Preliminary Shape Design for Screws and Helical Structures

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***Abstract*—Due to continuous development of additive technologies and perspectives for their practical application in many fields of engineering such as architecture and civil engineering, mechanical engineering, etc., the question of new approaches and requirements for design process is becoming a key point. In this framework the search for the rational or optimal shapes seems reasonable. In the paper, the investigation on the design of several types of helical surfaces which can be applied in different fields of mechanical industry is presented, along with the parametric equations for ruled helicoids which can be useful for further application using additive technologies. Different classifications of ruled helical surfaces are presented and compared from technical and mathematical points of view. There are also some recommendations for usage the proper types of helical structures in accordance with geometry investigation and effective material usage. The paper can be interesting for designers from mechanical and civil engineering, architecture and industry design.**

***Index Terms*—helicoids, screw, rational type, right helicoids, developable helicoids, convolute helicoids**

I. Introduction

Helical structures can be found in almost any field of engineering design and industry. The most popular examples are the screws, blades, some kinds of twisted beams. According to analytical geometry, these structures have the shape of helical ruled analytical surfaces which are generally called helicoids, but despite the wide application of these surfaces in industry, they are still not widely known by designers and engineers or have different classifications, names and general attitude.

However, helical ruled surfaces can suggest a reasonable variety of properties; they differ from each other and can be used more effectively when learning their geometry deeper. The aim of this paper is to show the variety of these surfaces and suggest the designers to use them after a certain work of rationalization. In this framework the search for the rational or optimal shapes seems reasonable.

II. Different Types of Ruled Helical Surfaces

There are various classifications and definitions for helicoids.

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A. Classifications in Mathematics and Open Access Resources

If we look into the open access resources, we will find several different mathematical definitions for helicoids.

According to the Encyclopedia of Mathematics (2014) [1]: helicoid is a ruled surface described by a straight line that rotates at a constant angular rate around a fixed axis, intersects the axis at a constant angle α, and at the same time becomes gradually displaced at a constant rate k along this axis. The equation of a helicoid in parametric form is [1]

$$\begin{aligned} x &= \rho \cos t \\ y &= \rho \sin t \\ z &= \rho \arctan \alpha + kt. \end{aligned} \tag{1}$$

If $\alpha=\pi/2$ the helicoid is called straight or right, otherwise it is called oblique.

According to Maple17 (2017) helicoid is “any solid or surface shaped like a screw thread”.

According to Wolfram MathWorld (2018) [2]: “The (circular) helicoid is the minimal surface having a (circular) helix as its boundary. It is the only ruled minimal surface other than the plane (Catalan, 1842 [3], do Carmo, 1986 [4]). For many years, the helicoid remained the only known example of a complete embedded minimal surface of finite topology with infinite curvature. However, in 1992 a second example, known as Hoffman's minimal surface and consisting of a helicoid with a hole, was discovered (Sci. News 1992). The helicoid is the only non-rotary surface which can glide along itself (Steinhaus 1999, p. 231 [5])”.

According to Wikipedia (2018) [6]: Helicoid “was described by Euler in 1774 and by Jean Baptiste Meusnier in 1776. Its name derives from its similarity to the helix: for every point on the helicoid, there is a helix contained in the helicoid which passes through that point”.

“A generalized helicoid is a surface in Euclidean space generated by rotating and simultaneously displacing a curve, the profile curve, along a line, its axis. Any point of the given curve is the starting point of a circular helix. If the profile curve is contained in a plane through the axis, it is called the meridian of the generalized helicoid. Simple examples of generalized helicoids are the helicoids. The meridian of a helicoid is a line which intersects the axis orthogonally. Essential types of generalized helicoids are ruled generalized helicoids

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(their profile curves are lines and the surfaces are ruled surfaces) and circular generalized helicoids (their profile curves are circles). In mathematics helicoids play an essential role as minimal surfaces. In the technical area generalized helicoids are used for staircases, slides, screws, and pipes".

According to the Encyclopedia of Analytical Surfaces by Krivoshapko and Ivanov [7]: "A helical surface is formed by a curve *L* in the process of its helical motion… A ruled helical surface is formed by a straight generatrix disposed in arbitrary position under its ordinary helical motion". Five types of ruled helicoids are presented in the Encyclopedia as the following: open (evolvent); convolute; pseudodevelopable; oblique; right.

B. Classifications in Engineering Papers

In engineering papers, the ruled helicoids are found to be classified as the following:

Right; Oblique; Developable; Pseudo-developable of general type offered by S.F. Pilipaka [8]; Pseudo-developable by Krivoshapko and Rynkovskaya in 2017 [9], where an overview of five types of ruled helical surfaces which can be used for helical conveyers, support anchors and screws is provided.

Right; Inclined; Deployable; Convoluted; Elliptic by Jean Paul in 2018 [10], where there is an attempt to review the geometry investigations of helicoids, but the given classification does not seem absolutely clear.

Archimedes's screw; evolvent screw; convolute screw by Kheifetc et al. in 2018 [11], where it is mentioned that "convolute screw has a range of advantages in comparison with Archimedes' screw".

There are also different names for the same types of helicoids, and the choice of the name generally is caused by the author's field of expertise [12], [13].

III. Geometry Investigations

The classification based on mathematical point of view along with other possible names of helicoids is shown in Table I.

Two general types differ depending on the generator position relatively to the axis: if the generator intersects the axis the helicoid is called the helicoid of a "closed type" (Fig. 2, *a*); if the generator never intersects the axis and it is a tangent to the inner cylinder the helicoid is called the helicoid of an "open type" (Fig. 2, *b*).

Figure 1. Screw piles.

TABLE I. Names of 5 Types of Helicoids in Mathematical and Engineering Resources

Ruled generalized helicoids					
General types	Closed		Open		
Particular types in Maths	Right closed type [6]; Helicoid [1]	Oblique closed type [6]	Right open type [6]	Oblique open type [6]; Convolute [13]	Tangent developable type [6]
Alternative types in Engineering	Right [8], [9]; Straight [11]	Oblique [7], [17]; Inclined [9]	Pseudo-developable [8], [16]; Right convolute [15]	Convolute, convolute screw [10]; Pseudo-developable of general type [8]	Developable [12]; Open (evolvent) [14]; Torso-helicoid [12]

Further classification depends on the angle of the generator slope: if the angle is equal to $\pi/2$ then the helicoid is called "right"; in other cases (when the slope angle is not equal to $\pi/2$) the helicoid is called "oblique".

However, there is also a special type of oblique open helicoids which is called "tangent open" (or developable, or evolvent, or torso-helicoid) and which is formed by the tangent lines to the helix of constant slope on a circular inner cylinder (Fig. 2, *b*).

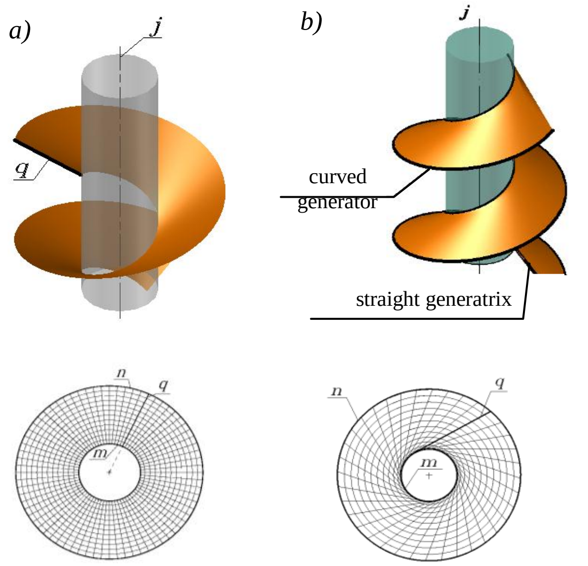

Figure 2. Helicoids: a) of a closed type (generatrix intersects the axis), b) of an open type – (generatrix is tangent to the inner cylinder).

According to the shown classifications, it seems reasonable to use two classifications depending on the purpose and field of application. For mathematics and analytical calculations the classification from [6] with right close type, oblique close type, right open type, oblique open type and developable may be used, while for mechanical and civil engineering the classification from [7] can be more useful: right, oblique, pseudo-developable, developable, convolute.

The parametric equations of five types of Helicoids for further practical application using additive technologies can be written in the following forms (Table II).

TABLE II. PARAMETRIC EQUATIONS FOR 5 TYPES OF HELICOIDS

Convolute	
$x = a\cos v - u\sin\alpha\sin v$	
$y = a\sin v + u\sin\alpha\cos v$	
$z = bv + u\cos\alpha$	
Developable	Pseudo-developable
$\sin\alpha = a/m$ $\cos\alpha = b/m$ $m = \sqrt{a^2 + b^2}$	$\alpha = \pi$
$x = a\cos v - \frac{au}{m}\sin v$	$x = a\cos v - u\sin v$
$y = a\sin v + \frac{au}{m}\cos v$	$y = a\sin v + u\cos v$
$z = cv + \frac{bu}{m}$	$z = cv$
Oblique	Right
$a = 0$	$a = 0$ $\alpha = \pi$
$x = u\sin\alpha\cos v$	$x = x(u,v) = u\cos v$
$y = u\sin\alpha\sin v$	$y = y(u,v) = u\sin v$
$z = cv + u\cos\alpha$	$z = z(u,v) = cv$

However, from the engineering point of view there can be suggested several other possible classifications such as:

1) Three types: “oblique”, “developable”, “convolute”, since two other types (“right” and “pseudo-developable” are the particular cases of oblique and convolute helicoids respectively). This classification is reasonable in the case when the analytical methods are used for calculations.

2) Two types: “oblique” and “convolute” (or closed and open) since they are the most general types which can be used in practice (without and with inner hole respectively). This classification is reasonable in the case of the specific requirements for the structure, but not for the angle of the generatrix inclination.

3) One type: “convolute”, because from mathematical point of view this surface includes all other types, however, it is not so obvious for engineering tasks and manufacturing.

The fact that some engineers separated the right and pseudo-developable helicoids in the separate types can be explained by two reasons:

1) the lack of analytical approaches for calculations (it was easier to develop analytical approaches to calculate these special types than the more general types: oblique and convolute helicoids; one can find only few attempts to calculate oblique helicoids, while there has not been found any analytical approaches to calculate the convolute helicoid);

2) the exploitation requirements (they are suitable for staircases).

Some geometric characteristics and specialties for five types of helicoids are shown in Table III.

TABLE III. GENERAL GEOMETRIC CHARACTERISTICS AND SPECIALTIES OF 5 TYPES OF HELICOIDS

Type of helicoid	Characteristics			
	Gaussian curvature	Middle curvature	Specialty	Analytical methods
Oblique	K<0	H≠0	General close type	+
Developable	K=0	H≠0	Developable surface	+
Convolute	K<0	H≠0	General open type	-
Right	K<0	H=0	Minimal surface	+
Pseudo-developable	K<0	H≠0	Particular case of convolute	+/-

Since engineers have obtained the instruments to define the surfaces in parametric equations in various commercial software based on the finite element methods such as Structural CAD, Ansys, Comsol, etc., there is the opportunity to calculate all five types and investigate their behavior, as well as create them in SolidWorks for further manufacturing using additive technologies.

However, the weak point of commercial software based on finite element methods is that the engineer has to create the new model (as well as the boundary conditions, materials and loading) any time when the new geometrical parameters are required, while analytical methods offer the opportunity to easily change the initial dimensions and other parameters. This disadvantage is especially inconvenient in the task of optimal design in accordance with specific exploitation parameters.

To avoid the problem mentioned above, several analytical approaches to calculate three types of helicoids (oblique, developable and right) have been developed (Krivoshapko, 2009 [14]; Rynkovskaya, 2015 [15]; Tupikova, 2016 [16]; Rynkovskaya and Ivanov, 2018 [17]). The semi-analytical approach to calculate a pseudo-developable helicoid has been delivered by Halabi in 2001 [18], but there have been found no analytical approaches to calculate convolute helicoids, the ruled helicoid of the most general type. The convolute helicoid has been investigated only by geometers [8]. The screws from composite materials have been investigated in [19]. The developed approaches will let a designer investigate stress-strain state of helicoids with various dimensions, and taking into account exploitation parameters choose the proper type and shape.

Thus, the type of a surface should be chosen according to at least four points:

- Exploitation parameters (with or without inner hole, with or without inclination slope);
- Stress-strain and stability behavior (for the particular physical and geometrical characteristics);
- The less weight of material;
- Manufacturing process (will it be obtained from the flat product or manufactured step-by-step with ruled

generatrixes or printed by additive technologies according to the model or parametric equations).

The investigation on comparison stress-strain analysis of different types of helicoids is presented in [20]. The material consumption can be estimated according to further information.

IV. Effective Material Usages

To illustrate the differences in the types of helical shapes mentioned above, the surface areas for the parts of four types (while convolute helicoid turns into developable one for the specific angle) are presented in Table IV.

The surface areas are calculated analytically according to geometric parameters such as parametric equations of the surfaces and their coefficient of the first quadratic form (E, G, F) by the following formula:

$$S = \iint \sqrt{EG - F^2} dudv \tag{2}$$

TABLE IV. The Ratios for Surface Areas in Comparison to Right Helicoid

Type of helicoid	Divergence, %
Oblique	13.0
Pseudo-developable	14.4
Convolute	15.8

As it can be seen from Table IV, right helicoids has the minimal surface area and requires less material (that is reasonable since this surface is a minimal surface according to Table 3); oblique and pseudo-developable helicoids have large surface areas, while the largest surface area is required for convolute helicoids. Thus, if we compare the helicoids with the same angle of generatrix inclination (right and pseudo-developable) then right helicoids will be more effective in terms of material usage.

However, as it is shown in [20], the stress-strain behaviour varies in the opposite way: the shells in the shape of open type helicoids generally show better stress-strain behaviour than the shell in the shape of close types. That means that the choice for the rational shape can be a more complicated task, will depend on the strict parameters and should take into account several parameters such as waste of material, specific exploitation requirements and conditions, stress-strain behaviour, etc.

V. Conclusion

Knowing the equations and differences for several helical structures, the designers can find the proper type of a surface and apply it to the manufacturing process such as parabolic bending from the flat product (for developable helicoids) or for the models in SolidWorks (for other types of helicoids) in order to obtain the product by the means of additive technologies.

Developable helicoid can find a variety of application in industry and manufacturing, since it can be obtained from the flat product as other developable surfaces, which are well-known and widely used, especially in design and mechanical engineering [21-24].

The equations shown in the paper as well as some basic characteristics can help the designers to better understand the geometry of helical products (screws) and can be useful for further investigation on optimization and rationalization of the products in the shape of helical surfaces.

It is sown that the type should be chosen according to a variety of parameters (from exploitation to stress-strain behavior and material effectiveness).

To conclude the differences in ruled helicoids classifications, the most interesting point is that according to mathematical point there can be named only one type – the convolute helicoid, since it is the most general type and its parametric equations suit to all other types. That is why, if anybody develop an analytical method to calculate the convolute helicoids (there has not been found any in the available sources), all five types of the helicoids stress-strain state may be completely investigated. Besides, it would be useful to develop some kind of recommendations for designers and engineers, how to optimize the structure according to the exploitation requirements using different ruled helicoids types.

Conflict of Interest

The author declares no conflict of interest.

Author Contributions

The author contributed to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript.

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