A Resource Management Problem under Different Assignment Rules in Cross Docks

Lijuan Zhang
DGUT-CNAM Institute, Dongguan University of Technology, Dongguan, China
Email: zhanglijuan@dgut.edu.cn

Benoît Trouillet and Frédéric Semet
Ecole Centrale de Lille, Lille, France
Email: {benoit.trouillet, frederic.semet}@ec-lille.fr

***Abstract*—In this paper, we address a resource management problem under the time window constraints in cross docks. The resources specify the operators working in cross dock terminal. The times as well as the resources required by each operation (unloading, sorting and loading) are considered. The objective aims to minimize the total resource number in cross dock in order to minimize the total cost, which includes labor cost and undelivered good penalty cost. We consider two different resource assignment rules: non-shared resource rules and shared resource rules. Two methods are proposed to deal with this problem: the mixed integer programming (MIP) formulation and the simulation using Petri net. First, the problem is formulated as a MIP model in which the resources at each work station are not shared. Then to evaluate the MIP model, the operations in cross dock are simulated using Petri net. Last, according the simulation results, a second MIP model with shared resources is proposed. The computational results show that the second MIP model outperforms the first one for organizing and distributing resources in cross dock.**

***Index Terms*—cross docks; resource management; MIP formulation; time windows; Petri net**

I. Introduction

A distribution center is a structure used to receive, temporarily store, and redistribute goods according to orders. In order to make the global logistic chain cost saving, efficient and effective, companies call for dynamic distribution centers. Distribution center operations consist of five basic functions: receiving, sorting, storing, picking, and shipping. Companies can cut their costs and improve their productivity by improving these five functions. The basic concept behind cross docks is to eliminate the two most costly operations in a distribution center by transferring the incoming goods directly from receiving docks to shipping docks. The activity of cross dock center is as follows: goods from suppliers are shipped into the cross dock center, then they are unloaded at the receiving doors, sorted according to their destinations and directly transferred to the shipping doors, after that, they are loaded in trucks and delivered to customers. Shipments typically spend a few hours in the distribution center.

In this paper, we address a specific cross dock layout with an I-shape and consider the door assignment and resource management problem under the time windows constraints when the number of trucks is larger than number of doors. As shown in Fig. 1, the layout of the cross dock is a rectangle physical structure where one receiving door faces to one shipping door. Here the transportations between two face-to-face doors are called vertical transportation, while the other transportations are called horizontal transportation. The vertical transportations are represented by vertical arrows. The horizontal transportations are represented by horizontal arrows and carried out between two vertical arrow columns. Between each two face-to-face doors, there are two scanners and temporary storage areas.

As the arrows shown in the Fig. 1, all the horizontal transportations are carried on between two consecutive doors. Goods are unloaded from inbound trucks, pass through the first scanner, and they are sorted according to their destinations. Three cases have to be considered. If the outbound trucks to their destinations are assigned to doors at the right of the current column, goods will be transferred to the first temporary storage area. If not, they will directly pass through the second scanner, and are sorted once more time. If outbound trucks to their destinations are at the left of the current column, they are transferred into the second temporary area. If not, they are directly transferred to the shipping doors. Goods in the first/second storage areas are scanned by the first/second scanners and sorted again.

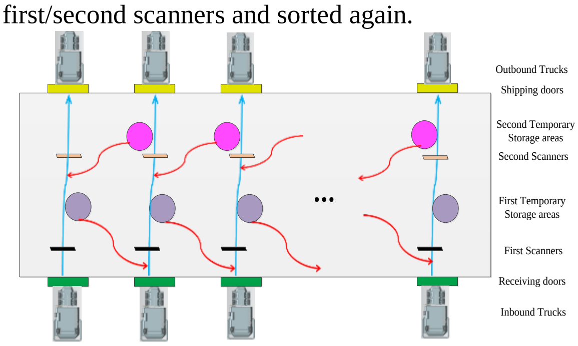

Figure 1. Cross dock layout and internal transportations

Manuscript received August 14, 2019; revised February 8, 2020.
Supported by Research Start-up Funds of DGUT (GC300502-60).

doi: 10.18178/ijmerr.9.4.596-602

Vertical transportation is the most efficient path for all the goods from receiving door to shipping door. In real-life cases, due to the truck time window constraints or to the door capacity constraints, goods are horizontally transported within the cross dock. Indeed, whatever the assignment of trucks to doors is, goods has to be transported from one side of the cross dock to the other. Therefore, the cost associated with the vertical transportation can be viewed as a fixed cost. At the opposite, horizontal transfers are impacted by the assignment and the associated cost must be minimized. In this article, we are interested in determining the resources required to perform the horizontal transfers.

The cross dock door assignment problem (CDDAP) aims to decide the sequence of inbound and outbound trucks at a given set of dock doors [1]. Several variants of the CDDAP have been addressed in the literature. Some of them can be viewed as extension of the Generalized Assignment Problem (GAP) [2].

A bilinear programming model based on GAP is formulated by [3] to describe a basic dock door assignment problem. In this formulation, one door must be assigned to one truck, and one truck must get assigned only one door, that is to say that the number of suppliers must be equal to number of receiving doors and the number of customers must be equal to the number of shipping doors. [4] improve the formulation by [3] so that the number of suppliers and receiving doors can be different and the number of customers and shipping doors can be also different. In both formulations, the number of trucks cannot exceed the number of doors, because they do not take in count the sequence of trucks that are assigned in the same door. Ref. [5] bring in time windows of trucks. Each truck has an arrival time at the cross dock and a departure time from the cross dock. Trucks have to be assigned according to their time windows, the number of trucks can exceed number of doors and two trucks cannot be assigned to the same door when their time windows overlap. Ref. [6], [7] and [8] improve their formulations. The main idea is that if there is no available door for some trucks with respect to their time windows, these trucks can be unassigned and a penalty is paid. However in their formulations, the function to compute penalty paid overestimates it. Moreover, the literatures above do not consider the activity of resources.

Certain authors also deal with the problems by simulation, and the most frequently-used tool is Arena. Ref. [9] and [10] have tried to study this problem using Petri net.

In this paper, we address a resource management problem under the truck time windows constraint in a specific cross dock. The activities of resources are considered. We distingue two types of resource assignment rules: non-shared rules with which one resource cannot be assigned to more than one work station and the resources at each work station cannot be shared by other work stations; shared rules with which all of the resources are shared by all of operations in terminal and the resources are not assigned to one work station. The MIP formulation with non-shared rules is presented in the Section 2. Section 3 details the strategy of modelling cross dock with Petri net and reports the simulation results to evaluate the MIP formulation. The improved MIP formulation is presented in Section 4. The computational results with the two MIP model are reported in Section 5, while conclusions are drawn in Section 6.

II. Formulations with Non-shared Resources

In this paper, we focus on a resource management problem for a specific cross dock terminal under capacity and time window constraints. We are interested in determining the resources required to perform the horizontal transfers. We can assume that the vertical transportations are automated by conveyers. No time cost and no resource cost are imposed at the opposite of horizontal transportations. Thus the distance between receiving door i and shipping door j $(distance_{ij})$ is proportional to abs(i-j).

The cross dock can be described by the following parameters:

- M: total number of inbound trucks(suppliers);
- N: total number of outbound trucks(customers);
- I: total number of receiving doors in cross dock;
- J: total number of shipping doors in cross dock;

In what follows, we assume that due to the parallel symmetric layout of cross dock, the number of receiving doors is equal to the number of shipping doors. We use the following notation:

- m: index of inbound trucks. $m \in \{1,\ldots, M\}$;
- n: index of outbound trucks. $n \in \{1,\ldots,N\}$;
- i: index of receiving doors. $i \in \{1,\ldots,I\}$;
- j: index of shipping doors. $j \in \{1,\ldots, J\}$;

Each receiving door and each shipping door are numbered according to their positions from left to right. If receiving door i and shipping door j are face to face, their indexes are equal, i.e $i=j$.

The goods transfers from suppliers to customers are identified by:

- w_{mn}: goods quantity which need to be delivered from supplier m to customer n;
- f_m: total goods quantity from supplier m, $m \in \{1,\ldots, M\}$;
- v_n: total goods demand by customer n, $n \in \{1,\ldots, N\}$;
- $penalty_{mn}$: penalty paid if goods from supplier m are not delivered to customer n;

Each truck arrives and leaves the cross dock within a time window, we denote:

- a_m: the arrival time of inbound truck m at the cross dock, $m \in \{1,\ldots, M\}$;
- d_m: the departure time of inbound truck m from the cross dock, $m \in \{1,\ldots, M\}$;
- a_n: the arrival time of outbound truck n at the cross dock, $n \in \{1,\ldots,N\}$;
- d_n: the departure time of outbound truck n from the cross dock, $n \in \{1,\ldots,N\}$;

We have, $a_m < d_m, a_n < d_n$, $m \in [1,\ldots, M]$, $n \in \{1,\ldots,N\}$;

The goods transfer with the cross dock are characterized thanks to the following parameters:

- $distance_{ij}$: distance between receiving door i and shipping door j;
- Δt_{ij}: time to transport goods from receiving door i to shipping door j. It varies with $distance_{ij}$;
- $q_{m,n,i,j} = 1$ if $d_n \geq a_m + \Delta t_{ij}$, otherwise $q_{m,n,i,j} = 0$, $m \in \{1,\ldots, M\}$, $n \in \{1,\ldots,N\}$, $i \in \{1,\ldots,I\}$, $j \in \{1,\ldots, J\}$;
- $q_{m,h} = 1$ if inbound truck m departs no later than inbound truck h arrives ($d_m \leq a_h$), otherwise $q_{m,h} = 0$, $m, h \in \{1,\ldots, M\}$;
- $q_{n,u} = 1$ if outbound truck n departs no later than outbound truck u arrives ($d_n \leq a_u$), otherwise $q_{n,u} = 0$, $n, u \in \{1,\ldots, N\}$;

Costs and velocities are associated with the resources used to unload/sort/load goods. Thus we have:

- c: cost per resource;
- p_1: unloading and loading velocity per resource per unit time;
- p_2: sorting velocity per resource per unit time.

We also define :

- t_g**:** Make all the $a_m s$ and $d_n s$ in an increasing order, and let t_g ($g=1,2,\ldots, (M+N)$) correspond to these $(M+N)$ numbers such that $t_1 < t_2 < \cdots < t_{(M+N)}$;

We define two types of decision variables. We have variables associated with the assignment of trucks to doors and variables for resource requirement.

- $x_{mi} = 1$ if inbound truck m is assigned to receiving door i, otherwise $x_{mi} = 0$, $m \in \{1,\ldots, M\}$, $i \in \{1,\ldots,I\}$;
- $y_{nj} = 1$ if outbound truck n is assigned to shipping door j, otherwise $y_{nj} = 0$, $n \in \{1,\ldots,N\}$, $j \in \{1,\ldots, J\}$;
- $z_{ijmn} = 1$ if inbound truck m is assigned to receiving door i and outbound truck n is assigned to shipping door j, otherwise $z_{ijmn} = 0$;
- r_i: number of unloading resources at receiving door i, $i \in \{1,\ldots,I\}$;
- s_j: number of loading resources at shipping door j, $j \in \{1,\ldots, J\}$;
- k_{ij}: number of transfer resources between column i and j for horizontal transportations, $i \in \{1,\ldots,I\}$, $j \in \{1,\ldots, J\}$;

The resources defined are not shared. Each resource can be assigned to only one work station.

The model is as follows:

$$\mathbf{min}\left(M_G \left(\begin{aligned} &\sum_{i=1}^{I-1}\sum_{j=i+1} c\cdot k_{ij} + \sum_{i=1}^{I} c\cdot r_i + \sum_{j=1}^{J} c\cdot s_j \\ &+\sum_{m=1}^{M}\left(\sum_{n=1}^{N} penalty_{mn}\cdot w_{mn}\cdot\left(1-\sum_{i=1}^{I}\sum_{j=1}^{J} q_{mnij}\cdot z_{ijmn}\right)\right)\end{aligned}\right) + \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{j=1}^{J} w_{mn}\cdot distance_{ij}\cdot z_{ijmn} \right) \quad (1)$$

Subject to:

$$\sum_{i=1}^{I} x_{mi} \leq 1, m \in \{1,\ldots,M\}, i \in \{1,\ldots,I\}; \quad (2)$$

$$\sum_{j=1}^{J} y_{nj} \leq 1, n \in \{1,\ldots,N\}, j \in \{1,\ldots,J\}; \quad (3)$$

$$z_{ijmn} \leq x_{mi}; \quad (4)$$

$$z_{ijmn} \leq y_{nj}; \quad (5)$$

$$z_{ijmn} \geq x_{mi} + y_{nj} - 1; \quad (6)$$

$$2(q_{m,h} + q_{h,m}) \geq x_{mi} + x_{hi} - 1, m, h \in \{1,\ldots, M\}, m \neq h; \quad (7)$$

$$2(q_{n,u} + q_{u,n}) \geq y_{nj} + y_{uj} - 1, n, u \in \{1,\ldots, N\}, n \neq u; \quad (8)$$

$$f_m * x_{mi} \leq p_1 r_i (d_m - a_m); \quad (9)$$

$$v_n * y_{nj} \leq p_1 s_j (d_n - a_n); \quad (10)$$

$$\sum_{l, l\leq i}\ \sum_{\substack{j,\\ j\geq i+1}}\ \sum_{\substack{m,\\ a_m\leq tg\\ d_m\geq tg}}\ \sum_{\substack{n,\\ d_n\geq a_m+\Delta t_{lj}\\ d_n\geq tg}} \frac{w_{mn}}{d_n - a_m}\left(z_{ljmn} + z_{jlmn}\right) \leq p_2 * \left(k_{i,i+1}\right), \forall i \in \{1,\ldots,I-1\},\ t_g,\ g \in \{1, \ldots, M+N\}; \quad (11)$$

The objective function is the combination of two terms. The first one (12) is the total cost which includes the labor cost and the penalty generated by delivery delays. The second term (13) is the total weighted travel distance.

$$\begin{aligned} &\sum_{i=1}^{I-1}\sum_{j=i+1} c\cdot k_{ij} + \sum_{i=1}^{I} c\cdot r_i + \sum_{j=1}^{J} c\cdot s_j \\ &+\sum_{m=1}^{M}\left(\sum_{n=1}^{N} penalty_{mn}\cdot w_{mn}\cdot\left(1-\sum_{i=1}^{I}\sum_{j=1}^{J} q_{mnij}\cdot z_{ijmn}\right)\right)\end{aligned} \quad (12)$$

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{j=1}^{J} w_{mn}\cdot distance_{ij}\cdot z_{ijmn} \quad (13)$$

In order to aggregate these two terms in a hierarchical manner, we introduce a factor M_G which is set to:

$$\begin{aligned} M_G &= \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{j=1}^{J} w_{mn}\cdot distance_{ij} \\ &= \sum_{m=1}^{M}\sum_{n=1}^{N} w_{mn}\cdot\sum_{i=1}^{I}\sum_{j=1}^{J} distance_{ij} \\ &= \sum_{m=1}^{M} f_m\cdot\sum_{i=1}^{I}\sum_{j=1}^{J} distance_{ij}\end{aligned} \quad (13)$$

The constraints can be explained as follows: Constraint (2) is to ensure that each inbound truck is not assigned to more than one receiving door. Constraint (3) is to make sure that each outbound truck is not assigned to more than one shipping door. Constraints (4) (5) (6) correspond to the linearization of $z_{ijmn} = x_{mi} * y_{nj}$. Constraint (7) is to make sure that one receiving door cannot be occupied by more than one inbound truck at a given time. Constraint (8) is to make sure that one shipping door cannot be occupied by more than one outbound truck at the same time. Constraints (9) specify that goods of inbound truck m have to be unloaded during the period $(d_m - a_m)$ at each receiving door where truck m is present. Constraints (10) express the same restriction for outbound trucks. Constraints (11) make sure that total quantity of goods transferred between two consecutive columns cannot exceed the sorting capacity. The left term is the average goods flow in unit time at period tg between column i and i+1, this flow cannot exceed the transferring capacity in unit time at period tg (right term). The time is divided into several periods by tg. The constraints on inbound truck m and outbound truck n is to control that the goods transferred between every two consecutive columns during each period must be those from the inbound trucks which arrive before tg and leaves after tg $(a_m \le tg, d_m \ge tg)$, and those to the outbound trucks which leaves after tg $(d_n \ge tg, d_n \ge a_m + \Delta t_{lj})$.

With this MIP model, the resources number required at each work station are separately obtained, and the total resource number is the sum of the resources at each work station. However, in real case, the resources at each work station are mixed and shared. For example, if there are not many tasks of unloading at receiving door, the resources assigned to the unloading can carry out other operations (sorting or loading) at other work stations. Thus in order to evaluate the solutions obtained with MIP model and study the behaviors of cross dock with different number of shared resources, we propose to simulate cross dock with Petri net.

III. Simulation with Petri Net

Petri net is a graphic and mathematic tool. The operations in cross dock are modelled with T-timed Petri net. The operations include assignment of trucks to doors, unloading, internal transportation and loading.

For the modelling, we make some hypothesis.

Hypothesis 1: The door assignment for the inbound trucks and outbound trucks are predefined.

Hypothesis 2: Pre-emption is not allowed. Unloading or loading of a truck cannot be interrupted.

Hypothesis 3: Before the earliest truck arrives, the cross dock is totally empty, there is not any work in cross dock.

We define the assignment priority rules for the trucks which share the same door:

Priority rule 1: The earlier the inbound truck arrives, the greater priority it has to be assigned to its door;

Priority rule 2: The earlier the outbound truck arrives, the greater priority it has to be assigned to its door;

We define the operation priority rules for the operations which share the same resources:

Priority rule 3: the unloading operations have greater priority than internal transportations, and interior transportations have greater priority than loading operations.

Priority rule 4: for all the unloading operations, the earlier the inbound truck arrives in cross dock, the greater priority it has for unloading.

Priority rule 5: for all the loading operations, the earlier the outbound truck arrives in cross dock, the greater priority it has for loading.

Priority rule 6: For all of the operations with the same priority, they are randomly operated.

Here we cite an example to present the results of simulation. We consider 3 receiving doors, 3 shipping doors, 4 inbound trucks with their time windows arrival time in cross dock $a_m = [16 \quad 20 \quad 39 \quad 30]$, depart time from cross dock $d_m = [28 \quad 36 \quad 50 \quad 40]$, and 4 outbound trucks with their time windows, arrival time in cross dock $a_n = [20 \quad 42 \quad 25 \quad 51]$, depart time from cross dock $d_n = [30 \quad 50 \quad 46 \quad 80]$. The unloading /loading/sorting velocity per resource per unit time is p_1=5. The goods quantity transferred from inbound truck m to outbound truck n is w_{mn}, and

$$w_{mn} = \begin{bmatrix} 33 & 15 & 52 & 22 \\ 18 & 34 & 25 & 20 \\ 31 & 24 & 14 & 38 \\ 29 & 35 & 25 & 44 \end{bmatrix}.$$

The door assignment is obtained with the formulation in Section 2. The inbound trucks *IT1* and *IT4* are assigned in receiving door *RD1*, *IT2* and *IT3* are assigned in *RD2*, the outbound trucks *OT1*, *OT2* and *OT4* are assigned in shipping door *SD1*, and *OT3* is assigned in *SD2*.

The detailed Petri net model for this case is presented by [10]. The simulation is carried out with different shared resources number (*N*) by software Tina. We note the beginning time and ending time of unloading/loading of each inbound/outbound truck and propose these time points as the new time windows for trucks, as shown in Table I.

We compare the time windows defined with those proposed and verify if they coincide in order to determine the resource number required for this terminal cited. It is obvious that the resource number inferior 6 is not feasible solution, while the resource number superior 6 is the feasible solution. However, to respect the time windows defined, 6 resources are sufficient.

The example is solved with the MIP formulation presented in Section 2, we obtain that it needs 11 resources. There is a big difference between the solution by MIP model and that by the simulation. That is because the rules of resource assignment are different: MIP with non-shared resources while the simulation with shared resources. We propose to improve the MIP model with shared resource rules, which is detailed in Section 4.

TABLE I. TIME WINDOWS PROPOSED

N		New proposed time windows				N		New proposed time windows			
1	a_m	16	42	90	62	7	a_m	16	20	39	30
	d_m	42	62	113	90		d_m	20	25	43	34
	a_n	156	191	167	213		a_n	26	44	27	51
	d_n	167	213	191	239		d_n	28	48	45	55
2	a_m	16	29	53	39	8	a_m	16	20	39	30
	d_m	29	39	64	53		d_m	20	24	42	34
	a_n	86	103	91	115		a_n	25	43	26	51
	d_n	91	115	103	128		d_n	27	47	44	55
3	a_m	16	25	40	31	9	a_m	16	20	39	30
	d_m	25	32	48	41		d_m	19	23	42	33
	a_n	62	74	66	82		a_n	24	43	25	51
	d_n	67	82	75	91		d_n	26	46	44	54
4	a_m	16	22	40	31	10	a_m	16	20	39	30
	d_m	23	28	46	38		d_m	19	22	42	33
	a_n	51	59	53	66		a_n	23	42	25	51
	d_n	54	66	60	73		d_n	25	46	43	54
5	a_m	16	21	40	31	11	a_m	16	20	39	30
	d_m	22	26	45	37		d_m	19	22	42	33
	a_n	30	51	39	56		a_n	23	42	25	51
	d_n	39	56	51	62		d_n	25	45	43	54
6	a_m	16	20	39	30	12	a_m	16	20	39	30
	d_m	21	24	43	35		d_m	19	22	41	33
	a_n	28	45	29	51		a_n	22	42	25	51
	d_n	30	50	46	56		d_n	24	45	43	54

IV. FORMULATION WITH SHARED RESOURCES

The strategy of improving the MIP model is to refine the time period. We bring in a parameter t_e .

- t_e : make all the $a_m s$, $d_m s$, $a_n s$ and $d_n s$ in an increasing order, and let t_e (e=1,2,…, t_{E}) correspond to these numbers such that $t_1 < t_2 < \cdots < t_{\mathrm{E}}$; $e \in [1, \dots, \mathrm{E}]$, and for $\forall\, e_1$, $e_2 \in [1, \dots, E]$, and $t_{e_1} \neq t_{e_2}$;

Using the same notations as in Section 2, we add some more notations:

- r_i^e : resources assigned to receiving door i for unloading between time period t_{e+1} and t_e;
- s_j^e : resources assigned to shipping door j for loading between time period t_{e+1} and t_e;
- k_{mnij}^e : resources assigned between column i and j for internal horizontal transporting goods from origin m to destination n between time period t_{e+1} and t_e;
- $resourcenumber_e$: total resources between t_{e+1} and t_e;
- *minresourcenumber*: minimal total resources;

Thanks to t_e, the time is divided into small periods. The resources at each work station are optimized period par period, and we find the resource required during each period. The max value of resource number of each period is the total resource number required.

The model is as follows:

$$\min \left(M_G \left(\begin{array}{l} c \cdot min\,resourcenumber \\ + \sum_{m=1}^{M} \left(\sum_{n=1}^{N} penalty_{mn} \cdot w_{mn} \cdot \left(1 - \sum_{i=1}^{I} \sum_{j=1}^{J} q_{mnij} \cdot z_{ijmn} \right) \right) \end{array} \right) + \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{j=1}^{J} w_{mn} \cdot distance_{ij} \cdot z_{ijmn} \right) \tag{15}$$

Subject to:

$$\sum_{i=1}^{I} x_{mi} \le 1, m \in \{1, \dots, M\}, i \in \{1, \dots, I\}; \tag{16}$$

$$\sum_{j=1}^{J} y_{nj} \le 1, n \in \{1, \dots, N\}, j \in \{1, \dots, J\}; \tag{17}$$

$$z_{ijmn} \le x_{mi}; \tag{18}$$

$$z_{ijmn} \le y_{nj}; \tag{19}$$

$$z_{ijmn} \ge x_{mi} + y_{nj} - 1; \tag{20}$$

$$2(q_{m,h} + q_{h,m}) \ge x_{mi} + x_{hi} - 1, \text{ for } m, h \in \{1, \dots, M\}, m \neq h; \tag{21}$$

$$2(q_{n,u} + q_{u,n}) \ge y_{nj} + y_{uj} - 1, \text{ for } n, u \in \{1, \dots, N\}, n \neq u; \tag{22}$$

$$f_m * x_{mi} \le p_1 * \sum_{\substack{e: \\ a_m \le t_e, \\ t_{e+1} \le d_m}} r_i^e (t_{e+1} - t_e); \tag{23}$$

$$v_n * y_{nj} \le p_1 * \sum_{\substack{e: \\ a_n \le t_e, \\ t_{e+1} \le d_n}} s_j^e (t_{e+1} - t_e) \tag{24}$$

$$w_{mn} \cdot q_{mnij} \cdot z_{ljmn} \le p_2 \cdot \sum_{\substack{e:a_m \le t_e,\\ t_{e+1} \le d_n}} k^{e}_{m,n,i,i+1}\left(t_{e+1}-t_e\right), \tag{25}$$

for $\forall i \in \{1,\dots,I-1\}, l \le i \le i+1 \le j$ or $j \le i \le i+1 \le l$;

$$\sum_{\substack{e':\\ t_{e'} \ge a_m\\ e' \le e}}^{e:\, t_e \le d_n} k^{e'}_{m,n,i,i+1}(t_{e'+1}-t_{e'}) \ge k^{e+1}_{m,n,i+1,i+2}(t_{e+2}-t_{e+1}), \tag{26}$$

for $\forall l \in \{1,\dots,I\}$, $\forall j \in \{1,\dots,J\}$, $l \le i \le i+1 \le j$, $\forall e \in \{1,\dots,E\}$;

$$\sum_{\substack{e':\\ t_{e'} \ge a_m\\ e' \le e}}^{e:\, t_e \le d_n} k^{e'}_{m,n,i,i+1}(t_{e'+1}-t_{e'}) \ge k^{e+1}_{m,n,i-1,i}(t_{e+2}-t_{e+1}), \tag{27}$$

for $\forall l \in \{1,\dots,I\}$, $\forall j \in \{1,\dots,J\}$, $j \le i \le i+1 \le l$, $\forall e \in \{1,\dots,E\}$;

$$\sum_{i} \sum_{e':e' \le e} r_i^{e'}(t_{e'+1}-t_{e'}) \ge \sum_{m:a_m \le t_e} \sum_{n:d_n \ge t_e} k^{e+1}_{m,n,i,i+1}(t_{e+2}-t_{e+1}), \tag{28}$$

for $\forall m \in \{1,\dots,M\}$, $\forall n \in \{1,\dots,N\}$, $\forall i \in \{1,\dots,I-1\}$, $\forall j \in \{1,\dots,J\}$, $\forall e \in \{1,\dots,E\}$;

$$\sum_{e':e' \le e} r_j^{e'}(t_{e'+1}-t_{e'}) + \sum_{m:a_m \le t_e} \sum_{n:d_n \ge t_e} \sum_{e':e' \le e} k^{e'}_{m,n,j-1,j}(t_{e'+1}-t_{e'}) + \sum_{m:a_m \le t_e} \sum_{n:d_n \ge t_e} \sum_{e':e' \le e} k^{e'}_{m,n,j,j+1}(t_{e'+1}-t_{e'}) \ge s_j^{e+1}(t_{e+2}-t_{e+1}), \tag{29}$$

for $\forall m \in \{1,\dots,M\}$, $\forall n \in \{1,\dots,N\}$, $\forall j \in \{1,\dots,J\}$, $j+1 \le J \,\&\&\, j-1 \ge 1$, $\forall e \in \{1,\dots,E\}$;

$$\textstyle\sum_i r_i^e + \sum_j s_j^e + \sum_{i,m,n} k^e_{m,n,i,i+1} \le resourcenumber_e, \text{ for } \forall e \in \{1,\dots,E\}; \tag{30}$$

$$minresourcenumber >= resourcenumber_e, \ \forall e \in \{1,\dots,E\}; \tag{31}$$

The objective function (15), the constraints (16), (17), (18), (19), (20), (21), (22) is as in the previous model in Section 2. Constraints (23)(24)(25) are similar with constraint (9)(10)(11), but the time for unloading, sorting and loading are refined to small periods by t_e. The objective of constraints (26) and (27) is to control the sequence of internal transportation. Inbound truck m is assigned at receiving door l, outbound truck n is assigned at shipping door n, and doors are indexed from left to right. Constraint (26) is under the condition that m is at the left of n ($l <= j$). Then goods from m to n are transported from left to right. The transportations from the farthest columns (i to i+1) start earlier than that from the closest columns (i+1 to i+2). Constraint (27) is subject to the fact that m is at the right of n ($l >= j$). The purpose of constraint (28) is to control that the unloading operation for an inbound truck starts before the sorting operation. The purpose of constraint (29) is to control that the loading operation for an outbound truck starts after the sorting operation. Constraint (30) determines the resource number at each period, and constraint (31) determine the minimal resource number required in the cross dock.

V. Computational Results

The two MIP models are installed in CPLEX 12.5. We obtain that by the MIP model proposed in Section 2, 11 resources are required for the case cited in Section 3. While by the MIP model proposed in Section 4, 6 resources are required, which coincides with that obtained by simulation. The distribution of resources in terminal is illustrated in Fig. 2.

For evaluating the performance of the two MIP formulations, 5 classes of instances with different sizes are generated. Each class includes 5 instances. The average value of each class is reported in Table II. The size of instances is characterized by M, N, I and J.

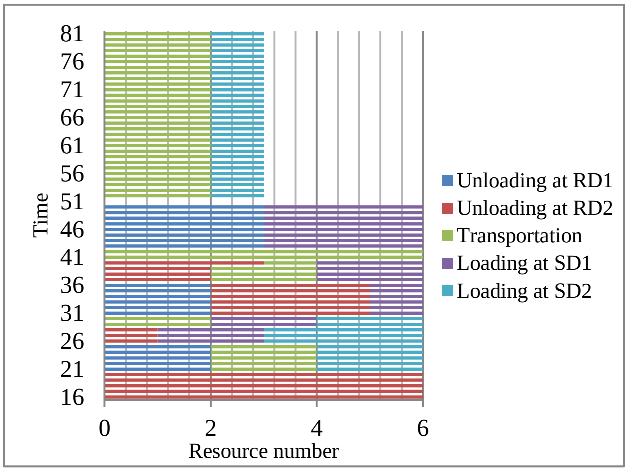

Figure 2. Resource distribution under shared rules

TABLE II. Performance of MIP Formulations

Size (M-N-I-J)	Resource number under non-shared rules	Resource number under shared rules
4-4-3-3	11	5
6-6-3-3	14	6
7-7-3-3	16	6
9-9-3-3	18	8
12-12-3-3	24	11

We can see that with the new formulation, the total resource number is much lower than that obtained by the previous one. The MIP formulation under shared resource rules outperforms the original MIP formulation under non-shared resource rules. But it is not to say that the MIP model with non-shared rules has no meaning. In opposite, with it, the resource number obtained at each work station represents the processing capacity of each work station. For example, if the resources are replaced by conveyors, the velocity of the conveyors can be determined by the capacity obtained.

VI. CONCLUSIONS

In this paper, we address a resource management problem under the time window constraints in cross docks. The resources specify the operators working in cross dock terminal. We consider two different resource assignment rules: non-shared resource rules and shared resource rules. Two MIP formulations with different rules are built, and the behavior of cross dock with shared resources is simulated with Petri net. The computational results show that the MIP formulation with non-shared resource rules can well present the processing capacity at each work station, while the one with shared resource rules can better organize and distribute resources.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Lijuan ZHANG conducted the research and wrote the paper; Benoît TROUILLET and Frédéric SEMET supervised and revised the paper.

ACKNOWLEDGMENT

This work was supported by Research Start-up Funds of DGUT (GC300502-60).

REFERENCES

[1] N. Boysen and M. Fliedner, "Cross dock scheduling: Classification, literature review and research agenda," *Omega*, vol. 38, no. 6, pp. 413–422, 2010.

[2] G. T. Ross and R. M. Soland, "A branche and bound algorithm for the generalized assignment problem," *Mathematical Programming*, vol. 8, 1975.

[3] L. Y. Tsui, C. Chang, "A microcomputer based decision support tool for assignment dock doors in freight yards," *Computers & Industrial Engineering*, vol. 19, pp. 309–312, 1990.

[4] Y. Zhu, P. M. Hahn, M. Guignard-spielberg, "New Approach for the Cross-Dock Door Assignment Problem," in *Proc. the XLI Brazilian Symposium on Operations Research*, pp. 1226–1236, 2009.

[5] A. Lim, H. Ma, Z. Miao, "Truck Dock Assignment Problem with Time Windows and Capacity Constraint in Transshipment Network Through Crossdocks," *Computational Science and Its Applications-ICCSA 2006*, pp. 688–697, 2006.

[6] A. Lim, H. Ma, Z. Miao, "truck dock assignment problem with operational time constraint within crossdocks," in *Proc. 19th International Conference on Industrial, Engineering and Other Applications of Applied Intelligent Systems*, 262–271, 2006.

[7] Z. Miao, A. Lim, H. Ma, "Truck dock assignment problem with operational time constraint within crossdocks," *European Journal of Operational Research*, vol. 192, no. 1, 2009.

[8] Z. Miao, S. Cai, D. Xu, "Applying an adaptive tabu search algorithm to optimize truck-dock assignment in the crossdock management system," *Expert Systems with Applications*, vol. 41, no. 1, pp. 16–22, 2014.

[9] B. Trouillet, "Petri Net Model and Mathematical Resolution for Cross Docking," *Conference on Intelligent Manufacturing Systems,* vol. 10, no. 1, 2009.

[10] L. Zhang, B. Trouillet, F. Semet, "A study of a cross-dock with predefined door assignment and truck time windows using Petri net," *Information systems, logistics and supply chain, ILS2016,* 2016.

Lijuan Zhang received her bachelor degree in engineering mechanics from Beihang University, Beijing, China, in 2010, her master degree in control science from Beihang University, Beijing, China, in 2013, an engineer degree from Ecole Centrale de Pekin, Beijing, China, in 2013, and a PhD in computer engineering from Ecole Centrale de Lille, France, in 2017. Her research interests include operation research, optimization algorithm. She is currently a lecturer in DGUT-CNAM institute, Dongguan University of Technology, Guangdong, China.

Benoît Trouillet his research interests include quality of service, flexible manufacturing systems, queueing theory, scheduling, Petri nets and optimization. Now he works in Ecole Centrale de Lille.

Frédéric Semet his main research activities are in the field of optimization, mostly in the area of logistics and transportation. He has been involved in a variety of projects for distribution companies. He is now a professor in Ecole Centrale de Lille.