadrotor model as well as the actuator parameters has not been presented in which the proposed system has not been critically analysed. Last but not least, the authors in [7] have presented similar work into analysing the motion of the UAV depending on the thrusts generated by the propellers, the parameters where presented as well the full dynamic theory. Though, the researchers focused on studying the motion in vertical direction where the results presented illustrate that, increasing the motor speed will enforce the vehicle to lift while reducing speed will cause it to descend.

In this paper, the quadrotor dynamics will be mathematically presented followed by proposing a system model that consists of suitable actuators, propellers and the physical model parameters. The aim of this paper is to provide the reader with an insight of the vehicle motion based on the thrusts generated from the angular velocity

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of each motor. By applying a certain voltage to each motor, the motion in terms of acceleration, speed and position of the vehicle can be theoretically calculated using kinematics approach, followed by a comparative study against the Simulation results. Primarily, designing control systems for these type of vehicles essentially relies on the dynamics presented. Hence, it is vital to carefully consider the mathematical model and ensure that the results attained must represent a real physical system. Both experimental and theoretical results are compared in which the performance will be justified in accordance with the errors attained.

The paper is structured as follows: In section II, the quadrotor dynamics are presented which holds the Euler's equation of motion, thrust control inputs and the full mathematical representation of the UAV in order for it to achieve the full six degrees of freedom. Section III presents the proposed dynamic model for this study which holds appropriate parameters for the propellers and the actuators. Section IV presents a discussion of the experimental and the theoretical results which consists of a comparative study based on the error margins. Finally, Section V concludes the work and the overall gained performance.

II. Quadrotor Dynamics

Quadrotors are commonly formed to operate in the '+' or 'x' configuration where the overall control authority for both configurations shows that the performance is identical [8]. Fig. 1 shows the basic UAV '+' configuration which will be considered in this paper, with motors 1 and 3 rotating in the clockwise direction which will be referred to as $Set_{1,3}\big|_{cw}$; similarly, motors 2 and 4 rotating counter-clockwise will be referred to as $Set_{2,4}\big|_{ccw}$. The Figure also shows the various basic flight directions that a UAV might describe subject to the speeds and spinning direction commands to the individual motors. For example in hovering position, the UAV will have all motors (in both sets) rotating at equal and opposite speeds, i.e. $\omega_{Set_{1=3}|cw} = \omega_{Set_{2=4}|ccw}$; and when the speeds of all motors are simultaneously increased or reduced, the UAV will hover at higher or lower altitudes respectively.

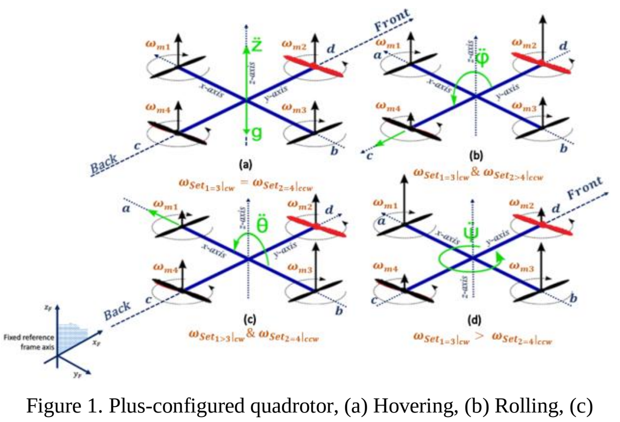

Figure 1. Plus-configured quadrotor, (a) Hovering, (b) Rolling, (c) Pitching, (d) yawing

The quadrotor is an under-actuated device and therefore can describe the basic roll, pitch and yaw motions as well as movements in the Z-directions. It is, however, incapable of performing pure movements in the x and y directions without accompanying rolling or pitching movements. It is in this section the quadrotor mathematical model will be developed and verified against others [9, 10, 11, 12]. The theory presented assumes that the drone is rigid and has a symmetric structure; thrust is produced by propellers of equal size with their rotors facing upward in the z-direction; and that all rotors have the same distances to the centre of mass. The behaviour of the quadrotor is determined by translations and rotations of the body inertial frame with respect to a fixed inertial frame. Mathematically, these are represented by the twelve states in x^T in Equation (1) which denote the quadrotor position and respective speeds with reference to the inertial fixed frame in the axis.

$$x^T = \{x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}\} \tag{1}$$

Rolling, Pitching and Yawing with respect to the fixed inertial frame may be described through the Euler angles transformation matrix shown in Equation (2), where s and c denote sin and cos, respectively. In order to ensure that the quadrotor reaches the desired positions without any loss of control, the angles must be bounded to $-\pi/_2 \leq \phi \leq \pi/_2$, $-\pi/_2 \leq \theta \leq \pi/_2$ and $-\pi \leq \psi \leq \pi$ [13]. Exceeding such angles would cause the UAV to over rotate resulting in great instability, whereby greater efforts from the controller are required to enforce the vehicle to regain stability.

$$\mathbb{R}_0^1(\phi,\theta,\psi) = \begin{pmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{pmatrix} \tag{2}$$

A. Euler's Rotation and Gyroscopic effects

The Euler's three rotational equation of motion for a rigid body is derived about the x, y and z axis respectively. As the body rotates and translates, any particles of mass will generally experience some acceleration due to a force. Thus, assuming that the centre of mass is present and that the body fixed axis are taken along principle axis of inertia, the sum of all moments about each axis is presented as [14]:

$$I_x\ddot{\phi} + \dot{\theta}\dot{\psi}(I_z - I_y) = \tau_x \tag{3}$$

$$I_y\ddot{\theta} + \dot{\phi}\dot{\psi}(I_x - I_z) = \tau_y \tag{4}$$

$$I_z\ddot{\psi} + \dot{\phi}\dot{\theta}(I_y - I_x) = \tau_z \tag{5}$$

When the UAV is hovering at a certain altitude, the axes of the rotors spinning at higher speeds simultaneously will always be coincident with the z axis of the body frame. However, if the vehicle rolls or pitches, the angular momentum of the motors also changes. Thus, a gyroscopic torque is generated on the vehicle frame as a result of the rotation [15, 16, 17] which is calculated as:

$$G_a = \sum_{i=1}^{4} J_r \, \dot{\omega}\Omega_i \tag{6}$$

Where J_r is the moment of inertia, $\dot{\omega}$ is the angular velocity of the body frame and Ω_i is the angular rate of the rotor i .

B. Thrust Control Inputs

Achieving desired positions can be reached in the Cartesian coordinate system via rolling, pitching, yawing or lifting. In order to do so, the thrust control inputs that will enforce the UAV to achieve such motions can be described by the following equations:

$$\begin{aligned} U_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 &= b(-\Omega_2^2 + \Omega_4^2) \\ U_3 &= b(-\Omega_1^2 + \Omega_3^2) \\ U_4 &= d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \end{aligned} \tag{7}$$

Where, U_1 is the total thrust provided by the four rotors; U_2, U_3 and U_4 are the respective roll, pitch and yaw moments. b and d are the thrust factor and the drag factor. The control action is dependent on the angular velocities of four independent motors noted as $\Omega_1, \Omega_2, \Omega_3$ and Ω_4. Ω_r Is the overall residual propeller angular speed which is considered in the gyroscopic effects as the UAV rotates:

$$\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \tag{8}$$

C. Quadrotor Equation of Motion

Upon defining the Euler rotational equations of motion, gyroscopic effects, aerodynamics factors and the thrust control inputs, the full dynamics model for the quadrotor in translational and rotational motion can be described through the transformation of equations (3), (4), (5), (6) and (7). Transposing the equations to achieve the acceleration is obtainable through the Euler equation of motion as:

$$\ddot{x} = \frac{1}{m}[\cos\phi sin\theta cos\psi - sin\phi sin\psi]\, U_1 \tag{9}$$

$$\ddot{y} = \frac{1}{m}[\cos\phi sin\theta sin\psi + sin\phi cos\psi]\, U_1 \tag{10}$$

$$\ddot{z} = -g + \frac{1}{m}[\cos\phi\cos\theta]\, U_1 \tag{11}$$

$$\ddot{\phi} = \frac{1}{I_x}[\dot{\theta}\dot{\psi}(I_z - I_y) - J_r\dot{\theta}\Omega + lU_2] \tag{12}$$

$$\ddot{\theta} = \frac{1}{I_y}[\dot{\phi}\dot{\psi}(I_x - I_z) + J_r\dot{\phi}\Omega + lU_3] \tag{13}$$

$$\ddot{\psi} = \frac{1}{I_z}[\dot{\phi}\dot{\theta}(I_y - I_x) + U_4] \tag{14}$$

Where m is the overall mass, g denotes gravity and l signifies the length from the rotor to the centre of mass.

III. THE PROPOSED QUADROTOR MODEL

Quadrotor models have been extensively examined by many researchers, since these vehicles are applied in many applications, their size and capabilities widely vary depending on the set application. In this research, the aim is to examine a common type of quadrotor that is widely used in outdoor and indoor applications such as the Draganflyer quadrotor as shown in Fig. 2 due to its simple design, lightweight and high rigidness [18, 19]. Some of the parameters that must be considered when studying quadrotors play a vital role in representing the performance of the vehicle. In other words, small changes to these parameters can greatly affect the stability and the response of the vehicle. Therefore, table I represents the parameters of the proposed physical model:

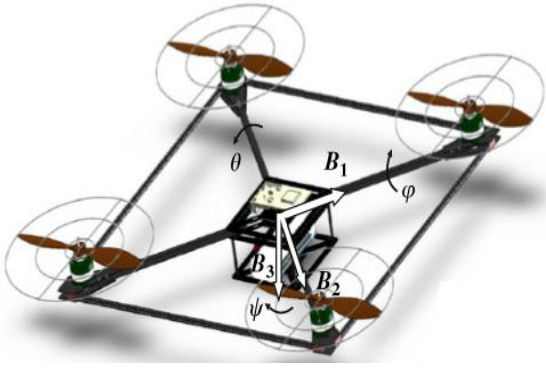

Figure 2. DraganFly Quadrotor [18]

TABLE I. QUADROTOR MODEL PARAMETER VALUES

Variable	Description	Value	Units
g	Gravity force	9.81	ms^{-2}
m	Quadrotor mass	0.53	Kg
I_x	Inertia around x-axis	5×10^{-3}	$Kg.m^2$
I_y	Inertia around y-axis	5×10^{-3}	$Kg.m^2$
I_z	Inertia around z-axis	8.9×10^{-3}	$Kg.m^2$
l	Length from rotor to cm	0.225	m

A. Propeller Characteristics

Quadrotors consist of propellers that are attached to the motor rods, they are commonly manufactured through two or more blades and a central hub that is placed coincident to the rotor rod. Reaching a desired altitude from ground level obliges that there must be an upward thrust in all actuators simultaneously. Each brushless motor (BLDC) generates a thrust which is theoretically expressed as [20, 21, 22]:

$$T_i = K\Omega_i^2 \tag{15}$$

Where T_i is the thrust moment for the corresponding BLDC motor i, Ω_i^2 is the angular speed of the BLDC motor i and K is a constant that represents either the thrust factor b or the drag factor d. The K constant is chosen according to the desired orientation of the quadrotor. In the form of Bernoulli's equation [23, 24, 25], one can come to conclude that the thrust and drag factor can be calculated as:

$$b = C_T \rho D^4 \tag{16}$$

$$d = C_P \rho D^5 \tag{17}$$

Where C_T is the thrust coefficient, C_P is the power coefficient, ρ is the air density and D is the propeller diameter. Assuming that a quadrotor is rotating about the z-axis, the propellers will generate a drag moment acting in the opposite direction of which it is turning in the horizontal direction [26]. Hence, the drag factor that determines the power required to spin the propeller is considered.

With regards to the variables mentioned in equations (16) and (17), the thrust and drag coefficients can be collected from the manufacturers or propellers datasheet. Numerous papers such as [18, 25, 27] provided the values of drag and thrust factors, these parameters describe the propellers used in their study. For example, a propeller that researchers may consider investigating is a carbon fibre T-Style 10x5.5 with the aerodynamics characteristics obtained from [20] and are depicted in table II:

TABLE II. T-STYLE 10X5.5 PROPELLER KEY PARAMETER

Parameter Names	***Symbol***	***Value***
diameter	d	$0.254\ m$
Thrust coefficient	C_T	0.121
Power Coefficient	C_P	0.0495
Air density	ρ	$1.255\ Kg/m^3$

Using the parameter values from Table II, the thrust and drag factor are calculated as:

$$b = C_T \rho D^4 = 6.317 \times 10^{-4} \tag{18}$$

$$d = C_P \rho D^5 = 1.61 \times 10^{-4} \tag{19}$$

B. Actuator Characteristics

With regards to selecting an appropriate motor for this study, the quadrotor is expected to orientate and move at the maximum pace without causing any complications to the vehicle such as tumbling or exceedingly losing stability due to speed. The motors must also be able to generate enough aerodynamic loads to lift the vehicle without using maximum actuator speed. Therefore, the selected actuator for this model are the BN12 BLDC motor [28].

TABLE III. BN12 BLDC MOTOR PARAMETERS

Parameter Name	***Value***	***Unit***
Nominal Voltage	12	$Volts$
Rated Speed	13,027	RPM
No-Load Speed	~15,900	RPM
Terminal Resistance	0.953	$Ohms$
Terminal Inductance	0.254	mH
Back EMF	0.0072	$volts/rad/sec$
Torque Constant	0.0072	Nm/amp
Rotor Inertia	2.82	$g - cm^2$

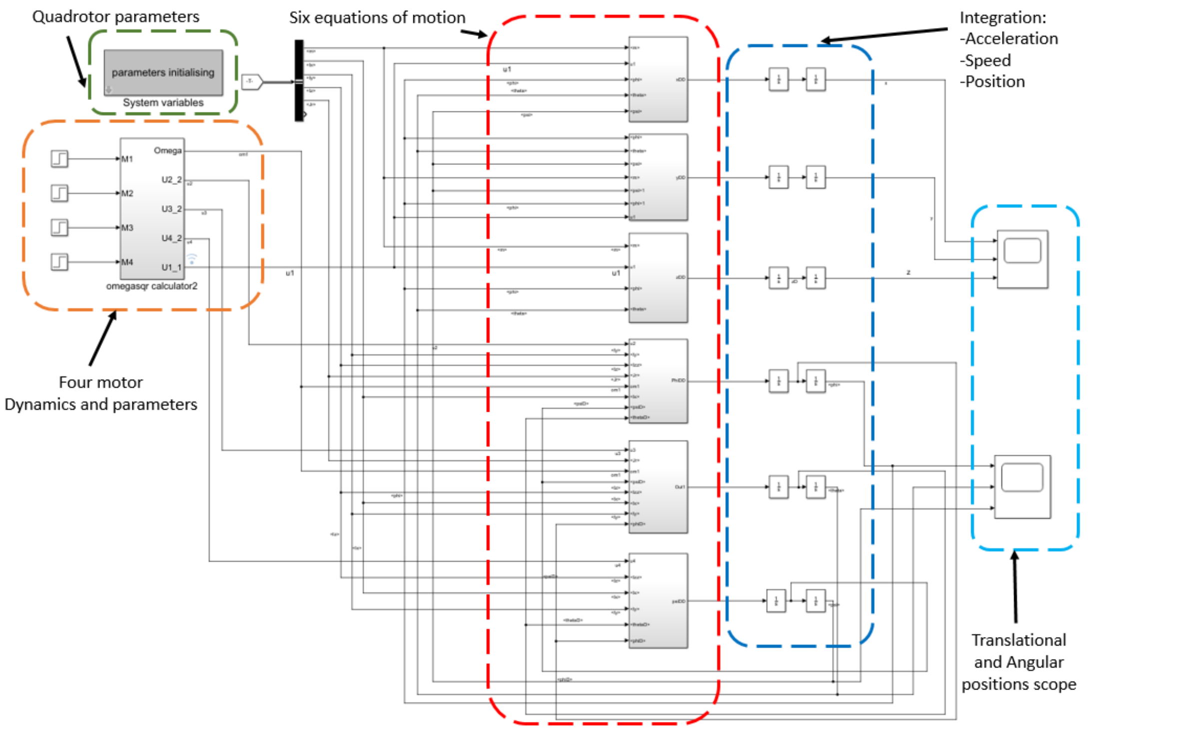

Figure 3. Simulink block diagram of the quadrotor dynamic model

IV. SIMULATION RESULTS AND DISCUSSION

The target of the Simulink block diagram is to verify the correctness of the quadrotor helicopter by calculating the expected altitude when the vehicle moves at a certain velocity. For ease of understanding, Fig. 3 was split into four regions, the first region (orange) allows users to input the specified voltages for each motor where the motor dynamics are processed, the angular speed achieved from these motors is then incorporated into the control mixer where the four inputs from equation (7) is calculated while considering the thrust and drag factors of the propellers. The results attained are the thrusts that are imported into the six equations of motion (red) enforcing the UAV to achieve angular and vertical motions. Finally, integrating the acceleration will present the speed of the specified state, integrating a second time will present the position of the UAV in the Cartesian coordinate system.

With regards to analysing the quadrotors motion specifically in the vertical direction, Increasing the speed of the actuators to achieve thrust will create motion in the desired trajectory. However, in order to ensure that the vehicle moves in the correct direction and that acceleration, velocity and position is overviewed and assessed while considering gravity, the geometry of motion or what is often referred to as kinematics will be considered. This method begins by describing the systems geometry as a mathematical problem where the initial conditions of any known position, speed and/or acceleration can be declared. Following that, these positions can be calculated using pre-defined equations to explain the behaviour of the system [29]. The kinematics formula is based on the presented variables below, if three out of the four variables is known, one can use the kinematic approach to determine the motion of the vehicle. These are:

$$v_f = v_i + at \tag{20}$$

$$\Delta x = \left(\frac{v_f + v_i}{2}\right) t \tag{21}$$

$$v_f^2 = v_i^2 + 2a\Delta x \tag{22}$$

$$\Delta x = v_i t + \frac{1}{2} a t^2 \tag{23}$$

Where, v_f is the final velocity, v_i is the initial velocity, t is the time interval, a is the constant acceleration and Δx defines the displacement of the vehicle.

By ensuring that the calculations presented above match the experimental results in Simulink, the performance will be based on the thrusts applied on the quadrotor. Hence, four studies will be carried out in order to analyse the behaviour of the vehicle. First, a 10N force that sums up the four motors will be applied on the vehicle for 1 second. This analysis is simply set in order to ensure that the kinematic equations applied matches the experimental results. Secondly, the angular velocity of each motor will be increased to achieve an overall force of 19.8N, where the expected motion of the UAV from ground levels is expected to rise faster. Applying the thrusts for 2 seconds then instantaneously reducing the speed to zero will cause the UAV to reach an equilibrium point in the air before descending due to gravity. Thirdly, in the process of analysing the motion, the velocity will be theoretically studied followed by a comparative study against the experimental results. Finally, the motion will be further analysed by initiating an overall thrust of 15N for 1 second followed by a second iteration of 15N at 3 seconds for 1 second again. This will demonstrate a motion such that the UAV will rise until the equilibrium point is reached then before descending, the vehicle will begin to rise again due to the force implemented on the second iteration.

Fig. 4 illustrates the experimental results attained from Simulink based on the block diagram presented in Fig. 3. Based on the results attained, we have specifically focused on critically analysing the vertical motion of the UAV in numerous ways. First, Fig. 4 (a) illustrates the general behaviour of the vehicle as a sum force from the four motors is set to 10N. By applying the kinematics approach, one can theoretically calculate the expected vertical motion the UAV can reach while considering gravitational acceleration.

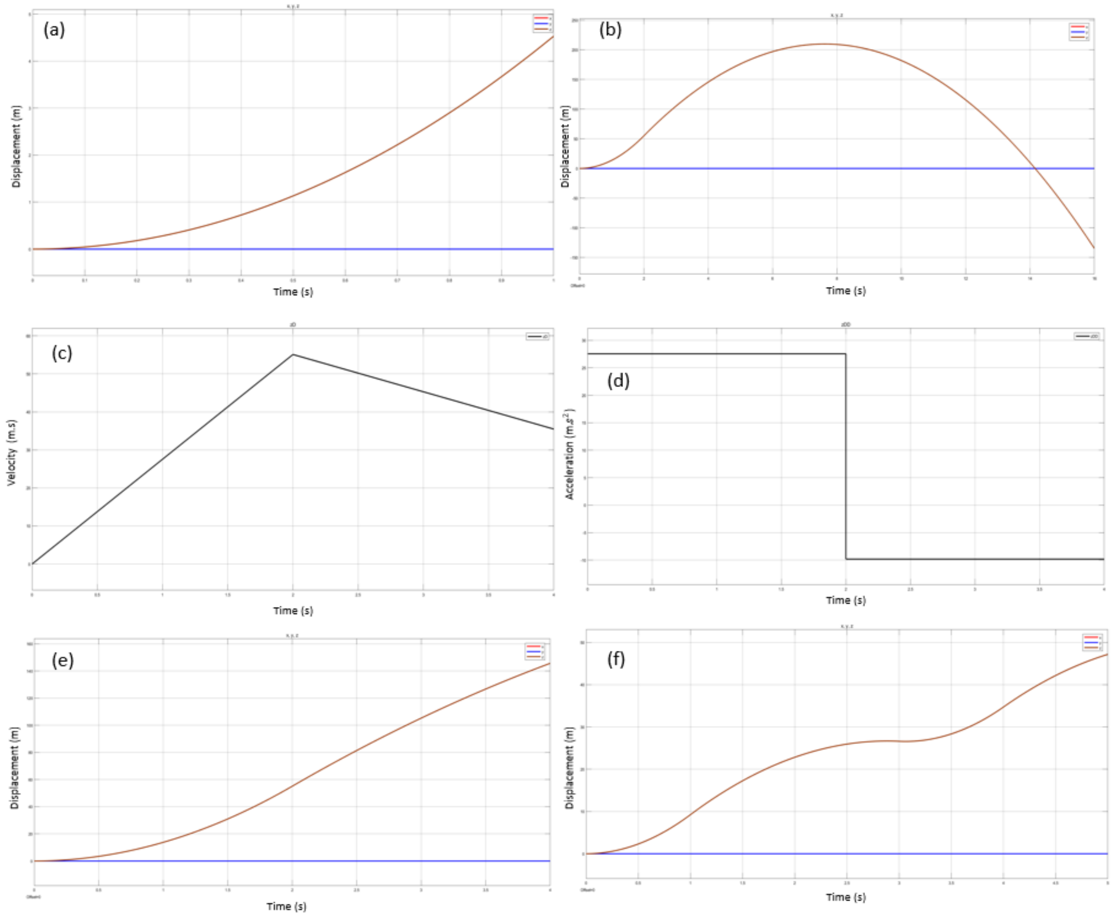

Figure 4. The vertical reaction of the UAV as Forces equivalent to four motors are applied on Simulink: (a) Applying 10N thrust for 1s, (b) Applying 19.8N thrust for 2s, (c) Maximum speed reached from the 19.8N force, (d) Acceleration achieved from the 19.8N Force, (e) Viewing the position of the UAV at 2s as 19.8N force is applied, (f) Applying 15N force for 1s then 15N at 3s for 1s

Second, Fig. 4 (b) shows the overall vertical motion if a 19.8N is applied for 2 seconds. By analysing the behaviour with an increased force of 9.8N compared to the previous study, the position of the UAV has dramatically increased which shows that the motion of the vehicle will increase as the force is also increased. Studying the behaviour of the physical system in Simulink illustrates that the final position reached at 16 seconds is -134.49 meters due to the gravitational acceleration which continuously causes the UAV to descend. Therefore, one can assume that the UAV has reached ground once a negative positional value is reached.

In order to justify the results theoretically using kinematics approach and newton's equation of motion, Fig. 5 illustrates a comparative study between the experimental results and the theoretical results. We can clearly see both results are promising in which the dynamic behaviour of the vehicle is correct and that the difference in error is very minimal. Fig. 5 (a) illustrates the motion of the UAV as a 10N force is applied for 1 second. Fig. 5 (b) illustrates the overall motion of the UAV as a 19.8N force is applied for 2 seconds. Both results illustrate that a maximum height of approximately 209 meters has been reached before the vehicle began descending due to gravitational acceleration. Fig. 5 (c) presents the velocity of the vehicle based on the motion in Fig. 5 (b) where after 2 seconds, the velocity began reducing due to no voltage being applied on the motors. Finally, Fig. 5 (d) presents a different study where the behaviour of the UAV changes as the forces are also changed at different interval.

V. Conclusion

In this paper, the quadrotor dynamic model has been presented which consisted of; the Euler's rotational equation of motion that are dependent on the thrusts generated by the actuators, the gyroscopic effects of the UAV as it rotates about a certain axis and the aerodynamic effects of the propeller. Considering all of these aspects for the quadrotor has allowed us to achieve the full dynamic model. Theoretical assumptions were implemented based on the proposed system where the results have been identified for the sole purpose of comparing such data against Simulink. The discussions of these results illustrates that the performance of the UAV in Simulink were very similar to the theoretical results. Further analysing the behaviour by changing the force or applying it at different intervals have presented various motions that satisfies the expected results from the dynamic system in which the behaviour of both models have demonstrated realistic responses. By introducing this method of assessing the dynamic system, many opportunities are now open into analysing the behaviour of the desired quadrotor model before implementing any controllers, saving time and costs from any imminent errors. Future contributions to this research will involve considering the behaviour of the vehicle as it rotates in various directions and the possibility of assessing two degrees of freedom simultaneously, that is, assessing how far will the UAV move along a lateral axis if it was to rotate a certain amount of degrees.

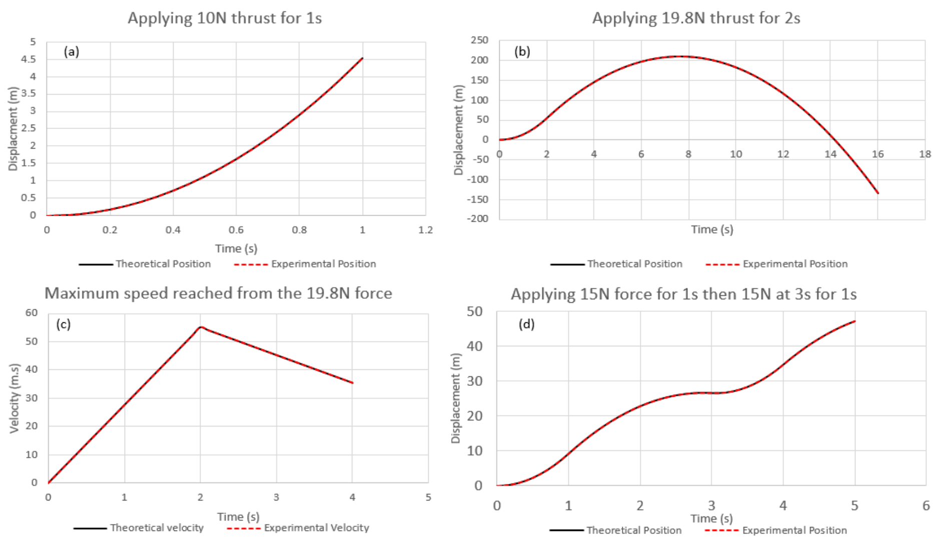

Figure 5. Theoretical results vs Experimental results, (a) Applying 10N thrust for 1s, (b) Applying 19.8N thrust for 2s, (c) Maximum velocity based on results attained on 19.8N thrust, (d) applying 15N at different iterations

Conflict of Interest

The authors declare no conflict of interest

Author Contributions

Moad Idrissi has contributed to this research by implementing the experimental and analytical work with the guidance and supervision from Dr Fawaz Annaz.

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Moad Idrissi has previously achieved a bachelor's Honor degree in Electronic Engineering in 2016. He then moved on to study an MSc in Embedded System at the University of Birmingham which was completed in 2017. Currently, Moad is now trying to achieve PhD within the field of UAVs in challenging environment at Birmingham City University.

Dr Fawaz Annaz had a long international career in Mechatronics Engineering, where he has taught and conducted research in many countries including the UK, Singapore, Japan, Malaysia and Brunei. In 1996, he received a PhD in Avionics that addressed 'Architectures Consolidation and Monitoring Methods in High Integrity Multi-Lane Smart Electric Actuators'. The research addressed the development of actuation architectures that were designed to move control surfaces similar to that of an aileron on the Sea-Harrier. This activity continued to grow under the international banners of High Integrity System Laboratory. Over the years, he has led several research groups, including the High Integrity System Laboratory in Japan and Malaysia, and the Mechatronics Group in Malaysia. He has also acted as a principal investigator and co-investigator to several international internal and external projects (including the Ministry of Education in Japan).