Synthesis of Four-Bar Linkage with Adjustable Crank Length for Multi-Path Generation

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***Abstract*—Synthesis of planar mechanism with adjustable crank length for generating multiple paths is presented. Least-square approximation problem is considered which allows carrying out approximate synthesis with unlimited number of desired coupler point positions and with unlimited number of prescribed trajectories. By reducing the task to synthesis of two-element link with variable binary link length, which is called RPR-module, the analytical solution is obtained to determine not only constant design parameters (mechanism link lengths) but the adjusting parameter values as well. Thus the number of design variables for non-linear optimization (applied to find the remaining parameters) will be decreased significantly. The applied method is exemplified by synthesis of the mechanism for variable straight line generation, where the required height of the end-effector is adjusted by adjusting the crank length. Combined with random search technique the method allows to find all local minimums of the optimized goal function and thus allows to take full advantage from the considered mechanism structure during design.**

***Index Terms*—adjustable mechanism, approximate synthesis, trajectories generation, least-square approximation**

I. Introduction

Adjustable mechanisms generating a variety of desired output motions allow to design simple and reliable robot manipulators, various reconfigurable or programmable mechanisms "with built-in intelligence" [1, 2]. The desired motion is provided in such mechanisms by the main (primary) actuator while the secondary one is responsible for varying (adjusting) the output motion, so each actuator is responsible for specified function. This peculiarity of such multi-function mechanisms can be effectively used by designing manipulators with simplified control and walking robot leg mechanisms with optimal power consumption [2, 3]. The main difficulties by synthesizing of the adjustable mechanisms are related to a limited number of output paths to be traced and the limited number of points along the desired paths, essentially increased number of design parameters [3-8]. Often more than one adjusting parameters are needed and additional mechanism is necessary to change the adjusting parameters [8-12]. Mechanism approximate synthesis methods were found to be the most effective concepts and first were applied for synthesizing of adjustable function-generators of both planar and spatial structure [13-15]. Later these methods were successfully applied for multiple paths generation [16-20]. The problem of analytical synthesis of planar linkage with the adjustable crank length is considered in this article.

The novelty and contribution of this article consists in the following. The analytical solution is obtained for least-square approximation problem that allows determining adjustable parameter values as well. Usually the adjusting parameter values are supposed to be given in generally used methods [16-19] or these parameters are determined when just two prescribed paths are given. For instance, if 10 desired trajectories have to be generated then 12 design parameters from the total 18 unknowns (link lengths) are determined analytically. Thus the dimensionality of the numerical optimization technique will be decreased down to 6 parameters ("non-linear variables") instead of 18 in the original task. Another advantage of the method is that there are no limitations on the number of prescribed positions on a given paths and no limitations on the number of desired paths as well. Finally, combined with Sobol & Statnikov' s random search technique the method allows to define all local minimums of the optimization task. Thus the method allows to take full advantage from the considered mechanism structure and to find out ("open up") by this way all functional abilities of the given mechanism scheme. The application of the method is demonstrated by synthesis of manipulator generating a variety of parallel straight line paths. Due to the decoupled motion of the end-effector, each actuator is responsible for specific function: the end-effector trace along horizontal path when only primary actuator is active, while the height of the end-effector is changed by the secondary one. This leads to simplified control and minimized power consumption by manipulating with heavy objects

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due to the "gravity independent" action of the main actuator (as the corresponding partial velocity of the end-effector is always orthogonal to the vertical axe of gravity action). The numerical results are discussed in order to bring together the best in applied designing techniques.

II. Basic Structural Schemes of Adjustable Four-Bar Linkages for Multi-Path Generation

Adjustable mechanisms for multiple path generation reproduce not only one prescribed output curve but a family of desired curves called curve series. The structural schemes of these mechanisms could be obtained from those of traditional path generator (Fig.1a) by adding the additional input joint (Fig.1b-d). Now we have one more moveable link and thus the mechanism degree of freedom is increased by one:

$$3n_1 - 2p_5 = 3 \cdot 1 - 2 \cdot 1 = 1 .$$

Depending on the location of this additional input joint we have various adjustable mechanism structures:
adjustable four-bar linkage ***(A)****BCDE* with adjustable fixed pivot *B* (Fig.1b), where *B* is main (primary) input joint and *A* is secondary one (*AB* is adjusting link);
adjustable four-bar linkage *A****(B)****CDE* with adjustable crank length *AC* (Fig.1c), where *A* is the primary input joint and *B* is secondary one (responsible to adjust crank length);
adjustable four-bar linkage *AB****(C)****DE* with adjustable coupler *BDP* (Fig.1d), where *A* is the primary input joint and *C* is secondary one (responsible to adjust coupler link lengths *BD* and *BP*).

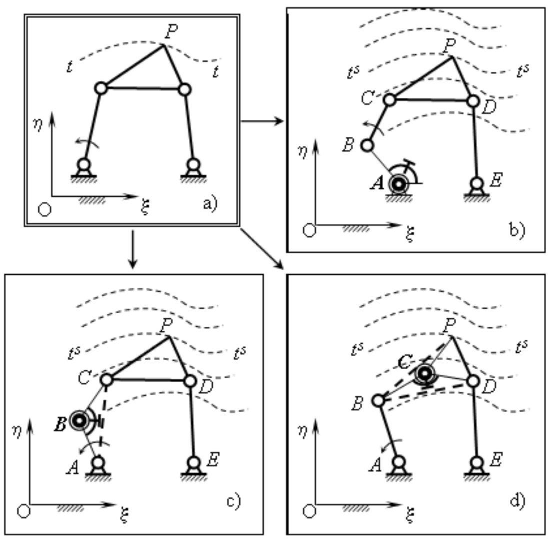

Figure 1. One-DOF four bar linkage (a) and adjustable mechanism structural schemes (b-d).

For instance, various kinematic diagrams of adjustable four-bar linkages generating horizontal straight lines (Fig.2) were obtained in [18, 19]. Fixed pivot *B* position of a four-bar linkage (Fig.1b) is adjustable in these schemes, the adjusting parameter is angular position of link *AB*. The end-effector *P* of four-bar linkage *BCDE* generates horizontal straight lines (approximately) and altitude (straight line height) change is carried out by changing the angular position of the adjusting link *AB*. Therefore link *AB* serves for changing vertical Cartesian coordinate of the end-effector while the influence of the second input link *ED* on this coordinate is negligible. These results in so-called "gravity independent" action of the input link *ED* while horizontal replacing of heavy load (mounted in point *P*), thus the power consumption of the actuator in joint *E* will be decreased.

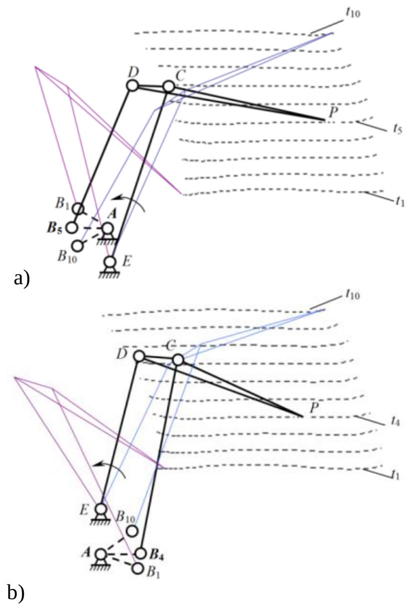

Figure 2. Adjustable four-bar linkages describing horizontal straight lines.

In accordance with the modular approach traditional one-DOF mechanism synthesis can be reduced to circle-fitting procedure (finding "circular point" on a coupler) and we deal with classical MMT problem called by "approximate synthesis of binary link (or two-element link)" or "RR-module synthesis". By the similar way the adjustable mechanism synthesis can be reduced to "synthesis of the approximating chain with the adjusting parameter" [18-20]. For instance, to synthesize the adjustable mechanism in Fig.2 the latter is considered as being composed of "variable chain" *EDP* and "approximating chain" *ABC* (so called "*RRR*-module") with the adjusting parameter – angular position of link *AB*:

- $Output('P') \xrightarrow{0,"P"} Var(3-4) \xrightarrow{0,3} RRR \bmod (1-2)$
- (See Fig.1b).

Given the output motion (a family of horizontal trajectories of the end-effector *P*) variable chain *4-3* (*EDP*) connects the end-effector *P* with frame (link *"0"*) and mechanism synthesis is reduced to approximate synthesis of the RRR-module *1-2*. By the same way the adjustable mechanism 1d synthesis is carried out by "formulae":

$$Output('P') \xrightarrow{0,"P"} Var(4-3) \xrightarrow{3,0} RRR \bmod (2-1)$$
(See Fig.1d).

One can easily observe that the difference between the latter schemes 1b and 1d consists just in the "inversion" of a *RRR*-module connection and the same structural module of type *RRR* is used to synthesis. Thus the same algorithms will be used for kinematic synthesis. Unlike these, the scheme 1c uses structural module of another type with variable length of a link *AC* (Fig.3), the latter is changed by the actuator mounted on the additional input joint *B* (*RPR*-module *1-2*)

$$Output('P')\xrightarrow{0,"P"}Var(4-3)\xrightarrow{0,3}RPR\,\mathrm{mod}(1-2)$$

(See Fig.1c).

Adjustable mechanisms for path generation of type 1c will be considered in this paper exemplified by the task of generation of the family of straight-line trajectories.

III. Analytical Synthesis of a RPR-Module

Let the end-effector P of four-bar linkage $ACDE$ have to trace along prescribed paths on a plane by adjusting link-length l_{AC} of link AC (Fig.1c). So the desired output motion of the end-effector P is given by planar trajectories t_s, $s=1,...,S$ and link-length $l_{AC}=l_s$ is constant on each *s*-th trajectory. If each of the desired trajectories are specified by N points, then the output motion is given by $N{\cdot}S$ points $P_{is}, i=1,...,N,\ s=1,...,S,$ with the given absolute coordinates $[\xi_{Pis},\eta_{Pis}]$ relative to the absolute reference frame $O\xi\eta$ (Fig.3).

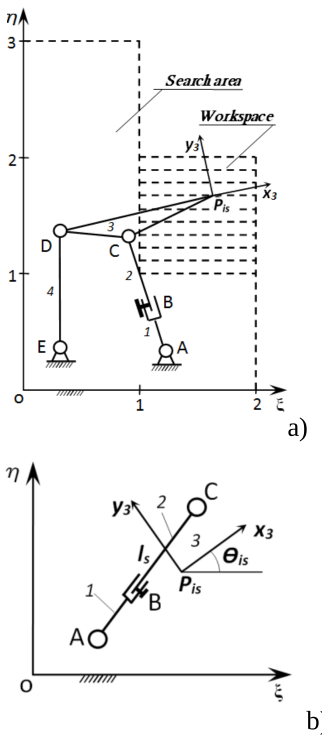

Figure 3. Adjustable four-bar linkage and concentric circles fitting.

In this section the variable chain *EDP* parameters $\mathbf{Q}=[\xi_E,\eta_E,l_{ED},l_{DP}]^T$ are supposed to be specified by designer. Let Px_3y_3 be the local coordinate system with the origin at point P and with the axis Px_3, which lies along the link DP (Fig.3a). As the dyad EDP parameters are known, the reference frame Px_3y_3 positions relative to the absolute reference frame $O\xi\eta$ can be simply found by solving inverse kinematics for the dyad EDP. So the mechanism synthesis task is reduced to synthesis of the kinematic chain *ABC*, called by RPR-module (Fig.3b).

The loci of the reference frame Px_3y_3 relative to the absolute reference frame $O\xi\eta$ is specified by S sets of N positions $\{\xi_{Pis},\eta_{Pis},\theta_{is}\}, i=1,...,N,\ s=1,...,S.$ When $S=1$ we deal with the traditional MMT problem of two-element link AC synthesis that connects planes $O\xi\eta$ and Px_3y_3: "circular point" C is sought on a plane Px_3y_3, this point follows circular curve (approximately) with respect to the reference frame $O\xi\eta$. Designing parameters are: the radius-vector $\mathbf{r}_C^{(3)}$ of point C on the plane Px_3y_3, the center $\mathbf{R}_A$ of this circle on the plane $O\xi\eta$ and the radius $R=l_{AC}$. But when more than one motions (i.e. the family or set of motion) have to be generated ($S>1$) then the series of point C trajectories have to be approximated by the concentric circle arcs with common center $\mathbf{R}_A$ and radius $l_s, s=1,...,S.$

The main constraint equation is

$$\Delta_{is}=(\mathbf{R}_{P_{is}}+\mathbf{\Gamma}(\theta)\mathbf{r}_C^{(3)}-\mathbf{R}_A)^2-(l_{AC})_S^2=0$$

This means that point C must lie on concentric circles, here $\mathbf{\Gamma}(\theta)$ is rotation matrix, $\mathbf{R}_P,\mathbf{R}_A,\mathbf{r}_C^{(3)}$ are the radius-vectors of joint centers P, A and C in the respective reference frames $O\xi\eta$ and Px_3y_3, Δ is the function of the approximation error. At the given quantities $\xi_{Pis},\eta_{Pis},\theta_{is}, i=1,...,N,\ s=1,...,S$ the problem of synthesis consists in the determining of the following design parameters:

ξ_A,η_A - the absolute coordinates of joint A;

$x_C^{(3)},y_C^{(3)}$ - local coordinates of joint C in the reference frame Px_3y_3; and the values $l_s, s=1,...,S$ of the adjusting parameter l_{AC}, which have to meet approximately constraint equations (1) for any $i=1,...,N,\ s=1,...,S.$

Following [15, 18-20], the approximation error Δ can be expressed as

$$\Delta_{is}^{(1)}=-2\mathbf{R}_{C_{is}}^T\mathbf{R}_A+(\mathbf{R}_A^2-l_S^2)+\mathbf{R}_{C_{is}}^2 \qquad (2)$$

$$\Delta_{is}^{(2)} = -2\mathbf{r}_{A_{is}}^T \mathbf{r}_C + (\mathbf{r}_C^2 - l_S^2) + \mathbf{r}_{A_{is}}^2 \tag{3}$$

where

$$\mathbf{R}_{C_{is}} = \mathbf{R}_{P_{is}} + \mathbf{\Gamma}(\theta)\mathbf{r}_C^{(3)}$$

$$\mathbf{r}_{A_{is}} = \mathbf{\Gamma}^T(\theta_{is})(\mathbf{R}_A - \mathbf{R}_{P_{is}})$$

In new variables

$$p_S^{(1)} = 0,5\cdot(\mathbf{R}_A^2 - l_S^2),\ p_S^{(2)} = 0,5\cdot(\mathbf{r}_C^2 - l_S^2),$$
$$s = 1,...,S,$$
$$p_{S+1} = \xi_A,\ p_{S+2} = \eta_A, p_{S+3} = x_C^{(3)}, p_{S+4} = y_C^{(3)},$$

the last equations can be written as

$$0,5\cdot\Delta_{is}^{(k)} = p_S^{(k)} + a_{is}^{(k)} p_{S+1} + b_{is}^{(k)} p_{S+2} + c_{is}^{(k)} = 0, \quad i = 1,...,N;\ s = 1,...S \tag{4}$$

where $k = 1$ or 2. The respective expressions for $a_{is}^{(1)},\ b_{is}^{(1)},\ c_{is}^{(1)}$ by $\mathbf{r}_C, \mathbf{R}_{Pis}, \theta_{is}$ and for $a_{is}^{(2)},\ b_{is}^{(2)},\ c_{is}^{(2)}$ by $\mathbf{R}_A, \mathbf{R}_{Pis}, \theta_{is}$ one can derive just as easily from (2) and (3) respectively.

Thus equations (1) are reduced to linear algebraic systems of $N - S$ equations on the parameters

$$\mathbf{P}^{(1)} = \left[p_1^{(1)},..., p_S^{(1)}, p_{S+1}, p_{S+2}\right]^T$$
$$\mathbf{P}^{(2)} = \left[p_1^{(2)},..., p_S^{(2)}, p_{S+3}, p_{S+4}\right]^T$$

$$\mathbf{A}^{(k)}\mathbf{P}^{(k)} = -\mathbf{b}^{(k)},\ \ k = 1,2 \tag{5}$$

Here

$$\mathbf{A}^{(k)} = \left[\left[\mathbf{D}_1^{(k)}\right]\left[\mathbf{D}_2^{(k)}\right]...\left[\mathbf{D}_S^{(k)}\right]\right]^{\mathrm{T}},$$
$$dim\mathbf{A}^{(k)} = NS\times(S+2),\ dim\mathbf{D}_s{}^{(k)} = (S+2)\times N,$$
$$s = 1,...,S$$

i-th column of the matrix $\mathbf{D}_s{}^{(k)}$ is $\mathbf{d}_{i,s}^{(k)} = \left[\mathbf{e}_s^{\mathrm{T}}, a_{is}^{(k)}, b_{is}^{(k)}\right]^{\mathrm{T}}$. Vector $\mathbf{e}_s$ is S -dimensional unit vector with the components $e_{\mu s} = \delta_\mu{}^s, \mu = 1,...,S.$

Vector $\mathbf{b}^{(k)}$ is NS -dimensional vector $\mathbf{b}^{(k)} = \left[\mathbf{b}_1{}^{(k)T}, \mathbf{b}_2{}^{(k)T},..., \mathbf{b}_S{}^{(k)T}\right]^T$, where $\mathbf{b}_s{}^{(k)} = \left[c_{1s}{}^{(k)}, c_{2s}{}^{(k)},..., c_{Ns}{}^{(k)}\right]^T; s = 1,...,S$.

If $S \ge 2$ and $N \ge 2$ (at least 2 series are given and at least 3 positions are specified for each of the series), then the number of equations NS in (5) is greater than the number of unknowns $(S+2)$ (for at least $S(N-1) > 2$). Thus the approximate solution is

$$\mathbf{P}^{(k)} \ = \ \mathbf{A}_p^{(k)-1}\ \mathbf{b}^{(k)} \tag{6}$$

where $\mathbf{A}_p^{-1} = \left[\mathbf{A}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}\mathbf{W}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}\mathbf{W}$ is "the left pseudo-inverse of $\mathbf{A}$ " [22], $\mathbf{W}$ is the diagonal matrix $diag\{w_1, w_2,..., w_{NS}\}$ of weighting factors; $dim\mathbf{W} = NS\times NS$. The solution $\mathbf{P}_0$ is found as the minimum of the Euclidean norm of the error $\mathbf{D} = \mathbf{AP} + \mathbf{b}$ in the approximation [21]

$$f \equiv \|\mathbf{\Delta}\|_2^2 = \mathbf{\Delta}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}\mathbf{W}\mathbf{\Delta} \tag{7}$$

The method of "kinematic inversion" proposed in [15] can be applied for minimization of (7) on all $(S+4)$ parameters $\xi_A, \eta_A, x_C^{(3)}, y_c^{(3)}, l_1,\ ..., l_S$. This method is based on the repetitive solving of the linear equation system (5) on the parameters $\mathbf{P}^{(k)}, k = 1,2$. In fact, this approach represents sectional descending minimization method and found to be of slow convergence for our problem. Instead, we propose to consider the modified objective function

$$f_1(\mathbf{X}_1) \equiv \min_{\mathbf{P}^{(1)}} f(\mathbf{X}_1, \mathbf{P}^{(1)}) = f^0(\mathbf{X}_1, \mathbf{P}^{(1)}(\mathbf{X}_1)) \ \Rightarrow \ \min_{\mathbf{X}_1} \tag{8}$$

where

$$\mathbf{X}_1 = \mathbf{P} / \mathbf{P}^{(1)},\ \mathbf{P}^{(1)}(\mathbf{X}_1) = -\mathbf{A}_p^{(1)-1}(\mathbf{X}_1)\cdot\mathbf{b}^{(1)}(\mathbf{X}_1).$$

By this way the dimensionality of the source minimization problem is reduced from (4+S) parameters down to just 2 parameters $\mathbf{X}_1 = \left[x_C^{(3)}, y_c^{(3)}\right]^T$.

IV. Synthesis of the Adjustable 4-Bar Linkage for Horizontal Straight Lines Generation

In the previous section the variable chain EDP parameters $\mathbf{Q} = [\xi_E, \eta_E, l_{ED}, l_{DP}]^T$ are supposed to be specified by designer and the mechanism synthesis task is reduced to synthesis of a RPR -module ABC . In order to optimize the parameters Q as well, we have to take into consideration the dependence $\theta_{is} = \theta_{is}(\mathbf{Q})$ in the equation (1), so the goal function will take the form

$$f_2(\mathbf{Q}, \mathbf{X}_1) \equiv \min_{\mathbf{P}^{(1)}} f(\mathbf{Q}, \mathbf{X}_1, \mathbf{P}^{(1)}) = f^0(\mathbf{Q}, \mathbf{X}_1, \mathbf{P}^{(1)}(\mathbf{X}_1, \mathbf{Q})) \Rightarrow \min_{\left[\mathbf{Q}^{\mathrm{T}}, \mathbf{X}_1^{\mathrm{T}}\right]^{\mathrm{T}}} \tag{9}$$

As $(S+2)$ parameters $\mathbf{P}^{(1)}$ are determined analytically, the numerical optimization is applied to determine only 6 parameters $\left[\mathbf{Q}^T,\ \mathbf{X}_1^{\ T}\right]^T$. But now we deal with constraint optimization problem in order to exclude break of a kinematic chain EDP while point P follows the given positions P_{is}. Thus the parameters $\mathbf{Q}$ have to comply with the restrictions

$$g_1 \geq 0, g_2 \geq 0 \tag{10}$$

where $g_1 = \min\left((\rho_{\min} - |q_3 - q_4|)/(\rho_{\max} - \rho_{\min}),\right.$ $\left.(q_3 + q_4 - \rho_{\max})/(\rho_{\max} - \rho_{\min})\right)$, $\rho_{\min} = \min\limits_{i,s} \rho_{is}$, $\rho_{\max} = \max\limits_{i,s} \rho_{is}$, ρ_{is} is the distance between given point P_{is} and the center of the frame E

$$\rho_{is} = \left\|\mathbf{R}_{P_{is}} - \mathbf{R}_E\right\|_2 = \tag{11}$$
$$= \left\{(\xi_{P_{is}} - \xi_E)^2 + (\eta_{P_{is}} - \eta_E)^2\right\}^{1/2}$$
$$g_2 = \ min\left(G_L, G_R\right),$$
$$G_L = \min_{r=1,6}((p_r - g_{Lr})/(g_{Rr} - g_{Lr})),$$
$$G_R = \min_{r=1,6}((g_{Rr} - p_r)/(g_{Rr} - g_{Lr})).$$

The second restriction $g_2 \geq 0$ provides the sought parameters to lie within the search area specified by the left and right boundaries g_{Lr}, g_{Rr} for r-th parameter variation: $g_{Lr} \leq p_r \leq g_{Rr}$, $r = 1,...,6$.

The "initial guess" $\left[\mathbf{Q}^T,\ \mathbf{X}_1^{\ T}\right]_0^T$ for optimization of the objective function (9) is varied using the generator of '$LP\tau$ -sequences' [23] (referred to below as LP -points) $\mathbf{T}^{(\alpha)} = \left[t_1^{(\alpha)},...,t_n^{(\alpha)}\right]^T$, $\alpha = 1,2,...,$ evenly distributed in n -dimensional cube $[0,1]^n$ (in our case $n = 6$). The generated points $\mathbf{T}^{(a)}$ are projected into the search area given by the left and right boundaries g_{Lr}, g_{Rr}, $r = 1,...,6$, as

$$p_{0r}^{\prime(\alpha)} = g_{Lr} + (g_{Rr} - g_{Lr}) \cdot t_r^{(\alpha)}, r = 1,...,6 \tag{12}$$

V. Numerical Example

Let the end-effector P have to trace along horizontal paths given by positions P_{is} relative to the reference frame $O\xi\eta$ (Fig.3a). The variation range for the parameters ξ_E, η_E variation are specified as two rectangles [0.0;1.0]x[0.0;3,0] and [1.0;2.0]x[0.0;1.0] at the workspace [1.0;2.0]x[1.0;2.0] (Fig.3a). The rest parameters are sought within the ranges:

$$0.5 \leq l_{ED_0} \leq 1.3,$$
$$0.5 \leq l_{DP_0} \leq 1.3,$$
$$-1.3 \leq x_{C_0}^{(3)} \leq 1.3,$$
$$-1.0 \leq y_{C_0}^{(3)} \leq 1.0\,.$$

Five local minimums were found for the optimization task and three clusters of solutions [24] with the following boundaries were separated:

- three local minimums within domain 1:

  $0.00 \leq \xi_E \leq 1.25$, $0.00 \leq \eta_E \leq 1.80$ (Fig.5a-c);

- one local minimum within domain 2:

  $0.00 \leq \xi_E \leq 1.00$, $1.80 \leq \eta_E \leq 3.00$ (Fig.5d);

- and one local minimum within domain 3:

  $1.25 \leq \xi_E \leq 2.00$, $0.00 \leq \eta_E \leq 0.70$ (Fig.5e).

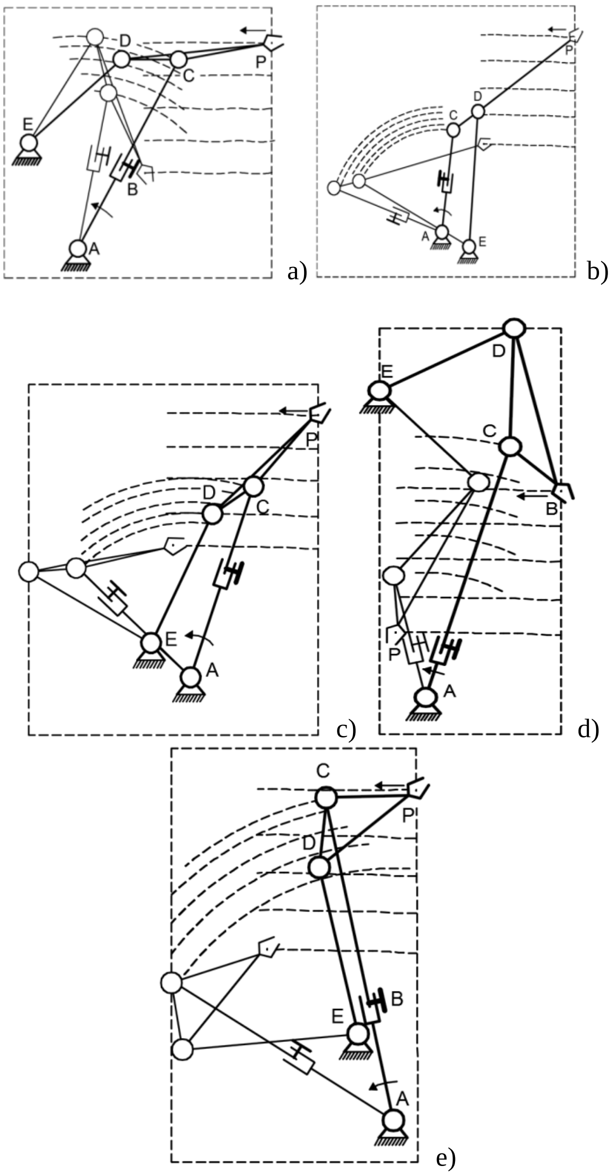

Figure 4. Adjustable mechanisms for multi straight-line generation.

The dimensions for the plotted mechanisms are brought together in Table I.

TABLE I. TYPE SIZES FOR CAMERA-READY PAPERS

Fig	X_E	Y_E	*ED*	*DP*	$x_C^{(3)}$	$y_C^{(3)}$	X_A	Y_A	l_{AC} range	Accuracy
5a	0,084	1,239	1,010	1,177	-0,750	-0,041	0,451	0,421	1,297÷ 1,699	2%
5b	0,854	0,044	1,249	1,271	-1,555	-0,014	0,617	0,151	1,196÷ 0,970	2,6%
5c	0,856	0,316	1,024	1,047	-0,710	-0,048	1,143	0,023	1,167÷ 1,497	2%
5d	0,907	2,743	0,924	1,196	-0,362	-0,218	1,162	0,566	0,851÷ 1,838	2%
5e	1,599	0,486	1,084	0,778	-0,479	0,323	1,831	-0,064	1,638÷ 2,090	1,4%

A large variety of solutions was found within the domain 1 (52%). Two kinds of kinematic diagrams were found within domains 1 and 3. While increasing the working height of wrist point P kinematic chain ABC usually behaves as abductor, “link” AC is lengthened (Fig.4a, 4c, 4e). But in some other solutions the same kinematic chain acts as adductor, “link” AC is shortening in order to follow the upper trajectories (Fig.4b, 4d). For instance, mechanism in Fig.4c seems to be similar to Fig.4b, it may seem that the difference consists only in the relative location of frames A and E. But the mechanisms act in significantly different way.

VI. CONCLUSION

The adjustable four-bar mechanism approximate synthesis problem with changeable crank length is considered when the coupler point (end-effector) traces along a set of desired paths on a plane. The method based on least square approximation and allows designing the mechanism with no limitations on the number of desired paths and no limitations on the number of prescribed positions on each path as well. Due to the proposed analytical solution for $(S+2)$ variables, the number of unknowns is reduced essentially (from $(S+8)$ variables down to 6, where $S \geq 10$ is the number of given paths). The modified goal function is introduced to find non-linear parameters and gradient based optimization techniques of higher order are applied instead of traditionally used kinematic inversion method. The method is exemplified by synthesizing the mechanism for variable straight line generation with adjustable height of the horizontal path. Combined with random search technique the method allows to find all local minimums of the optimized goal function and thus allows to take full advantages from the considered mechanism structure.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Sayat Ibrayev is a primary author who has taken the most of the responsibility of performing and conceiving the design analysis of the manuscript; Assylbek Jomartov is the corresponding author of this research work. He is in charge of conceptual generation, final editing of the manuscript; Amandyk Tuleshov participated in the synthesis of planar mechanism with adjustable crank length for generating multiple paths; Nutpulla Jamalov analyzed the data obtained from the tests carried out on the numerical experiment; Aidos Ibrayev, Gaukhar Mukhambetkalieva, Gulnur Aidasheva, Aziz Kamal are PhD students. They are participated in a numerical experiment. All the authors had approved the final version

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