Effect of Machine-Tool Rigidity on Geometric Error Formation in Turning Operation

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***Abstract*—In machining operations, the rigidity of the machine-tool-workpiece system is one of the main criteria that have high influence the process capability in terms of part accuracy and process precision. In this study, a special mathematical model was developed for predicting the profile error received on a cylindrical part during turning operation as a result of variation in machine-tool rigidity during the feed movement of the lathe cutting tool. A graphical presentation of error formation caused by elastic squeezing of shaft sections under the action of the cutting force is generated in this work. The study can present a guide for engineers and technical interested in determining the error expected during machining of precise shafts and other long mechanical components having relatively low stiffness.**

***Index Terms*—cutting tools, cutting force, rigidity, precision machining, geometric error in turning**

I. Introduction

In machining operations, the rigidity of the machine - tool-workpiece system is one of the main criteria that have high influence on the process capability since it determines the level of machining accuracy under cutting load in the steady state operation. Studies of various types of machine tools have shown that the total deformation of a workpiece and machine-tool under the action of cutting force is more dependent on the surface deformation at the joints of kinematic pairs rather than the machine-tool parts [1-3]. The contact deformations at the joints are random in nature and largely depend on the quality of the surface (accuracy, surface shape and roughness of the contacting surfaces). Therefore, in addition to the kinematic accuracy test, rigidity should be controlled for each machine-tool separately in order to achieve an objective assessment not only from the point of the machine-tool design, but also in terms of the level of quality of its manufacture.

However, it is extremely difficult to determine the compliance of the surfaces of parts in the joints. In addition, it is not possible to theoretically calculate the rigidity of the machine-workpiece system due to the probabilistic nature of the manifestation of the operational properties of the contacting surfaces, as well as the actual dimensions of the guaranteed allowances of the machine components. Therefore, the rigidity of the mechanical components of a machine-tool (rigidity of the tailstock, spindle, caliper, etc.) is mainly determined experimentally and based on the obtained experimental data, mathematical models are developed, which can be used as the basis for analyzing the rigidity of machine tools and determining necessary measures to develop recommendations for increasing the rigidity and processing accuracy during various metal cutting operations.

II. Theoritical Background

The rigidity of a machine-tool system is defined by its ability to resist the action of deforming forces resulting from metal cutting. The quantitative rigidity characteristic is determined as the ratio of the deforming force to the displacement caused by the action of this force.

$$j = \frac{P}{y}, N/mm \qquad (1)$$

where P is the deformation force, N, and y is deformation deflection resulting from force applied, mm.

The inverse value of stiffness is called ductility:

$$d = \frac{1}{j} = \frac{y}{P}, mm/N \qquad (2)$$

A machine-tool is considered a complex system. Therefore, depending on the direction of change in the magnitude of cutting forces and the position of the nodes in them, different joint surfaces can be affected in various ways according to different stiffness values. Therefore, when testing the rigidity of a machine-tool, it is necessary to get as close as possible to the most realistic typical cases of in metal cutting and processing. In lathe operation, the direction of the loading force is selected based on the analysis of angles formulated as a result of lathe tool setting (Fig. 1):

$$\alpha = arctg\frac{Py}{Pz} \qquad (3)$$

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$$\beta = arctg\,\frac{Px}{Pz} \qquad (4)$$

where Px, Py, Pz are the cutting force components.

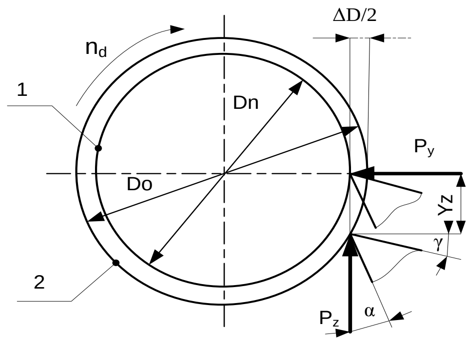

Figure 1. Schematic explaining errors occurring due to forced release of workpiece in a vertical plane under effect of force *Pz*.

To simplify testing, we can assume that β is neglected since the rigidity of the shaft in the longitudinal direction is considered relatively high, and also the Px component has a minimal effect on the machining accuracy in lathe cutting setups.

III. Effect of Cutting Force Copmponents

The effect of the component cutting forces Pz and Py on the machining accuracy can be estimated in advance by a calculation method. Since the directions of the forces Pz and Py are perpendicular to each other, they will influence the deformation of the machine elements and the part in different ways. It is believed that the force component Py, has an order of a magnitude much smaller than that of the cutting force component Pz, [Py = (0.12 .. 0.18) Pz]. Therefore, Pz has a dominant influence on the release of the workpiece, since it is perpendicular to the part in the horizontal plane. However, it is necessary to take into account the influence of both force components in the analysis.

In the beginning of the analysis, suppose that under the influence of the cutting force component Pz, the part shifted in the vertical plane due to the flexibility of the workpiece from position 1 to position 2 by the value of y_z (Fig. 1). This will cause the diameter of the part to increase by ΔD = Do – Dn, where Dn is the diameter to which the tool is initially set; Do is the diameter of the workpiece, increased as a result of being squeezing under the action of the force component Pz.

From the analysis of the scheme, presented in Fig. 1, we can find the change in the diameter of the workpiece during its deformation in the vertical direction under the influence of the force Pz:

$$\Delta Dz = 2\sqrt{\left(\frac{Dn}{2}\right) + y_{3\breve{a}}^{2}} - Dn = (Do - Dn) \qquad (5)$$

The deformations of the workpiece y_{hsz} and y_{hsy} under the influence of the component forces Pz and Py are mutually perpendicular with respect to each other, however, they are calculated using the same formulas taking into account all the component deformations in each element in the workpiece and machine tool technological system.

We set the mathematical relationship that can be used for determining the total release of the workpiece under the influence of the cutting force as a function of the movement of the tool along the *z*-axis of the workpiece. The ZY origin is in line with the left end of the workpiece. At the beginning of a shaft turning, when moving the tool in the feed direction from the tailstock to the headstock, the resulting cutting force Py or Pz, will cause an elastic squeezing action of tailstock tip point by y_{hsy} or y_{hsz} (Fig. 2)

At the same time, the caliper is squeezed in the direction of the *y* axis along with the cutter by the value of y_p. Squeezing the caliper in the direction of the *z* axis is assumed to be infinitely large. The value of squeezing the caliper will be almost constant throughout the process, since the depth of cut, changeable in connection with the release of the part is insignificant compared with the designated depth of cut, so the force of will not change significantly.

Stiffness of the tailstock and caliper have different numerical values, that is, yts $\neq$ ys, where, yts is the deformation of tailstock tip; ys is the caliper deformation. Due to these squeezes, under the influence of the component of the cutting force Py, there will be an increase in the diameter of the turned shaft compared to the theoretical targeted tuned size by the amount:

$$\Delta Dy = 2(y_{tsy} + y_y) \qquad (6)$$

With further shaft turning and due to the movement of the cutter in the feed direction along the workpiece, the effect of the cutting force on the amount of deformation of the tailstock tip will decrease linearly. Designating the coordinate system ZY, as shown in Fig. 2, we will have the following expression for the deformation of the tailstock tip from moving the cutting force along the coordinate *z*:

$$y_{tsy} = \frac{P_y.Z}{j_{tsy}.L} \qquad (7)$$

where *L* is length of the turned shaft and *z* is current coordinate of the tool tip position, measured from the left end of the shaft.

Similarly, we calculate the deformation of y_{tsz} in the vertical plane, caused by the force Pz, which is included in equation (5).

The values of the y_{tsy} and y_s cannot be analytically calculated due to the large number of factors affecting the spin process. These include the state of the surfaces of the contacting parts that makes up the workpiece / machine-tool system, the gap value between the contacting parts which always has a random character, etc. However, these values can only be determined experimentally for each particular machine tool.

$$y_{tsy} = \frac{2P_y.Z}{j_{tsy}.L} \qquad (8)$$

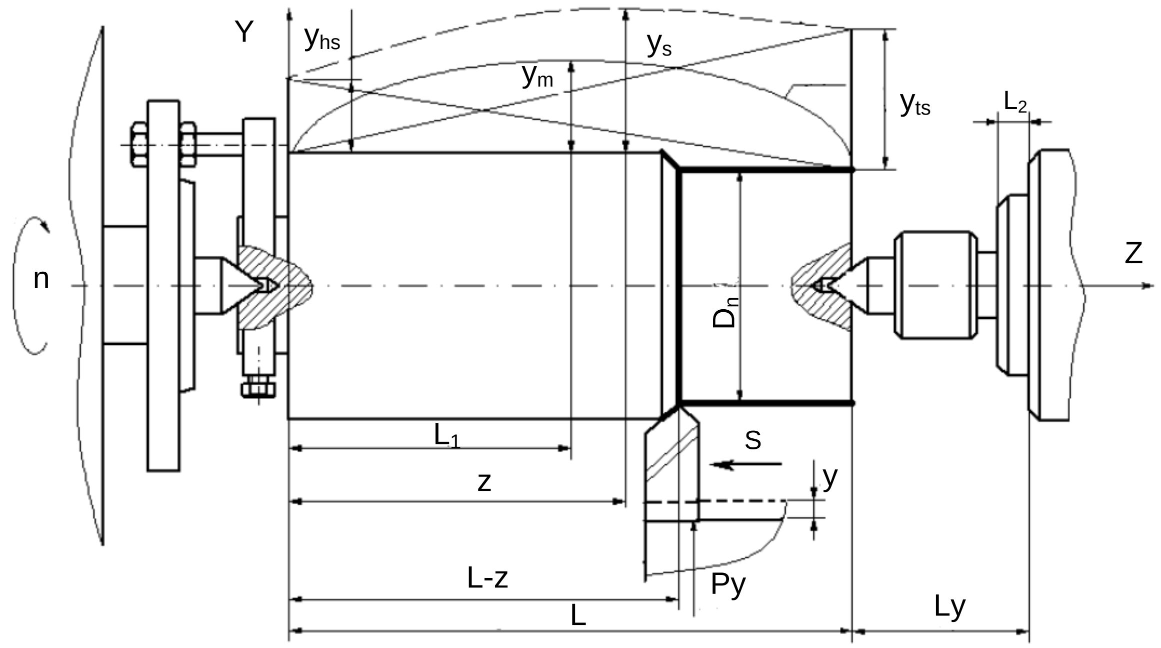

Figure 2. Schematic of formation of errors caused by elastic squeezing of shaft sections under the action of force Py

IV. Mathematical Model Stiffness Prediction in Turning Operation

All that has been said and for deformation of the headstock. Therefore, the dependence of the deformation of the headstock on the cutting force Py and Pz moving in the direction of the z axis has the same equation (7), namely:

$$y_{hsy} = \frac{P_y (L - Z)}{j_{hs}.L} \quad (9)$$

$$y_{hsz} = \frac{P_z (L - Z)}{j_{hs}.L} \quad (10)$$

Depending on the position of the cross section along the turned, to which the cutting force is applied, the value of the total squeezing of the headstock and tailstock which have deformation in the superimposed form of the workpiece, is equal to:

$$Y_{3y} = \frac{P_y}{3EI_b} \times \frac{Z^2 (L - Z)^2}{L} \quad (11)$$

$$Y_{3Z} = \frac{P_Z}{3EI_b} \times \frac{Z^2 (L - Z)^2}{L} \quad (12)$$

where $I_b \approx 0.05\ Db^4$ is the inertia moment of the cross section of a solid shaft (for the annular section $I_c \approx 0{,}05$ Db4 (1- α^4), where α = d / Db; d - is the inner diameter of the annular shaft cross-section, *E* is the elastic modulus of the material being processed E = 2,1x10^5 MPa.

As a result, the total change in the diameter of the part in the load application section corresponding to the z, we may obtain the following relationship:

$$\Delta D_S = 2P_y \left[y_{tsy}(z) + y_{hsy}(z) + y_{3y}(z) + y_s \right] + \Delta D_Z \quad (13)$$

Substituting into this formula the dependencies obtained earlier, we obtain:

$$\Delta D = 2P_y \left[\frac{z}{L.j_{ts}} + \frac{L - z}{L.j_{hs}} + \frac{z^2 (L - z)^2}{3LEI_b} + \frac{I}{j_c} \right] + \sqrt{\left(\frac{Dn}{2}\right)^2 + y_{tsz}^2} - Dn \quad (14)$$

where deformation segment y_{tsz} in the direction of the application of cutting force Pz, is equal to

$$y_{pz} = 2P_z \left[\frac{z}{L.j_{ts}} + \frac{L - z}{L.j_{hs}} + \frac{z^2 (L - z)^2}{3LEI_b} \right] \quad (15)$$

It is noticeable that the maximum deflection of the part will not be in the middle of the shaft span, since the total curve of the shaft deflections is not symmetrical due to the fact that the rigidity of the tailstock is less than that of the headstock.

In the above formulas, the values of the rigidity of the tailstock j_{ts}, the headstock j_{pb}, and the caliper j_c are determined experimentally (or given in the catalogue of a machine-tool), the force of Py and Pz are calculated using the formulas [4 -11]:

$$Py = C_p y^{yp} t^{yp} HB^{yp} \quad (16)$$

$$Pz = C_p y^{zp} t^{zp} HB^{zp} \quad (17)$$

where C_p is the specific heat of the material.

Table I shows the results of these cutting force components calculated when turning shafts made from different materials, mainly steels, cast iron, and nonferrous materials such as bronze and aluminum alloys.

TABLE I. CUTTING FORCE COMPONENTS FOR SOME ENGINEERING MATERIALS

Component cutting force	Material tool	Metal to be processed							
		Steel, aluminum alloys				Cast iron, bronze			
		C_p	x_p	y_p	z_p	C_p	x_p	y_p	z_p
P_x	Hard alloy	0,21	1,2	0,65	1,5	0,51	1,2	0,65	1,1
	High speed steel	0,21	1,2	0,65	1,5	1,6	1,2	0,65	1,1
P_y	Hard alloy	0,027	0,9	0,75	2	0,45	0,9	0,75	1,1
	High speed steel	0,027	0,9	0,75	2	1,3	0,9	0,75	1,3
P_z	Hard alloy	35,7	1	0,75	0,75	51,4	1	0,75	0,55
	High speed steel	27,9	1	0,75	0,35	63,5	1	0,75	0,55

The literature states that the curve described by equation (14) can be either convex or concave. If the rigidity of the machine elements is large enough, and the rigidity of the workpiece is small (machining of a long and thin shaft), then the release of the y_{hs} and y_{ts} are small, and the *yzy* is significant. As a result, the workpiece will have a form of a barrel shape (convex), and, conversely, with a sufficiently rigid workpiece and a small amount of deflection, the form of the workpiece will turn into "corset-like", with the smallest diameter value in the middle part of the workpiece.

The difference between the maximum machining diameter and the adjusting shaft diameter determines the accuracy of the resulting size of the machined shaft. We can determine the *z* coordinate corresponding to the maximum deflection of the shaft in two ways: graphically, by plotting a curve using equation (14) or analytically. To determine the *z* coordinate corresponding to the maximum deflection of the shaft by an analytical method, we find the derivative of equation (14) with respect to *z*, neglecting the effect on the deflection of the shaft in the vertical plane:

$$\frac{d\Delta D}{dz} = 2P_y\left[\left(\frac{1}{L.j_{ts}} - \frac{1}{L.j_{ts}}\right) + \frac{2zL}{3EI_b} + \frac{2z^2}{EI_e} + \frac{4z^2}{3LEI_b}\right] \quad (18)$$

The written equation with the condition $\frac{d\Delta D}{dz} = 0$ is cubic, having the following form:

$$a.z^3 + b.z^2 + c.z + d = 0 \quad (19)$$

where:

$$a = \frac{8P_y}{3LEJ_b}, b = \frac{4P_y}{EJ_b}, c = \frac{4P_y}{3EJ_b}, d = \frac{2P_y}{L}\left(\frac{1}{j_{ts}} - \frac{1}{j_{hs}}\right)$$

The same derivative can be found using the appropriate calculation procedures when using computers and MathCAD software.

Substituting the found value of the shaft deflection into equation (14) for determining the total shaft squeezing, we can obtain the maximum tolerance deviation for this operation without taking into account other factors affecting the machining accuracy (dimensional tool wear, thermal deformation and fluctuation, non-uniformity of the workpiece material, non-consisting tool feed movement, error of mounting, etc.).

Therefore, to calculate the maximum allowable release of the part $[\Delta D_{ad}]$, regardless of the position of the cutter along the shaft length, and converting equation (14) relative to the feed, we can obtain a functional dependence for their determination:

$$S = \left[\frac{\Delta D_{ad}}{20(C_p t^x HB^n K)\left(\frac{z}{Lj_{ts}} + \frac{L-z}{Lj_{hs}} + \frac{4z^2(L-z)^2}{3LEI_b} + \frac{1}{j_c}\right)}\right]^{\frac{1}{Ypy}} \quad (20)$$

V. CONCLUSIONS

The following conclusions are withdrawn from this study:

- A graphical presentation of error formation caused by elastic squeezing of shaft sections under the action of force Py was demonstrated.
- In this study, a mathematical model that can be used for brief description of the mathematical model for determining the stiffness of a lathe machine-tool when turning a shaft was developed.
- The study can present a guide for engineers and technical interested in determining the error expected during machining of precise shafts and other long mechanical components having relatively low stiffness.

CONFLICT OF INTEREST

The authors declare that there has been no conflict of interest in carrying out the research activities related to this article.

AUTHOR CONTRIBUTIONS

This article is a product of our collaboration effort between Nelson Mandela University (South Africa) and Lebanese University (Lebanon). The first two authors, Dr. El-Dahabi and Prof. Dib from the Lebanese University, set up the paper plan and conducted the first steps of the mathematical model. The rest of the authors participated in further developing the mathematical model. They also participated in writing the literature survey of the paper and improved its linguistic level.

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