Abstract—In the present work, computations of performance of symmetric hybrid worn slot-entry journal bearing under turbulent lubrication have been investigated. The model of abrasive wear given by Dufrane has been used to determine the wear on the surface of bearing because of starting and stopping operations. For wear and turbulent lubrication, the Reynolds equation has been modified using Constantinescu turbulent model. This modified equation has been solved by applying the finite element method. The effects of wear and turbulent lubrication on the characteristics of bearing have been evaluated for different Reynolds number and wear depth parameter. Minimum thickness of the fluid film and damping coefficients are more for the unworn bearing having slot width ratio (SWR) of 0.25 than the bearing with SWR of 0.5 and 0.75 operates at a constant value of Reynolds number but these values reduce for bearings as wear depth parameter increases. Further, it is also observed that as wear depth parameter value increases, stability threshold speed gets reduced for bearing operates at slot width ratio 0.25, 0.5 and 0.75 for constant Reynolds number.

Index Terms—Hydrostatic journal bearing, turbulence, wear, finite element method

I. INTRODUCTION

For a heavy varied dynamic loading in the direction of rotation, the hybrid journal bearings are used in many industrial applications. In recent years, Shires and Dee [1] applied the bearing with a slot in the purely hydrostatic application by originally developing the slot entry bearings. Rowe et al. [2] computed and made the comparison of characteristics of slot-entry bearings with other bearings by taking different power ratios. Later on, Rowe et al [3] reported the slot-entry bearings with better performance than other bearings. Sharma et al. [4] analyzed slot entry bearing in hydrostatic and hybrid mode of operation by having bearings shell flexibility. The effect of wear due to start/stop operation on journal bearings has been examined by many researchers. The analysis for wear on bearing surface was done by Dufrane et al. [5]. Later on, Hashimoto et al. [6] reported the effect of wear on various performance parameter of the hydrodynamic bearing. Kumar and Mishra [7] determined the wear influence on bearing by using Constantinescu’s theory of turbulent flow.


From literature, it is found that the no research work for symmetric slot-entry journal worn bearing operating under turbulent regime have been carried out. Therefore, for a realistic prediction of bearing performance, the combined effect of wear is essential to be considered in the analysis. This present work is focused on the combined effect of turbulence and wear on slot entry journal bearings.

II. ANALYSIS
The Reynolds equation used for computation of pressure in both laminar and turbulent flow regimes is given as:

\[
\frac{\partial}{\partial x} \left( \frac{n^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{n^3}{\mu} \frac{\partial p}{\partial y} \right) = \frac{n}{2} \frac{\partial^2 h}{\partial x^2} + \frac{\partial h}{\partial t}
\]  \tag{1}

\(G_\alpha\) and \(G_\beta\) are coefficients for turbulent flow.

These coefficients depend on local Reynolds number \(R_e\) and calculated as

\[
G_\alpha = 12 + 0.026(R_e)^{0.8265}
\]  \tag{2}

\[
G_\beta = 12 + 0.0198(R_e)^{0.741}
\]  \tag{3}

The values of these coefficients are equal for laminar flow i.e. \(G_\alpha = G_\beta = 12\) and \(R_e = 0\).

**A. Nominal Fluid-Film Thickness**

It depends on the geometry of bearing with a slot for the journal center position \(X_j\) and \(Z_j\) in steady state (Fig. 1) is given as

\[
\tilde{h}_0 = 1 - X_j \cos \alpha - Z_j \sin \alpha
\]  \tag{4}

**B. Computation of Thickness due to Worn Bearing Geometry**

For worn slot bearing geometry (Fig. 2), the fluid film thickness is given by Dufrane et al. [5] as

\[
\tilde{h} = \tilde{h}_0 + \frac{\partial \tilde{h}}{\partial \alpha} = 1 - X_j \cos \alpha - Z_j \sin \alpha + \frac{\partial \tilde{h}}{\partial \alpha}
\]  \tag{5}

\(\partial \tilde{h}/\partial \alpha\) is included to the nominal film thickness of fluid due to wear as

\[
\frac{\partial \tilde{h}}{\partial \alpha} = \delta_w - 1 - \sin \alpha ; \text{ for } \alpha_b \leq \alpha \leq \alpha_e
\]  \tag{6}

\[
\frac{\partial \tilde{h}}{\partial \alpha} = 0; \text{ for } \alpha < \alpha_b \text{ or } \alpha > \alpha_e
\]  \tag{7}

The value of angles \(\alpha_b\) and \(\alpha_e\) are at starting and finishing of the footprint respectively have been computed by putting Eq. (6) into Eq. (7) as

\[
\sin \alpha = \delta_w - 1
\]  \tag{8}

**C. Finite Element Formulation**

Using the finite element formulation and Galerkin’s technique considering 4-noded quadrilateral iso-parametric elements, the matrix form of Eqn. (1) is given as

\[
\begin{bmatrix} F^e \end{bmatrix} [\bar{p}]^e = \{Q\}^e + \Omega \{\bar{R}_H\}^e + \tilde{X}_j [\bar{R}_{xj}]^e + \tilde{Z}_j [\bar{R}_{zj}]^e
\]  \tag{9}

\[
\begin{align}
\bar{R}_{ij} &= \int_{A^e} \bar{R}_{ij} \frac{1}{\mu} \left[ \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{1}{\alpha^2} \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right] d\alpha d\beta \\
\bar{R}_{xij} &= \int_{A^e} \bar{R}_{xij} N_i \cos \alpha \ d\alpha d\beta \\
\bar{R}_{zij} &= \int_{A^e} \bar{R}_{zij} N_i \sin \alpha \ d\alpha d\beta \\
\tilde{Q}_j &= \int_{A^e} \tilde{Q}_j \frac{1}{\mu} \left[ \frac{\partial \bar{p}}{\partial x} + \frac{1}{\alpha^2} \frac{\partial \bar{p}}{\partial y} \right] N_i d\tau - \frac{1}{2} \int \tilde{h} \frac{N_i d\tau}{\mu}
\end{align}
\]  \tag{9a-9e}

**D. Equation of Flow-Through Restrictor**

The value of the flow \(Q_{in}\) is calculated for slot bearing given as

\[
Q_{in} = \frac{1}{12\rho} \left( P_S - P_h \right) \frac{a_s R^3}{V_0}
\]  \tag{10}

In non-dimensional form, the flow by slot restrictor is given as

\[
\bar{Q}_R = \bar{C}_{SR} (1 - \bar{p}_c)
\]  \tag{11}

where \(\bar{C}_{SR}\) is slot-restrictor design parameter and this parameter is given as

\[
\bar{C}_{SR} = \frac{\pi \text{ SWR} k}{36} \frac{(a_s c)}{\bar{V}_S} \left( \frac{Z}{c} \right)^3
\]

**E. Stiffness and Damping Coefficients**

The coefficients of stiffness and damping of fluid film are determined as
The coefficient of fluid film stiffness and damping are given in matrix form as [12]:

\[
\begin{bmatrix}
\tilde{S}_{11} & \tilde{S}_{12} \\
\tilde{S}_{21} & \tilde{S}_{22}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial F_x}{\partial \tilde{x}_1} & \frac{\partial F_x}{\partial \tilde{z}_1} \\
\frac{\partial F_z}{\partial \tilde{x}_1} & \frac{\partial F_z}{\partial \tilde{z}_1}
\end{bmatrix}
\]

(12)

\[
\begin{bmatrix}
\tilde{C}_{11} & \tilde{C}_{12} \\
\tilde{C}_{21} & \tilde{C}_{22}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial F_x}{\partial \tilde{x}_j} & \frac{\partial F_x}{\partial \tilde{z}_j} \\
\frac{\partial F_z}{\partial \tilde{x}_j} & \frac{\partial F_z}{\partial \tilde{z}_j}
\end{bmatrix}
\]

(13)

Threshold speed of journal is given as:

\[
\tilde{\omega}_\text{th} = \sqrt{\frac{\tilde{C}_1}{\tilde{S}_{11}}}^{1/2}
\]

(14)

III. COMPUTATIONAL METHOD

For computing the performance characteristics, trial values of \( \tilde{x}_j, \tilde{z}_j \) are taken as an input. After applying boundary conditions to the Eqn. (9), it is solved together with slot restrictor flow Eqn. (11) for computing nodal pressures. After computing the nodal pressures, characteristics of slot-entry bearing in a hybrid mode of operation are determined using the relevant expressions given in ref [11].

IV. VALIDATION

FEM is applied for determining the solution from governing equations of slot unworn/worn bearing operating under the turbulent regime. A program is developed in FORTRAN based on the analysis. The validity of obtained results from the generated program is shown in Fig.3 for Somerfield number \( \tilde{S}_0 \) which is compared by already published results of Ref. [7] for unworn/worn hydrodynamic journal bearing operating in turbulent regime. Performance characteristics of a slot-entry worn bearing under turbulent lubrication are simulated numerically for selecting bearing parameters on the basis of literature.

V. COMPUTED RESULTS AND DISCUSSION

Fig.4 presents variation for minimum thickness of fluid film \( \tilde{h}_\text{min} \) with Reynolds number \( \tilde{R}_e \). The unworn bearing with slot width ratio (SWR) = 0.25 has a larger value of the thickness of fluid film operating at a constant value of \( \tilde{R}_e \) than the worn/unworn bearings with SWR=0.5 and SWR=0.75. The film thickness reduces for worn bearings with SWR= 0.25, 0.5 and 0.75 operates at constant Reynolds number as compared with unworn bearing. Further, the value of \( \tilde{h}_\text{min} \) increases for these bearings when operates under the turbulent regime. For wear depth parameter \( \delta W = 0.25 \), a decrease of 8.46% in the film thickness \( \tilde{h}_\text{min} \) is found for the Reynolds number \( \tilde{R}_e = 10000 \) for SWR=0.75 bearing than SWR=0.25 bearing. Further, it may be noted that the due to the influence of the turbulence, the value of \( \tilde{h}_\text{min} \) increases for SWR=0.25, 0.5 and 0.75 bearings operates in laminar as well as turbulent regime.

Fig. 5 and Fig. 6 show the variation of horizontal and vertical stiffness of fluid film ($\tilde{S}_{11}$, $\tilde{S}_{22}$). From Fig. 5 and 6, it is noted that the value of $\tilde{S}_{11}$ and $\tilde{S}_{22}$ increases for certain value of Reynolds number $R_e = 2500$ and then it decreases with increasing value of Reynolds number for worn and unworn bearings. As slot width ratio of bearing increases, the stiffness $\tilde{S}_{11}$ decreases at a constant value of Reynolds number and the stiffness $\tilde{S}_{22}$ is more for unworn bearing with SWR=0.25 than SWR=0.5 and 0.75 bearings. Further, it is interesting to notice that the stiffness $\tilde{S}_{22}$ value is more at wear depth parameter $\delta w = 0.25$ for the bearing with SWR=0.25 for constant Reynolds number than unworn bearing under the laminar regime. The least value for vertical stiffness $\tilde{S}_{22}$ is observed for unworn and worn bearing with SWR=0.75 operates for constant Reynolds number.

![Figure 6. Variation of $\tilde{S}_{22}$ with $R_e$](image)

![Figure 7. Variation of $\tilde{C}_{11}$ with $R_e$](image)

![Figure 8. Variation of $\tilde{C}_{22}$ with $R_e$](image)

Fig. 7 and Fig. 8 presents the values for coefficients (horizontal and vertical) of fluid film ($\tilde{C}_{11}$, $\tilde{C}_{22}$) with Reynolds number ($R_e$). It is found that the values of $\tilde{C}_{11}$ and $\tilde{C}_{22}$ is larger for unworn/worn bearings with SWR=0.25 as compared to unworn/worn bearings with SWR=0.5 and 0.75 bearings under the turbulent regime. The damping coefficients of the fluid film increases with increasing value of Reynolds number for worn/unworn bearings with SWR=0.25, SWR=0.5 and 0.75. Further, as the wear depth parameter increases, the damping coefficients $\tilde{C}_{11}$ and $\tilde{C}_{22}$ reduces when operates at constant value of $R_e$ for bearings with SWR=0.25, SWR=0.5 and 0.75.

Variation for threshold speed ($\tilde{\omega}_{th}$) is shown in Fig. 9. The value of $\tilde{\omega}_{th}$ is lower for unworn/unworn bearing with slot width ratio 0.75 than the bearing with slot width ratio 0.25 under the turbulent regime. The threshold speed is 3.05% at $\delta w = 0.5$ for bearing with SWR=0.75 for Reynolds number $R_e = 5000$ than unworn bearing with SWR=0.25.

![Figure 9. Variation of $\tilde{\omega}_{th}$ with $R_e$](image)
VI. CONCLUSIONS

The following conclusions are drawn from the above results:

- It has been observed that for increasing value of wear depth parameter, the value of $h_{\text{min}}$ gets reduced for bearings at a constant value of Reynolds number. However, for worn/unworn bearings, the value of $h_{\text{min}}$ is more for slot width ratio = 0.25 as compared to slot width ratio = 0.5 and 0.75 under turbulent regime. Further, the bearing with slot width ratio = 0.25 has the higher value of $h_{\text{min}}$ as compared to bearings with slot width ratio = 0.5 and 0.75.

- The coefficients of stiffness ($C_{11}, C_{22}$) values increase under laminar regime while it decreases in the turbulent regime for unworn/worn bearings. It has been observed that the combined effect of turbulence and wear reduces the values of damping coefficients for bearings with a slot width ratio = 0.5 and 0.75. The coefficient of stiffness $C_{22}$ has higher value for the bearing with slot width ratio = 0.5 for wear depth parameter $\delta w = 0.25$ operates under turbulent regime. The worn/unworn bearings with slot width ratio = 0.25 has the higher values of $C_{11}, C_{22}$ as compared to bearings with slot width ratio = 0.5 and 0.75 under laminar/turbulent regime.

- It has been observed that as wear depth increases, stability threshold speed($\tilde{\omega}_{th}$) reduces for bearings under the turbulent regime. It has been observed that for a turbulent regime, the value of $\tilde{\omega}_{th}$ is lower for bearings than the bearings under the laminar regime. Further, the bearing with slot width ratio = 0.25 has the higher $\tilde{\omega}_{th}$ value up to certain value of Reynolds number as compared to slot width ratio = 0.5 and 0.75 bearings.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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