Design of Tendon-Driven Mechanism by Using Geometrical Condition

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Abstract— **A tendon-driven robot has an advantage of setting mass distribution and facilitation of the motion and so on. This paper explains a procedure of designing the tendon-driven robot using single actuator. The joint torque is generated by the single actuator and wire that passes through the pulleys attached on the links. In order to setting pulley position, a geometrical condition to realize the desired motion is adopted. For evaluation of the proposed method, a physical monopod robot is developed for vertical jumping. We compared calculated and physical force on the contacting point of the robot, and observed the jumping motion.**

Index Terms—**tendon driven mechanism, geometrical condition, Jacobian matrix**

I. Introduction

A Wire driven or tendon-driven robot whose joint is driven through a wire has been discussed in many aspects of robotics area. Many manipulators and hands are driven by the wire [1][2][3]. One of the advantages of the wire-driven or tendon-driven mechanism is that an actuator is set apart from the joint. In case of the robot finger, a volume of the finger is very limited, and it is difficult to set actuator to drive the finger joint in the finger link.

Some studies have developed legged robots driven by the wire [4][5][6]. The advantage of the tendon-driven leg is that the mass and the inertia of the leg link can be small by setting actuator apart from the leg link. Seok and Kim et al. pointed out that the mass and inertia should be small to swing the leg for faster locomotion [7]. For the jumping robot in which all joints are driven by actuators one by one, the large landing impact is occurred due to heavy mass of the actuators, and it leads breaking the joint and actuator. On the other hand, the tendon-driven mechanism with small number of the actuator avoids generating such heavy landing impact. Higashimori et al. explained the wire-driven mechanism in which multiple numbers of joints are driven by single actuator though it is adopted for the robot hand [2]. One of the mechanical characteristics of the tendon-driven mechanism is that multiple joints are driven simultaneously by pulling the wire that passes through via points, that is, pulleys, attached on the links. Therefore, it is possible to reduce the number of actuator.

As some studies have pointed out, the position of the pulley through which the wire passes influences a motion of the robot because the force by the wire is added on the pulley and the joint torque in case of the rotational joint is changed by the position of the pulley. Higashimori et al. derived the equation of the tendon-driven mechanism that explains the relationship between the position of the pulley and each joint torque. In many studies, the position is determined by the revision from the iterative trials because the mechanism of the motion is so complex and then it is difficult to explain the relationship between task performance and position of the pulley by using a formulation

In this paper, we deal with a jumping robot that generates vertical force to the ground. We revise Higashimori's tendon-driven mechanism by a single actuator [2], and derive a formulation in order to generate desired direction and magnitude of the kicking force for the vertical jump. For verification, we develop a physical jumping robot, and observed whether the desired force is generated at the contacting point of the foot. Although this paper discusses the vertical force, that is, the horizontal force is restrained, the proposed method can determine not only magnitude of the force but also direction of the force.

II. Robot Model

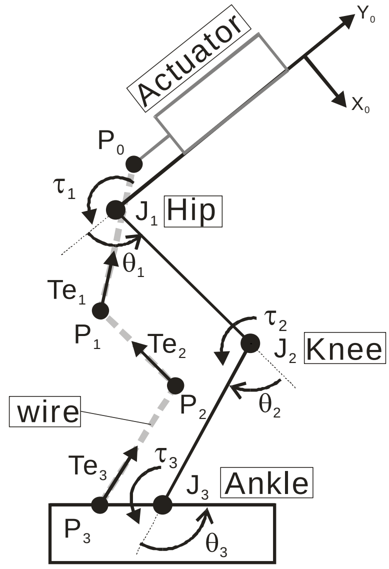
A. Framework and Tendon-Driven Mechanism [8]

Fig. 1(a) shows the frame of the robot and the driving mechanism consisting of a single actuator and a wire set. Fig. 1(b) shows position of the pulley based on the coordinate system Σ_k: the origin is set on the tips of the foot, the X_k axis is set along the foot link, and the Y_k axis is perpendicular to the X_k axis. The model has four links: the trunk, thigh, shank, and foot. It has three joints: the hip, knee, and ankle. The lengths of the links are denoted by l_1 – l_4, respectively, and torques of the hip, knee, and ankle are denoted by τ_1, τ_2, and τ_3, respectively. The angle is denoted by θ_1, θ_2, and θ_3. For derivation of the relationship between the positions of the pulley and a reaction force, we set the coordinate system Σ_0: the origin is set on the tips of the trunk, the Y_0 axis is set along the trunk link, and the X_0 axis is perpendicular to the Y_0 axis, as shown in the figure. The positions of the pulley and end point of the wire are labeled P_i (i = 0, 1, 2, 3) and the joints are labeled J_i (i = 1, 2, 3). Note that the point where

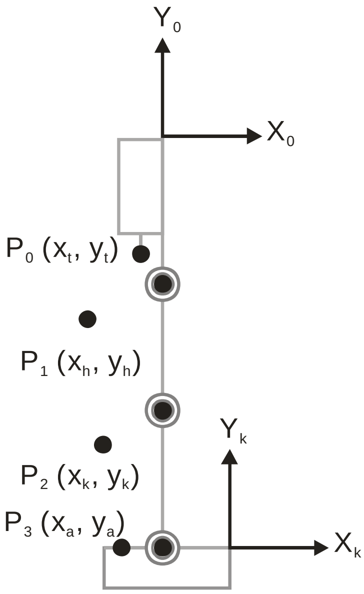
Manuscript received April 5, 2019; revised March 9, 2020.

doi: 10.18178/ijmerr.9.5.759-763

the wire is attached to the actuator is P_0 and the other endpoint of the wire is P_3. P_1 is attached to the thigh and P_2 is attached to the shank link. In this paper, the position of the attaching point P_0 did not move when the actuator pulled the wire, although the actual linear actuator does move while pulling the wire. J_1, J_2, and J_3 correspond to the hip, knee, and ankle joint, respectively. e_i (i = 1, 2, 3) is a unit vector of tensional force on the pulley P_i and T is the magnitude of pulling force of the actuator.



(a) Wire, pulley, and joint configuration



(b) Position of the pulley with respect to Σ_k

Figure 1. Joint configuration and position of pulley

B. Condition for the Vertical Jumping

In order to derive the numerical model, we have some assumption. One is that the model is constrained on the sagittal plane. The other is that the interaction between the robot and the ground is occurred instantly and then the joint angles are not changed during the lifting off motion and the reaction force is added at the toe. We also have an assumption that the link moments are ignored. We also applied the condition that the reaction force on the toe should be vertical and the projection of the center of mass (CoM) on the ground should correspond to the toe; otherwise, the robot tumbles while in the air. In order to set CoM on the ground, joint angles $\theta_1\ \theta_2\ \theta_3$ are determined. In this paper, the angles are set as $61.35°, -63.03°$, and $123.54°$, respectively. For discussing the position of the pulley, we assume that the weights of pulley and wire are zero.

C. Mathematical Model [8]

Based on the condition mentioned above, we examined the position of the pulley that provides a vertical reaction force without a horizontal force being exerted on the toe following the assumption.

Figure 2(a) shows the relationship between the joint torques and the force on the toe. The horizontal and vertical forces generated by the joint torque are expressed as F_x and F_y, respectively. Following the principle of virtual work, the relationship between the joint torque $(\tau_1, \tau_2, \tau_3)^T$ and $(F_x, F_y)^T$ is expressed as

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \mathrm{J}^{\mathrm{T}} R \begin{pmatrix} F_x \\ R_y \end{pmatrix}, \mathrm{J} = \begin{pmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \end{pmatrix}, \qquad (1)$$

where (x, y) is the position of the toe as

$$x = l_2 \sin\theta_1 + l_3 \sin(\theta_1 + \theta_2) + l_4 \sin(\theta_1 + \theta_2 + \theta_3),$$
$$y = -l_1 - l_2 \cos\theta_1 - l_3 \cos(\theta_1 + \theta_2) - l_4 \cos(\theta_1 + \theta_2 + \theta_3).$$

R is the rotation matrix from the force based on Σ_0 to $(F_x, F_y)^T$,

$$\mathrm{R} = \begin{pmatrix} \sin\Theta & \cos\Theta \\ -\cos\Theta & \sin\Theta \end{pmatrix}, \Theta = \theta_1 + \theta_2 + \theta_3$$

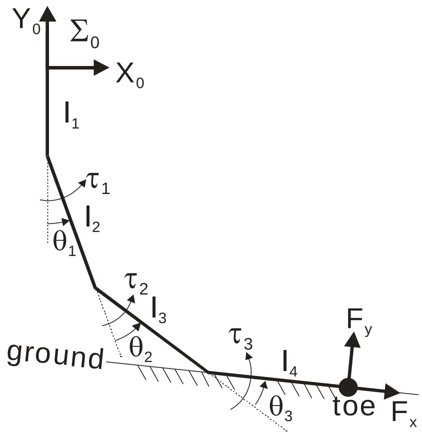
The joints are driven by the force applied on the pulley. The joint torque is determined by the amount of pulling force T, the length of moment arm between the joint and the pulley, and the angle between the moment arm and the direction of pulling force as shown in Fig. 2(b). Higashimori et al. derived the torques for such a tendon-driven mechanism with a single wire and actuator [2]. In their study, the radius of the pulley is assumed to be non-zero, that is, the pulley was assumed to have a certain volume. For simple expression, in this paper, it was assumed that the radius of the pulley is infinitely small. Following this assumption, the relationship between the position of pulley and torques are expressed by modifying Higashimori's equation as

$$\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = -\mathrm{T} \begin{pmatrix} (\mathbf{p_1} - \mathbf{j_1}) \otimes \mathbf{e_1} - (\mathbf{p_2} - \mathbf{j_2}) \otimes \mathbf{e_2} \\ (\mathbf{p_2} - \mathbf{j_2}) \otimes \mathbf{e_2} - (\mathbf{p_3} - \mathbf{j_3}) \otimes \mathbf{e_3} \\ (\mathbf{p_3} - \mathbf{j_3}) \otimes \mathbf{e_3} \end{pmatrix}, \qquad (2)$$

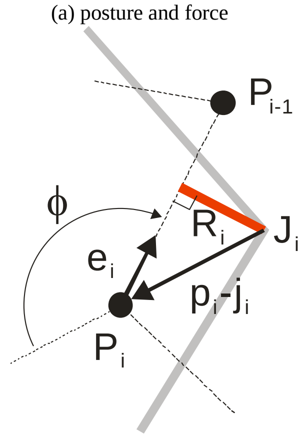
where $\mathbf{p_i}$ (i = 1, 2, 3) is a position vector of P_i, $\mathbf{j_i}$ is that of joint J_i, e_i is a unit vector from P_i to P_{i-1} (see Fig. 1). $\otimes$ is the operator as

$$(x_1, y_1)^T \otimes (x_2, y_2) = x_1 y_2 - y_1 x_2.$$

By combining Equations (1) (2) and (3), the relationship between force (F_x, F_y) and position of pulley P_i is explained.



(a) posture and force



(b) distance R_i

Figure 2. Mathematical model

D. Geometrical Condition for the Vertical Jumping

Considering that the vector e_i in Equation (2) is the unit vector, the term of $(\mathbf{p_i}-\mathbf{j_i})\otimes\mathbf{e_i}$ (i = 1, 2, 3) in Equation (2) is deformed as

$$(\mathbf{p_i}-\mathbf{j_i})\otimes\mathbf{e_i} = |\mathbf{p_i}-\mathbf{j_i}|\sin\phi \triangleq R_i, \qquad (3)$$

where ϕ is the angle between $\mathbf{p_i}-\mathbf{j_i}$ and $\mathbf{e_i}$ (see Fig. 2(b)). Therefore, the term R_i is expressed by the length between the line of P_{i-1} and P_i and the joint J_i.

As explained above, in order to achieve vertical jumping, the horizontal force F_x should be zero. Therefore, by substituting $F_x = 0$ in Equation (1) and combining the equations (1) (2) and (3), the length of R_i is expressed as

$$\begin{pmatrix} R_1-R_2 \\ R_2-R_3 \\ R_3 \end{pmatrix} = \mathrm{K}\begin{pmatrix} l_2\cos(\theta_2+\theta_3)+l_3\cos\theta_3+l_4 \\ l_3\cos\theta_3+l_4 \\ l_4 \end{pmatrix}, \qquad (4)$$

where $\mathrm{K} = -F_y/T > 0$ is the ratio of the force of pulling wire T and the reaction force from the ground −Fy. Considering that the distance R_i is the distance from joint to the line P_i-P_{i-1}, the line P_i-P_{i-1} is determined by calculating R_i. From the third row, the length R_3 is obtained when K is determined. From the second row, R_2 is automatically obtained when the joint angles that satisfy the condition of the CoM are determined. From the first row, R_3 is also obtained. Because the positions of end points P_0 and P_3 are fixed, the positions of P_1 and P_2 are constrained by the condition of R_1, R_2, and R_3. By using such geometrical condition, pulley position that generates desired force on the toe can be derived such as

1. Calculate a constant value $K=-F_y/T$ by determining $F_y$and T
2. Circle C_i (i=1,2,3) whose radius is R_i and center is the joint J_i is set (see Fig.3)
3. A line t_1 that tangents circle C_1 and passes through P_0 is set. t_1 is the set of candidate of pulley position P_1.
4. A line t_3 that tangents circle C_3 and passes through P_3 is set. t_3 is the set of candidate of pulley position P_2.
5. Pulley position P_1 is determined on the line t_1, or the position P_2 is determined on the line t_3

6-1. When P_1 is determined, a line t_2 that tangents circle C_2 and passes through P_1 is set, and P_2 is determined as an intersection point of t_2 and t_3

6-2. When P_2 is determined, a line t_2 that tangents circle C_2 and passes through P_2 is set, and P_1 is determined as an intersection point of t_1 and t_2

Although the force F_x is set zero because this paper deals with the vertical jumping, this approach can be adopted for determining the position of the pulley in order to achieve an arbitral direction of the force of the end effector (F_x, F_y).

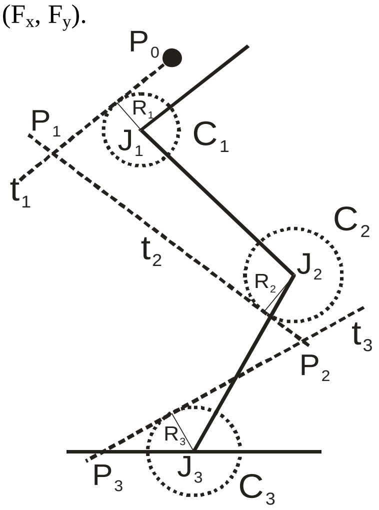


Figure 3. Circle C_i and tangential line t_i

III. Physical Robot

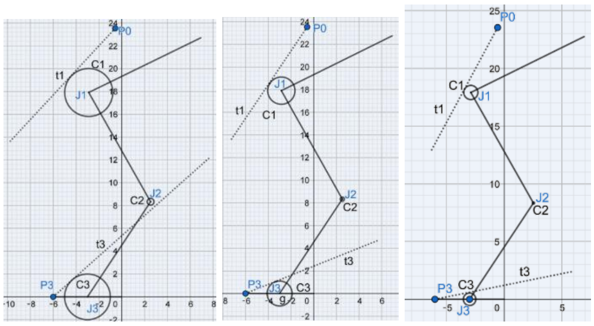
A. Configuration of the Robot

Fig. 4 shows a developed prototype of the single leg jumping robot. The links are made of light ABS and PLA by using 3D printer (XYZprinting Da Vinci 1.0 pro). The actuator is a pneumatic cylinder (SMC CDUJB6-20DM) that generates the tensional force as T = 11.0 N when the pressure of the supply air is 0.7 MPa. The lengths of the trunk, thigh, shank and foot is 0.135m, 0.11m, 0.1m, and 0.03m, respectively, i.e. l_1=0.135, l_2=0.11, l_3=0.1, and l_4=0.3 m. The position of pulley P_0 that corresponds to the point generating the tensional force by the actuator, and the position of pulley P_3 that corresponds to the terminal point of the wire attaching on the foot is set as (-0.04, 0.275) [m] and (-0.06, 0.0) [m] when the joint angles are $\theta_1 = 0.0, \theta_2 = 0.0, \theta_3 = \frac{\pi}{2}$ rad, respectively.

Figure 4. Developed 3-DoF jumping robot

B. *Numerical Solution of the Pulley Position*

By setting joint angles θ_1, θ_2, and θ_3 and position of pulley P_0 and P_3, the geometrical condition of P_1 and P_2 are determined by setting the vertical force F_y. Fig. 4 shows the circles C_i and the tangential line l_i when the force F_y is -7 N (Fig. 5(a)), -4 N (Fig. 5(b)), and -2 N (Fig. 5(c)) and T is fixed as 10 N. Seen from the figures, the circle C_i and the tangential line l_i depends on the vertical force F_y as well as the tensional force T. Table I shows a relationship between the vertical force F_y and the radius of the circles C_1, C_2, and C_3 when the tensional force of the actuator T is fixed as 10 N.



(a) F_y = -7N (b) F_y=-4N (c) F_y = -2N

Figure 5. Circles Ci and the tangential line li depending on the vertical force F_y as well as tensional force T (T is fixed as 10 N)

TABLE I. RELATIONSHIP BETWEEN THE VERTICAL FORCE FY AND THE RADIUS OF CIRCLES C1, C2, AND C3. THE TENSIONAL FORCE T IS FIXED AS 10 N. THE UNIT OF FY AND RADIUS ARE [N] AND [MM], RESPECTIVELY.

F_y	C_1	C_2	C_3
-9.0	27.6	3.9	24.6
-8.0	24.5	3.5	21.8
-7.0	21.4	3.0	19.1
-6.0	18.4	2.6	16.4
-5.0	15.3	2.2	13.6
-4.0	12.2	1.7	10.9
-3.0	9.2	1.3	8.2
-2.0	6.1	0.9	5.5
-1.0	3.1	0.4	2.7

IV. EXPERIMENT

A. *Measurement of Vertical Force*

In order to confirm the relevance of the pulley position that is determined by the desired vertical force, the force is measured. Fig. 6 shows the experimental setup to measure the vertical force F_y. The position of the pulley is calculated following previous procedure by the desired force Fy (Fy = -6.5, -6.8, -8.4 N). The force is measured by a scale under the foot assuming that the force is generated on the toe. The tensional force T is 10 N. Table 1 shows position of the pulley determined by the vertical force. It also shows the measured vertical force. As shown in the table, the measured vertical force almost matches with the desired force. In the measurement, we confirmed that the foot did not slide on the scale, i.e., the horizontal force F_x did not occurred.

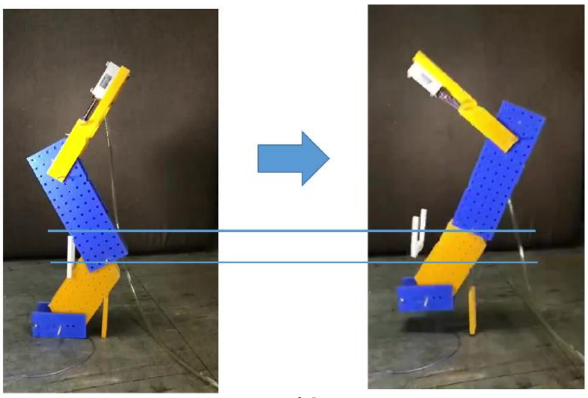
Figure 6. Experimental setup

TABLE II. PULLEY POSITION AND DESIRED AND MEASURED VERTICAL FORCE

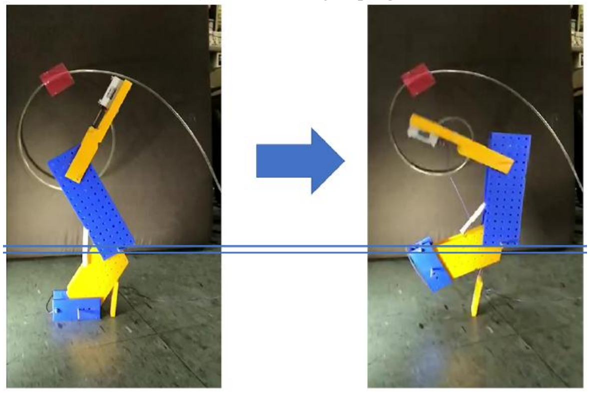
desired F_y [N]	x_k [m]	y_k [m]	x_h [m]	y_h [m]	measured F_y [N]
-6.5	-0.03	0.09	-0.01	0.24	-6.1
-6.8	-0.01	0.09	-0.03	0.23	-6.5
-8.4	-0.03	0.10	-0.04	0.23	-8.4

B. *Vertical Jumping*

By setting the pulley position, we observed the robot jumping. Fig. 7 (a) shows the successful jumping when the pulley position was set following the proposed geometrical condition ((x_k, y_k) = (-0.01, 0.09) [m], (x_h, y_h) = (0.03,-0.23) [m], Fy = -6.8N). For comparison, we set another pulley position that did not follow the condition ((x_k, y_k) = (-0.01, 0.06) [m], (x_h, y_h) = (-0.03,-0.23) [m]). The jumping motion is shown in Fig. (b). As shown in Fig. 7(a), the knee joint lifted up to the vertical direction. This means that the force was occurred toward vertical direction, and then the robot jumped. On the other hand, as shown in Fig. 7(b), the knee joint did not lift up toward vertical direction, and it moved toward horizontal direction. This means that the horizontal force is occurred on the toe, and then the robot failed to jump.



(a) Successful jumping



(b) Failed jumping

Figure 7. Jumping experiment

V. Conclusion

In this paper, we proposed the geometrical condition that determines pulley position for generating the desired reaction force on the toe. In order to achieve the desired vertical jumping, we figured out the condition of the vertical jumping that the projection of the center of mass before jumping is set on the toe, and the horizontal force should not be generated. For the first condition, the posture of the robot is determined. For the second condition, we adopted Higashimori's equation of the wire-driven mechanism by a single actuator. We also derived from the equation to the geometrical condition in order to generated desired horizontal and vertical forces. For the vertical jumping, by substituting the horizontal force into zero, the pulley position is determined by setting the magnitude of the desired vertical force. In the experiment, we confirmed that the measured and desired vertical force are matched, and the robot driven by the wire jumps vertically.

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